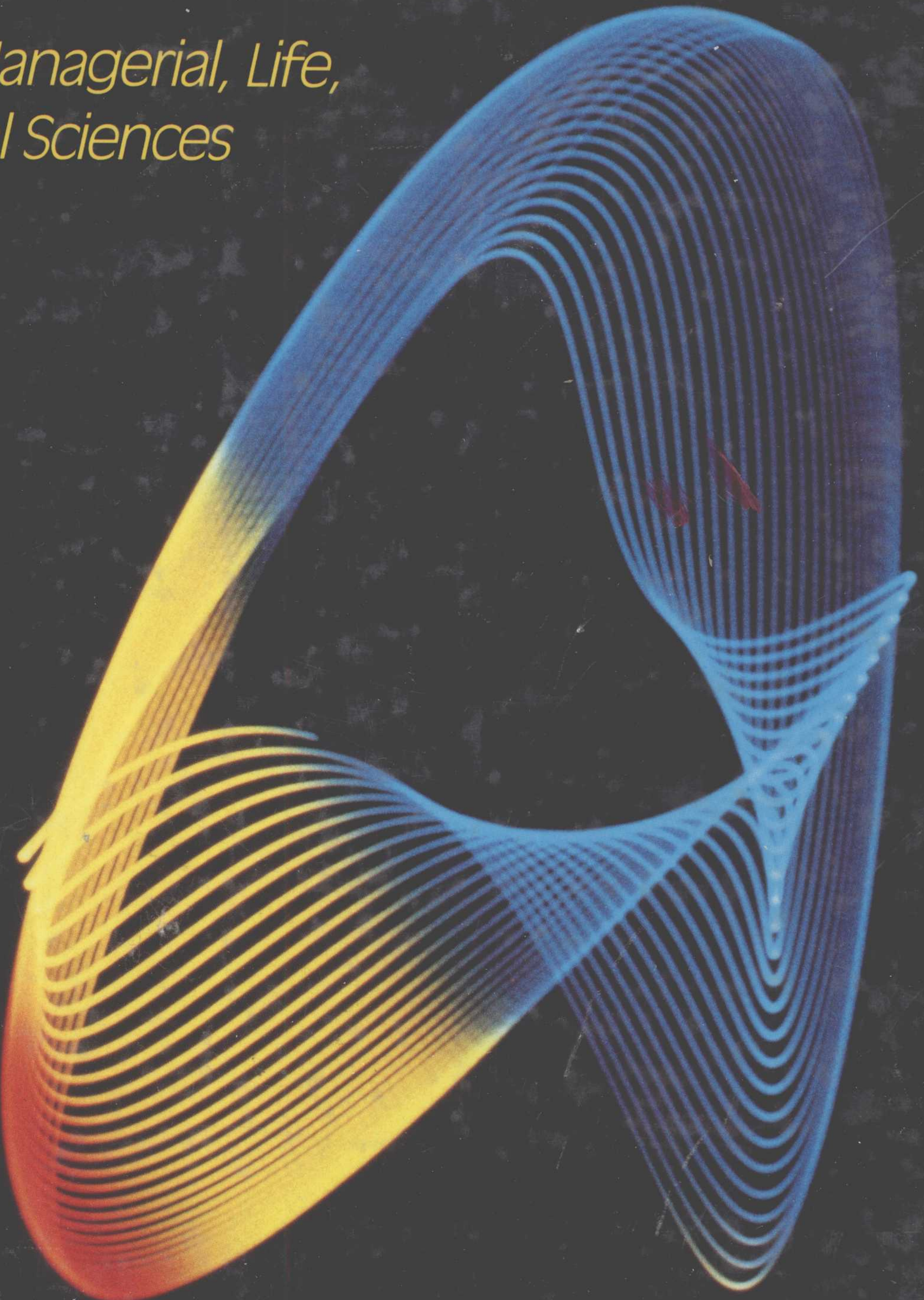
Calculus

for the Managerial, Life, and Social Sciences

Tan



S. T. TAN
Stonehill College

Calculus for the Managerial, Life, and Social Sciences

Prindle, Weber & Schmidt, BOSTON

PWS PUBLISHERS

Prindle, Weber & Schmidt • Duxbury Press • PWS Engineering • Breton Publishers
20 Park Plaza • Boston, Massachusetts 02116

PWS Publishers is a division of Wadsworth, Inc.

Library of Congress Cataloging-in-Publication Data

Tan, Soo Tang.
Calculus for the managerial, life, and social sciences.

