DYNAMIC MODELLING AND CONTROL OF NATIONAL ECONOMIES

Edited by
J. M. L. JANSSEN L. F. PAU A. J. STRASZAK

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INTRODUCTION

This Conference was the third in the series of IFAC/IFORS conferences on dynamic modelling and control of national economies, following the first in Warwick, United Kingdom (1973), and the second, also sponsored by IIASA, in Vienna, Austria (1977). The third conference was also sponsored by the Society for Economic Dynamics and Control as a participant. According to present plans the fourth conference will be held in the USA in 1983.

On behalf of the Polish National Member Organization of IFAC, the conference was organized by the Systems Research Institute of the Polish Academy of Sciences. As hosts of about 182 participants from 27 countries they made very succesful local arrangements.

The wide variety of contributions to the conference, including 2 round-table discussions, some 60 papers divided over plenary sessions and many parallel sessions, and lively discussions, were evidence of the strength and growth in the areas of economic modelling, control techniques ; and their use for planning and decision making.

The International Program Committee under the chairmanship of Professor Z. Pawlowski emphasized very strongly that the selected papers should include numerical results and experience of practical implementation.

Whereas the individual papers in these proceedings dealing with special subjects may appear to be widely scattered over the whole field, in the round tables and discussions a pattern emerged of what the successes and limitations were of this approach, thus pointing also at areas for further research. This will be summarized below in the sections on

- Interaction between decision making and model building
- Enhancing model credibility and applicability.

The third section gives a summary of the main topics of the conference, since reports of the discussions in the various sessions were not available in full detail. However, thanks to the efforts of Dr. Cichocki and Dr. Owsinski, detailed reports of the two round-table discussions have been included in this volume.

For various administrative and editorial reasons not all papers presented at the conference could be included in the proceedings. Other papers were included after minor or major revision of the text as originally submitted, often complicated by problems of language or channels of communication. The editors very much appreciate the co-operation of the authors in this matter.
They sincerely hope that a reasonable balance has been struck between perfection and timeliness.

The papers have been grouped in accordance with the sessions of the conference.

INTERACTIONS BETWEEN DECISION MAKING AND MODEL BUILDING.

Decision making is usually a very complex process, of which model building is one of many parts, and model builders are few of many actors. It is their specific responsibility to clarify complex issues by means of formal methods and in this manner to supply the other actors with additional information. In this connection several points were made:

- The buying / selling relationship between the model builders and the others may lead to selling models that provide justification for pre-conceived ideas, rather than models that are well-tested results of scientific investigations under clearly specified assumptions.
- There is a difference in background and perspective between the model builders and those who are responsible for executing the decisions. The question was raised to what extent model builders could become involved in taking the actions, so that they would get personal experience of real-life mechanismes. This would also reduce the danger of building "a big model for the boss", not necessarily agreeable to the rank and file.
- The gap between model builders and others might also be bridged by "interpreters" who derive their credibility from working in responsible

advisory positions close to the decision makers, as well as from being knowledgeable in the process of model building through education and experience. On the one hand their participation in decision and policy generation should expose them to the facts of life. On the other hand, they should keep close contacts with the community of model builders.
- Decision making at a central position in combination with model building with a corresponding degree of aggregation, does not necessarily lead to the expected decentralized actions.
- In decision making, empirical rules of thumb may be used by lack of something better. It is one of the roles of model building to verify or reject these rules, with the hopeful ultimate result of improved designs of qualitative or quantitative instruments for decision making.

ENHANCING MODEL CREDIBILITY AND APPLICABILITY.

In the Scientific community two views prevail, and both are legitimate:
- Model building as a scientific activity for its own sake. Then applicability is not the major issue. Credibility requires that models can stand the test of scrutiny by the scientific community.

  Thus, it was said that
  * no model should be published without a track record
  * a multiplier analysis should be made
  * the data base should be made available.
- Model building as part of the decision-making process. This type of model building was, in fact, the theme of this conference. Applicability is a major issue, and effectiveness dominates scientific rigour: "the final test of a theory is its ability to solve the problems that originated it" (George Dantzig).

In connection with the second view several points were made:
- Economic models were said to be notoriously unstable, in the sense that changing economic conditions require frequent structural and numerical model changes. Consequently, there are diminishing returns on increasing the size of economic models.
- There may be competing models, on the basis of intuitively equally obvious assumptions, but showing different system behaviour. Model selection by statistical tests is usually not conclusive.
- In forecasting, univariate models may be competitive with economic models. However, univariate models say nothing of causal relations between control actions and responses. Since in day-to-day activities control is to be effected by coherent sequential decisions, development of theory and building tools for short-term causal models needs more attention.
- Models, even competing models, have an educational impact on the organization, even if none of them will eventually be used.
- The importance of the time horizon was stressed. At least the first step should be taken in the right direction; this direction needs to be known within a certain range. In this sense the long-term goals and effects are a crucial factor. Model builders should be aware of possible conflicts between short-term decisions and long-term goals.
- Good data quality is required because of the "garbage in garbage out" rule. The question is to what extent model builders should contribute to the specifications and rules at the data collection level.

