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Holger H. Hoos
David G. Mitchell (Eds.)

Theory and Applications of Satisfiability Testing

7th International Conference, SAT 2004
Vancouver, BC, Canada, May 2004
Revised Selected Papers

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Preface

The 7th International Conference on Theory and Applications of Satisfiability Testing (SAT 2004) was held 10–13 May 2004 in Vancouver, BC, Canada. The conference featured 9 technical paper sessions, 2 poster sessions, as well as the 2004 SAT Solver Competition and the 2004 QBF Solver Evaluation. It also included invited talks by Stephen A. Cook (University of Toronto) and Kenneth McMillan (Cadence Berkeley Labs). The 89 participants represented no less than 17 countries and four continents. SAT 2004 continued the series of meetings which started with the Workshops on Satisfiability held in Siena, Italy (1996), Paderborn, Germany (1998) and Renesse, The Netherlands (2000); the Workshop on Theory and Applications of Satisfiability Testing held in Boston, USA (2001); the Symposium on Theory and Applications of Satisfiability Testing held in Cincinnati, USA (2002); and the 6th International Conference on Theory and Applications of Satisfiability Testing held in Santa Margherita Ligure, Italy (2003).

The International Conference on Theory and Applications of Satisfiability Testing is the primary annual meeting for researchers studying the propositional satisfiability problem (SAT), a prominent problem in both theoretical and applied computer science. SAT lies at the heart of the most important open problem in complexity theory ($\mathcal{P}$ vs $\mathcal{NP}$) and underlies many applications in, among other examples, artificial intelligence, operations research and electronic design engineering. The primary objective of the conferences is to bring together researchers from various areas and communities, including theoretical and experimental computer science as well as many relevant application areas, to promote collaboration and the communication of new theoretical and practical results in SAT-related research and its industrial applications.

The 28 technical papers contained in this volume were selected as follows. Of the 72 technical papers submitted to the SAT 2004 conference, 30 were accepted for full presentation at the conference, and a further 18 were selected for posters and short presentations. These selections were made by the Program Committee based on a strict peer-review process, in which each submission received between two and four reviews by Program Committee members or auxiliary reviewers. Authors of accepted papers were invited to submit extended versions of those papers to this volume. From these submissions, 24 were selected for inclusion in this volume, based on another round of rigorous peer reviews. Two additional papers report on the 2004 SAT Solver Competition and the 2004 QBF Solver Evaluation. These were prepared, by invitation, by the organisers of the respective events. Furthermore, authors of the three SAT solvers which placed first in one or more categories of the 2004 SAT Solver Competition were invited to submit papers. Among these latter, one team of authors declined (because their work is presented in part in a previous publication, and in part in another paper included in this volume). These invited papers were peer-reviewed according to the same standards as the other papers in this volume.

We are very grateful to the many people who contributed to the organisation of SAT 2004, most of whom are listed on the following pages. We thank in particular Dave Tompkins for help in preparing this volume. We also thank the authors, presenters and all other attendees for making SAT 2004 a successful and memorable event.

Vancouver, Canada, 8 April 2005 Holger H. Hoos and David G. Mitchell

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Mapping Problems with Finite-Domain Variables to Problems with Boolean Variables*

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Abstract. We define a collection of mappings that transform many-valued clausal forms into satisfiability equivalent Boolean clausal forms, analyze their complexity and evaluate them empirically on a set of benchmarks with state-of-the-art SAT solvers. Our results provide empirical evidence that encoding combinatorial problems with the mappings defined here can lead to substantial performance improvements in complete SAT solvers.

1 Introduction

In the last few years, the AI community has investigated the generic problem solving approach which consists of modeling hard combinatorial problems as instances of the propositional satisfiability problem (SAT) and then solving the resulting encodings with algorithms for SAT. The success in solving SAT-encoded problems depends on both the SAT solver and the SAT encoding used. While there has been a tremendous advance in the design and implementation of SAT solvers, our understanding of SAT encodings is very limited and is yet a challenge for the AI community working on propositional reasoning.

In this paper we define a collection of mappings that transform many-valued clausal forms into satisfiability equivalent Boolean clausal forms and analyze their complexity. Given a combinatorial problem encoded as a many-valued clausal form, the mappings defined allow us to generate six different Boolean SAT encodings. We evaluated empirically the Boolean SAT encodings generated for a number of combinatorial problems (graph coloring, random binary CSPs, pigeon hole, and all interval series) using Chaff [21] and Siege_v4.[1] Our results provide empirical evidence that encoding combinatorial problems with the mappings defined here can lead to substantial performance improvements in complete

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[1] Siege_v4 is publicly available at http://www.cs.sfu.ca/~loryan/personal

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SAT solvers. The behaviour of different SAT encodings of graph coloring and all interval series instances on local search solvers was analyzed in [1, 23].

These results are part of a research program about many-valued satisfiability that our research group has developed during the last decade (see e.g. [2, 5, 9, 11, 18, 20]). Our research program is aimed at bridging the gap between Boolean SAT encodings and constraint satisfaction formalisms. The challenge is to combine the inherent efficiencies of Boolean SAT solvers operating on uniform encodings with the much more compact and natural representations, and more sophisticated propagation techniques of CSP formalisms.

We have used before mappings between many-valued clausal forms and Boolean clausal forms to identify new polynomially solvable many-valued SAT problems [7, 19], to known which additional deductive machinery is required to design many-valued SAT solvers from Boolean SAT solvers [7, 10], and to solve many-valued SAT encodings with Boolean SAT solvers [3, 4]. We invite the reader to consult two survey papers [8, 17] that contain a summary of our previous work.