Brief ed. of: Applied calculus. 1986.
Includes index.
1. Calculus. I. Tan, Soo Tang. Applied calculus.
II. Title
QA303.T142 1986 515 86-91467
ISBN 0-87150-003-5

Printed in the United States of America.
87 88 89 90 91 — 10 9 8 7 6 5 4 3 2

Sponsoring Editor: *Chuck Glaser*
Production Coordinator: *Elise Kaiser*
Interior Illustration: *Scientific Illustrators*
Interior Design: *Helane Manditch-Prottas; Jean Coulombre*
Cover Design: *Elise Kaiser*
Composition: *Polyglot Pte. Ltd.*
Text Printer/Binder: *Halliday Lithograph*
Cover Printer: *John P. Pow Company, Inc.*

Cover photo courtesy of *Harold M. Lambert Studios, Inc./ E.P. Jones, Boston*

Preface

Calculus for the Managerial, Life, and Social Sciences is a brief edition of the author's *Applied Calculus*. This book is suitable for use in a one-semester or two-quarter introductory calculus course for students in the managerial, life, and social sciences. It has been written with two main goals in mind: to write a textbook that is readable by the student, and to make the book a useful teaching tool for the instructor. It is hoped that these goals have been met by the many features found in the book, some of which are listed below.

Prerequisites The only prerequisite for understanding this book is a year of high school algebra. An algebra review section is included in the appendix of the book for any student who wishes to review this material.

Level of Presentation Our approach is intuitive. However, special care has been taken to ensure that this does not compromise the mathematical content or accuracy of the material presented. Proofs of certain results are given, but may be omitted if desired. The problem-solving approach is stressed throughout the book. Numerous examples and solved problems are used to motivate each new concept or result in order to facilitate the student's comprehension of the new material. Figures are used extensively to help the student visualize the concepts and ideas being presented.

Applications The text is application-oriented. Many interesting, relevant, and up-to-date applications are drawn from the fields of business, economics, social and behavioral sciences, life sciences, physical sciences, and other fields of general interest. These applications are to be found in the illustrative examples in the main body of the text as well as in the exercise sets. In fact, one goal of the text is to include (whenever feasible) at least one real-life application in each section.

Exercises Each section of the text is accompanied by an extensive set of exercises that contains an ample set of problems of a routine, computational nature, which will help the student master new techniques. The routine problems are followed by an extensive set of application-oriented problems that test the student's mastery of the topics. Each chapter of the text also contains a set of review exercises. Answers to all odd-numbered exercises appear in the back of the book.

Coverage of Topics The book contains more than enough material for the usual applied calculus course. Thus, the instructor may be flexible in choosing the topics most suitable for his or her course.

Acknowledgments I wish to express my personal appreciation to each of the following reviewers, whose many suggestions helped make a much improved end product: Professor Daniel Anderson, University of Iowa; Professor Charles Clever, South Dakota State University; Professor William Coppage, Wright State University; Professor Lyle Dixon, Kansas State University; Professor Bruce Edwards, University of Florida at Gainesville; Professor Charles S. Frady, Georgia State University; Professor Howard Frisinger, Colorado State University; Professor Larry Gerstein, University of California at Santa Barbara; Professor Lowell Leake, University of Cincinnati; Professor Maurice Monahan, South Dakota State University; Professor Lloyd Olson, North Dakota State University; Professor Richard Porter, Northeastern University; and Professor Thomas N. Roe, South Dakota State University.

I also wish to thank Reverend Robert J. Kruse, Academic Dean of Stonehill College, for his enthusiastic support of this project. My thanks also go to the editorial and production staffs of PWS Publishers: Chuck Glaser, Elise Kaiser, and Jean Coulombre, for their assistance and cooperation in the development and production of this book. ◆

Contents

1. *Preliminaries*

1.1 REAL NUMBERS AND COORDINATE SYSTEMS

The system of real numbers plays a fundamental role throughout this book. This system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division. We shall assume that the reader is familiar with the rules governing these algebraic operations (see Appendix I).

It is convenient and fruitful to have a geometrical representation of the set of real numbers. Such a representation is called a **number line** and is constructed as follows: Arbitrarily select a point on a straight line to represent the number 0. This point is called the **origin**. If the line is horizontal then a point at a convenient distance to the right of the origin is chosen to represent the number 1. This determines the scale for the number line. Each positive real number x lies x units to the right of 0 and each negative real number $-x$, lies x units to the left of 0.