The paper is structured as follows. In Section 2, we formally define the syntax and semantics of the many-valued clausal forms used in the paper. In Section 3, we define six mappings that transform many-valued clausal forms into satisfiability equivalent Boolean clausal forms. In Section 4, we report the empirical investigation conducted to assess the performance of those mappings.

2 Many-Valued Formulas

We first formally define the syntax and semantics of signed CNF formulas, and then present monosigned and regular CNF formulas, which are the subclasses of signed CNF formulas that are considered in this paper.

Definition 1. *A* truth value set *N is a non-empty finite set $\{i_1, i_2, \ldots, i_n\}$ where $n \in \mathbb{N}$. The cardinality of N is denoted by $|N|$. A total order $\leq$ is associated with N, which may be the empty order.*

Definition 2. *A* sign *is a set $S \subseteq N$ of truth values. A* signed literal *is an expression of the form $S : p$ where S is a sign and p is a propositional variable. The* complement *of a signed literal $S:p$, denoted by $\overline{S}:p$, is $(N \setminus S):p$. A* signed clause *is a disjunction of signed literals. A* signed CNF formula *is a conjunction of signed clauses. The* size *of a signed clause C, denoted by $|C|$, is the total number of literals occurring in C, and the size of a signed CNF formula Γ, denoted by $|\Gamma|$, is the sum of the sizes of the clauses of Γ.*

Definition 3. *An* interpretation *is a mapping that assigns to every propositional variable an element of the truth value set. An interpretation I* satisfies *a signed literal $S:p$ iff $I(p) \in S$,* satisfies *a signed clause C iff it satisfies at least one of the signed literals in C, and* satisfies *a signed CNF formula Γ iff it satisfies all clauses in Γ. A signed CNF formula is* satisfiable *iff it is satisfied by at least one interpretation; otherwise it is* unsatisfiable.

Definition 4. *A sign S is* monosigned *if it either is a singleton (i.e. it contains exactly one truth value) or the complement of a singleton. A monosigned sign S is* positive *if it is identical to $\{i\} : p$, and is* negative *if it is identical to $\overline{\{i\}} : p$ for some $i \in N$. A signed literal $S : p$ is a* monosigned literal *if its sign S is monosigned. A signed clause (a signed CNF formula) is a* monosigned clause *(a* monosigned CNF formula*) if all its literals are monosigned.*

Definition 5. *Given a monosigned CNF formula Γ, the* domain of a variable *p occurring in Γ is $N_\Gamma(p) = \{i \in N \mid \{i\} : p$ or $\overline{\{i\}} : p$ occur in $\Gamma\}$ if $N_\Gamma(p) = N$, and $N_\Gamma(p) \cup \{j\}$, where j is any element of $N \setminus N_\Gamma(p)$, otherwise. The* Boolean signature *of Γ is $\Sigma = \{\{i\} : p \mid \{i\} : p$ or $\overline{\{i\}} : p$ occur in $\Gamma\}$.*

Definition 6. *For all $i \in N$, let $\uparrow i$ denote the sign $\{j \in N \mid j \geq i\}$, where $\leq$ is the total order associated with N, and let $\overline{\uparrow i}$ denote the complement of $\uparrow i$. A sign S is* regular *if it either is identical to $\uparrow i$ (positive) or to $\overline{\uparrow i}$ (negative) for some $i \in N$. A signed literal $S : p$ is a* regular literal *if its sign S is regular. A signed clause (a signed CNF formula) is a* regular clause *(a* regular CNF formula*) if all its literals are regular.*

Definition 7. *Given a regular CNF formula Γ, the* domain of a variable *p occurring in Γ is $N_\Gamma(p) = \{i \in N \mid \uparrow i : p$ or $\overline{\uparrow i} : p$ occur in $\Gamma\}$. The* Boolean signature *of Γ is $\Sigma = \{\uparrow i : p \mid \uparrow i : p$ or $\overline{\uparrow i} : p$ occur in $\Gamma\}$.*

Example 1. Suppose that $N = \{1, 2, 3, 4\}$. Then, we have that the signed clause $\{1, 2, 3\}{:}p_1 \vee \{4\}{:}p_2$ can be represented as a monosigned clause by $\overline{\{4\}}{:}p_1 \vee \{4\}{:}p_2$, and as a regular clause by $\overline{\uparrow 4} : p_1 \vee \uparrow 4 : p_2$.

The notation used in this paper is the one used in the many-valued logic community, which is the notation we used in our previous work on many-valued satisfiability. Since some readers can find hard to read that notation, we next show how to encode a graph coloring problem as a signed CNF formula.

Example 2. Given an undirected graph $G = (V, E)$, where V is the set of vertices ans E is the set of edges, the 3-colorability problem of G is encoded as a signed CNF formula as follows: for each edge $[u, v] \in E$, we define three signed binary clauses

$$(\{2,3\}{:}u \vee \{2,3\}{:}v) \wedge (\{1,3\}{:}u \vee \{1,3\}{:}v) \wedge (\{1,2\}{:}u \vee \{1,2\}{:}v)$$

and take as truth value set $N = \{1, 2, 3\}$.[2] The intended meaning of the previous signed clauses is that there are no two adjacent vertices with the same color.

Signed CNF formulas and their subclasses have been studied since the early 90's by the research community working on automated theorem proving in many-valued logics [6, 13, 15, 16, 22]. A few years later, Frisch and Peugniez [14] used the term non-Boolean formulas to refer to signed CNF formulas.

[2] These clauses are represented as monosigned clauses by $(\overline{\{1\}} : u \vee \overline{\{1\}} : v) \wedge (\overline{\{2\}} : u \vee \overline{\{2\}} : v) \wedge (\overline{\{3\}} : u \vee \overline{\{3\}} : v)$.