In this manner a one-to-one correspondence is set up between the set of real numbers and the set of points on the number line, with all the positive numbers lying to the right of the origin and all the negative numbers lying to the left of the origin (see Figure 1.1).

In much of our later work we shall restrict our attention to certain subsets of the set of real numbers. For example, if x denotes the number of cars rolling off an assembly line each day in an automobile assembly plant, then x must be nonnegative, that is, $x \geq 0$. Taking this example one step further, suppose management decides that the daily production of cars in the plant is not to exceed 200; then x must satisfy the inequality $0 \leq x \leq 200$. More generally, we shall be interested in the following subsets of real numbers.

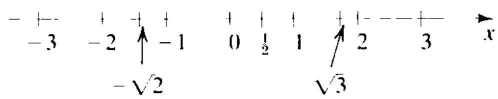


Figure 1.1

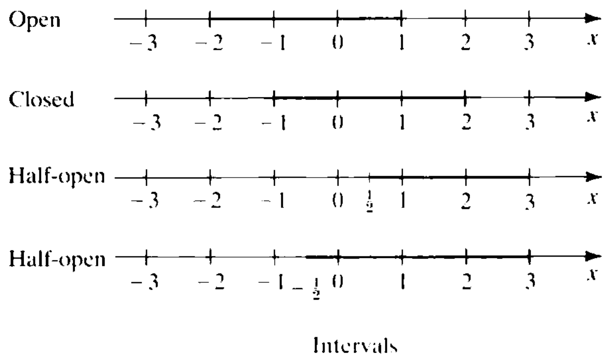


Figure 1.2

The set of all real numbers x satisfying $a < x < b$, where a and b are fixed and distinct numbers, is called an **open interval** (a, b). Thus, the interval (a, b) consists of the set of all numbers lying strictly between a and b. A set of real numbers x satisfying the inequalities $a \leq x \leq b$ is called a **closed interval** $[a, b]$. (The square brackets indicate that the endpoints a and b belong to the set.) A set of real numbers x satisfying the inequalities $a \leq x < b$ or $a < x \leq b$ is called a **half-open interval** $[a, b)$ or $(a, b]$. A single term, **interval**, encompasses open, closed, and half-open intervals. Figure 1.2 illustrates the intervals $(-2, 1)$, $[-1, 2]$, $(\frac{1}{2}, 3]$ and $[-\frac{1}{2}, 3)$.

In addition to the **finite intervals** defined above we will encounter **infinite intervals**. Examples of infinite intervals are the half-lines (a, ∞), $[a, \infty)$, $(-\infty, a)$, and $(-\infty, a]$ defined by the set of all real numbers satisfying $x > a$, $x \geq a$, $x < a$, and $x \leq a$, respectively. The symbol ∞, called infinity, is not a real number. It is used here only for notational purposes in conjunction with the definition of infinite intervals. The notation $(-\infty, \infty)$ is used for the set of real numbers x since, by definition, the inequalities $-\infty < x < \infty$ hold for any real number x. The infinite intervals $(2, \infty)$ and $(-\infty, \frac{1}{2}]$ are depicted in Figure 1.3.

In practical applications, the subsets of real numbers are often found by solving one or more inequalities involving a variable x. In such

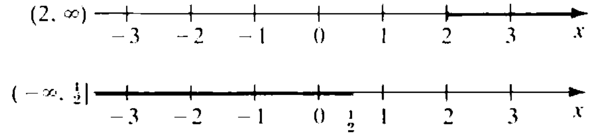


Figure 1.3

situations, the following properties of inequalities may be used to advantage (where a, b, and c are any real numbers).

		Example
Property 1	If $a < b$ and $b < c$, then $a < c$.	$2 < 3$ and $3 < 8$, so $2 < 8$
Property 2	If $a < b$, then $a + c < b + c$.	$-5 < -3$, so $-5 + 2 < -3 + 2$; that is, $-3 < -1$
Property 3	If $a < b$ and $c > 0$, then $ac < bc$.	$-5 < -3$, and since $2 > 0$, we have $(2)(-5) < (2)(-3)$; that is, $-10 < -6$
Property 4	If $a < b$ and $c < 0$, then $ac > bc$.	$-2 < 4$, and since $-3 < 0$, we have $(-2)(-3) > (4)(-3)$; that is, $6 > -12$

Similar properties hold if each inequality sign "$<$" is replaced by "$\geq$", "$>$" or "$\leq$".

EXAMPLE 1.1 Find the set of real numbers satisfying $-1 \leq 2x - 5 < 7$.

Solution Adding 5 to each member of the given double inequality, we obtain

$$4 \leq 2x < 12.$$

Next, multiplying each member of the resulting double inequality by $\frac{1}{2}$ yields

$$2 \leq x < 6$$

and the solution is the interval $[2, 6)$. ■

EXAMPLE 1.2 Solve the inequality $x^2 + 2x - 8 < 0$.

Solution Observe that $x^2 + 2x - 8 = (x + 4)(x - 2)$ so that the given inequality is equivalent to the inequality $(x + 4)(x - 2) < 0$. Since the product of two real numbers is negative if and only if the two numbers have opposite signs, we solve the inequality $(x + 4)(x - 2) < 0$ by studying the signs of the two factors $x + 4$ and $x - 2$. Now, $x + 4 > 0$ when $x > -4$, and $x + 4 < 0$ when $x < -4$. Similarly, $x - 2 > 0$ when $x > 2$, and $x - 2 < 0$ when $x < 2$. These results are summarized graphically in Figure 1.4.

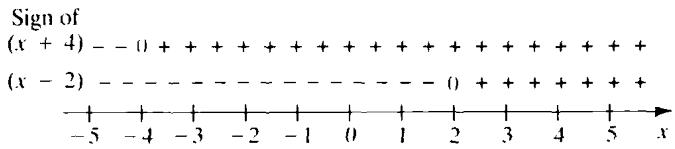


Figure 1.4

From Figure 1.4 we see that the two factors $x + 4$ and $x - 2$ have opposite signs when and only when x lies strictly between -4 and 2. Therefore, the required solution is the interval $(-4, 2)$. ■

EXAMPLE 1.3 Solve the inequality $\dfrac{x+1}{x-1} \geq 0$.

Solution The quotient $(x + 1)/(x - 1)$ is strictly positive if and only if both the numerator and the denominator have the same sign. The signs of $x + 1$ and $x - 1$ are shown in Figure 1.5.

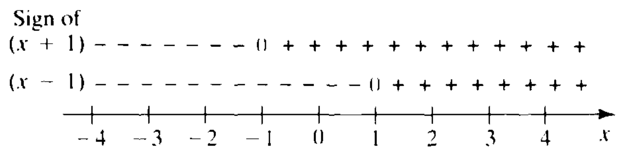


Figure 1.5

From Figure 1.5 we see that $x + 1$ and $x - 1$ have the same sign when and only when $x < -1$ or $x > 1$. The quotient $(x + 1)/(x - 1)$ is equal to zero when and only when $x = -1$. Therefore, the required solution is the set of all x in the intervals $(-\infty, -1]$ and $(1, \infty)$. ■

EXAMPLE 1.4 The management of Colbyco, a giant conglomerate, has estimated that x thousand dollars is needed to purchase

$$100{,}000(-1 + \sqrt{1 + 0.001x})$$

shares of the common stock of the Starr Communications Company. Determine the amount of money Colbyco needs in order to purchase at least 100,000 shares of the stock.

Solution The amount of cash required to purchase at least 100,000 shares is found by solving the inequality

$$100{,}000(-1 + \sqrt{1 + 0.001x}) \geq 100{,}000.$$

Proceeding, we find

$$\begin{aligned} -1 + \sqrt{1 + 0.001x} &\geq 1 \\ \sqrt{1 + 0.001x} &\geq 2 \\ 1 + 0.001x &\geq 4 \\ 0.001x &\geq 3 \\ x &\geq 3000 \end{aligned}$$

so that the amount of cash needed by Colbyco is at least \$3,000,000. ■

The **absolute value** of a number a (denoted by $|a|$) is defined as follows:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0. \end{cases}$$

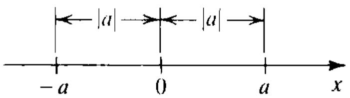


Figure 1.6

Since $-a$ is a positive number when a is negative, it follows that the absolute value of a number is always nonnegative. For example, $|5| = 5$ and $|-5| = -(-5) = 5$. Geometrically, $|a|$ is the distance between the origin and the point on the number line representing the number a (see Figure 1.6).

The following properties hold for arbitrary numbers a, b, c.

		Example
Property 5	$\lvert -a\rvert = \lvert a\rvert$	$\lvert -3\rvert = -(-3) = 3 = \lvert 3\rvert$
Property 6	$\lvert ab\rvert = \lvert a\rvert \lvert b\rvert$	$\lvert (2)(-3)\rvert = \lvert -6\rvert = 6 = \lvert 2\rvert \lvert -3\rvert$
Property 7	$\left\lvert \frac{a}{b}\right\rvert = \frac{\lvert a\rvert}{\lvert b\rvert}, (b \neq 0)$	$\left\lvert \frac{(-3)}{(-4)}\right\rvert = \left\lvert \frac{3}{4}\right\rvert = \frac{3}{4} = \frac{\lvert -3\rvert}{\lvert -4\rvert}$
Property 8	$\lvert a + b\rvert \leq \lvert a\rvert + \lvert b\rvert$	$\lvert 8 + (-5)\rvert = \lvert 3\rvert = 3 \leq \lvert 8\rvert + \lvert -5\rvert = 13$

The last property is called the **triangle inequality**.

EXAMPLE 1.5 Solve the inequalities $|x| \leq a$ and $|x| \geq a$, where a is a nonnegative number.

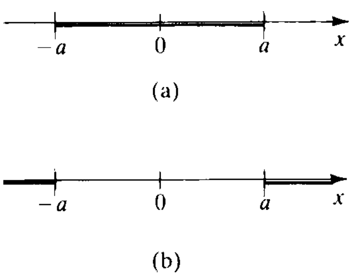


Figure 1.7

Solution First we consider the inequality $|x| \leq a$. If $x \geq 0$, then $|x| = x$ so that $|x| \leq a$ implies $x \leq a$ in this case. On the other hand, if $x \leq 0$ then $|x| = -x$ so that $|x| \leq a$ implies $-x \leq a$ or $x \geq -a$. Thus, $|x| \leq a$ means $-a \leq x \leq a$ (see Figure 1.7(a)). To obtain an alternative solution, observe that $|x|$ is the distance from the point x to 0 so the inequality $|x| \leq a$ implies immediately that $-a \leq x \leq a$.

Next, the inequality $|x| \geq a$ states that the distance from x to 0 is greater than or equal to a. This observation yields the result $x \geq a$ or $x \leq -a$ (see Figure 1.7(b)). ■

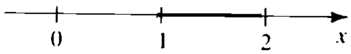


Figure 1.8

EXAMPLE 1.6 Solve the inequality $|2x - 3| \le 1$.

Solution The inequality $|2x - 3| \le 1$ is equivalent to the inequalities $-1 \le 2x - 3 \le 1$ (see Example 1.5). Thus, $2 \le 2x \le 4$ and $1 \le x \le 2$. The solution is therefore given by the set of all x in the interval $[1, 2]$ (see Figure 1.8). ■

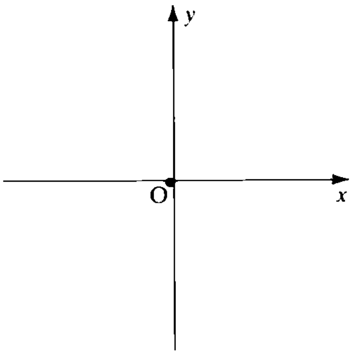


Figure 1.9

Earlier we saw how a one-to-one correspondence between the set of real numbers and the points on a straight line leads to a coordinate system on a line (a one-dimensional space). A similar representation for points in a plane (a two-dimensional space) is realized through the **Cartesian coordinate system**, which is constructed as follows: Take two perpendicular lines, one of which is normally chosen to be horizontal. These lines intersect at a point O called the **origin** (see Figure 1.9). The horizontal line is called the **axis of abscissas**, or more simply the **x-axis**. The vertical line is called the **axis of ordinates**, or the **y-axis**. A number scale is set up along the x-axis with the positive numbers lying to the right of the origin and the negative numbers lying to the left of the origin. Similarly, a number scale is set up along the y-axis with the positive numbers lying above the origin and the negative numbers lying below the origin.

Observe that the number scales on the two axes need not be the same. Indeed, in many applications different quantities are represented by x and y. For example, x may represent the number of typewriters sold and y the total revenue resulting from the sales. In such cases it is often desirable to choose different number scales to represent the different quantities. Observe, however, that the zeros of both number scales coincide at the origin of the two-dimensional coordinate system.

A point in the plane can now be represented uniquely in this coordinate system by an **ordered pair** of numbers, that is, a pair (x, y) where x is the first number and y the second. To see this, let P be any point in the plane (see Figure 1.10). Draw perpendiculars from P to the x-axis and y-axis, respectively. Then the number x is precisely the number corresponding to the point on the x-axis at which the perpendicular through P hits the x-axis. Similarly, y is the number corresponding to the point on the y-axis at which the perpendicular through P crosses the y-axis.

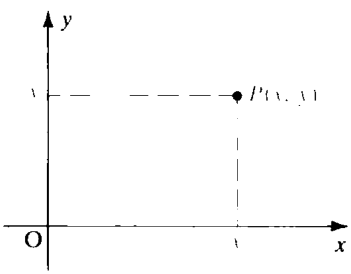


Figure 1.10

Conversely, given an ordered pair (x, y) with x as the first number and y as the second number, a point P in the plane is uniquely determined as follows: Locate the point on the x-axis represented by the number x, and then draw a line through that point parallel to the y-axis. Next, locate the point on the y-axis represented by the number y and draw a line through that point parallel to the x-axis. The point of intersection of these two lines is the point P (see Figure 1.10).

In the ordered pair (x, y), x is called the **abscissa** or **x-coordinate**, y is called the **ordinate** or **y-coordinate**, and x and y together are referred to as the coordinates of the point P.

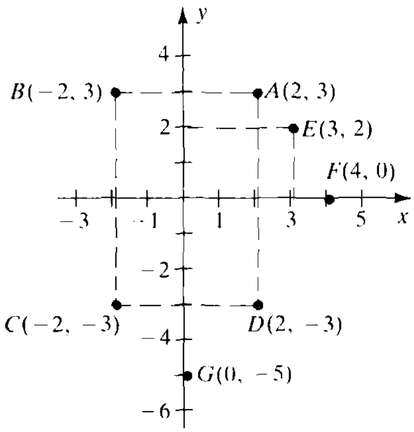


Figure 1.11

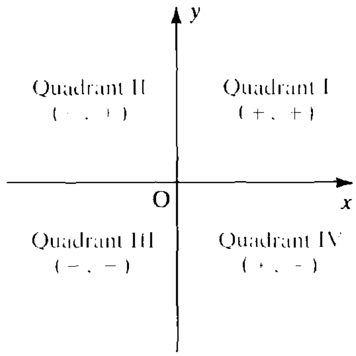


Figure 1.12

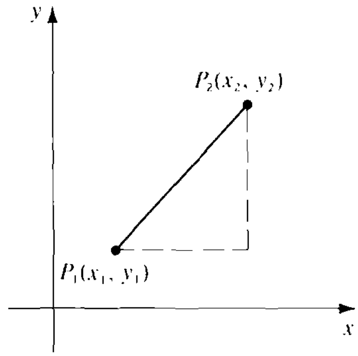


Figure 1.13

The points $A = (2, 3)$, $B = (-2, 3)$, $C = (-2, -3)$, $D = (2, -3)$, $E = (3, 2)$, $F = (4, 0)$, and $G = (0, -5)$ are plotted in Figure 1.11. The fact that, in general, $(x, y) \neq (y, x)$ is clearly illustrated by points A and E in Figure 1.11.

The axes divide the plane into four quadrants. Quadrant I consists of the points P with coordinates x and y (denoted by $P(x, y)$) satisfying $x > 0$ and $y > 0$; Quadrant II, the points $P(x, y)$ where $x < 0$ and $y > 0$; Quadrant III, the points $P(x, y)$ where $x < 0$ and $y < 0$; and Quadrant IV, the points $P(x, y)$ where $x > 0$ and $y < 0$ (see Figure 1.12).

One immediate benefit arising from the use of the Cartesian coordinate system is that the distance between any two points in the plane may be expressed solely in terms of their coordinates. To this end, let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be two points lying in the plane (see Figure 1.13). Then the distance between P_1 and P_2, denoted by $|P_1P_2|$, is, by Pythagoras's theorem, given by the distance formula

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \tag{1.1}$$

For a proof of this result see Exercises 1.1, problem 58.

EXAMPLE 1.7 Find the distance between the points $P_1(-4, 3)$ and $P_2(2, 6)$.

Solution We have by the distance formula (1.1),

$$\begin{aligned} |P_1P_2| &= \sqrt{[2 - (-4)]^2 + (6 - 3)^2} \\ &= \sqrt{6^2 + 3^2} \\ &= \sqrt{45} \\ &= 3\sqrt{5}. \end{aligned}$$

■