The two views can be harmonized by maintaining scientific rigour as far as possible, since nothing is more practical than good theory. To achieve this, a long-term effort is required geared at specific education (graduate and post-graduate) in dynamic modelling and control of economic systems. This education should be offered in economics, engineering and social studies.

MAIN TOPICS OF THE CONFERENCE

Theory of modelling

The papers covered a broad class of problems arising in both the theory and the practice of modelling various economic systems.

The main issues discussed were:
- model representations (digraphs, two-level control systems, hybrid models, subjective information modelling);
- solution concepts (experimential solutions, pseudo-inverses, game theory, stochastic solutions and disturbances, general equilibrium models).

National models

The main issues raised were:
- changing model structures;
- modelling of institutional intervention in countries with planned economies;
- choice of weights in the criterion functions;
- use of explicit inequality constraints versus replacement by penalties in the criterion function;
- effect of data inaccuracies on feed-back control;
- interpretation of residuals and dummy variables in estimated relations;
- estimation of rational expectations and uniqueness of a response to external shocks.

Sectoral Models

Presentations covered such sectors as energy, health care, and agriculture. They addressed mainly the issues of interaction between sectors and decision levels, as well as sector stability.

Regional models

The main emphasis was on:
- the co-ordination between national and regional planning processes;
- the effects of demographic changes and migrations (especially on urban areas);
- the existence of multiple technology levels distributed within the same sectors throughout regions (e.g. agriculture, energy);
- the environmental effects.

The need was stressed for this area to make a larger use of interactive tools, of gaming approaches, and of multiple-criteria planning.

Monetary and fiscal models

This session included three nonlineair aggregate models of the effects of financial controls (monetary, fiscal) on economic activity, with optimization and simulation results. The issues of intermediate targets and causality relations were also discussed.

Econometric forecasting and estimation

Progress was reported on a variety of econometric estimation procedures: regression methods; spectral analysis; forecasting error compensation. A large interest was shown in various uses of adaptive estimation, e.g. by Kalman filtering.

Optimization methods

Besides a presentation of various algorithms, a number of possible criteria were discussed: state trajectories; integral performance indexes; qualitative criteria.

Software tools for macroeconomic models and analysis

Besides a methodological analysis of structural and parametric sensitivity, a number of estimation, optimization and data-base handling packages were described. The discussion went on about ways of designing user-friendly packages for economists.

CONCLUSION

This conference again witnessed the role and importance of bringing together model builders, decision makers, economists, and engineers, from various countries and organizations, for the sake of progress in this important area.
Since dynamic modelling and control of national, regional, and sectoral economies is an art built on science, discussion and presentation of a multiplicity of approaches are essential for good decision making and international co-operation.

THE EDITORS:

J.M.L. Janssen

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CONTENTS

ECONOMETRIC FORECASTING AND ESTIMATION

Contents

NON-PRICE CONTROL
Report on a Research Trend

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Abstract. The paper reports on a line of studies conducted by 8 Hungarian researchers. Their work deals with simple (vegetative) control mechanisms of a multi-agent economy with the following characteristics: control signals are mostly of the non-price type (stocks, orders), norms or normal paths are used as command signals, linear feedback control is applied. The 20 models are analysed with respect to stability and viability of the system as well as to the degree of decentralization of information flow and decision structure. The differences among the approaches lie in the control signals, the response functions, time lags, deterministic *vs* stochastic behaviour, constant *vs* variable parameters, etc.

Keywords. Economics; models; multivariable control systems; stock-control; stability; decentralized control.

INTRODUCTION

In this paper a sketchy overview of a research trend is given dealing with different control mechanisms of the economy. The purpose of the research is to lay foundations to the study of some components of the economic control systems - more or less neglected in the literature, where prices were considered as the typical information source on which economic decisions are based.

The contributors to this research are: A. Bródy, I. Dancs, L. Hunyadi, Zs. Kapitány, J. Kornai, B. Martos, A. Simonovits, J. Sivák.

The results are being published in the forthcoming book: J. Kornai and B. Martos (Editors): Non-Price Control (1981) Akadémiai Kiadó, Budapest and North-Holland Publ. Co. Amsterdam. The present paper is essentially a digest of this book.

THEORETICAL BACKGROUND

Antecedents

The four lines of thinking which preceeded and influenced our research are depicted in Fig. 1. The drawback of this foursome inspiration is that we must often apply many unrealistic assumptions inherited from these various disciplines.

Some Basic Concepts and Characteristics

Real sphere vs control sphere. Following Kornai (1971) we always make a clear distinction, separation of the real processes of the economy (as production, transfer of goods, consumption) and the control processes accompanying them (as information processing, decision making, fiscal and monetary processes etc.). Accordingly from an organizational point of view we assume each organization to include a real unit in which the real processes take place and a control unit which directs the real processes within the organization and may be in an informational connection with control units of other organizations. Moreover organizations are classified in two groups: real organizations whose main activity is to perform real tasks (as productive firms, households, etc.) and control organizations whose real activity is negligible and are organized to perform control functions (as a statistical office, a planning office, a bank, a ministry, etc.).

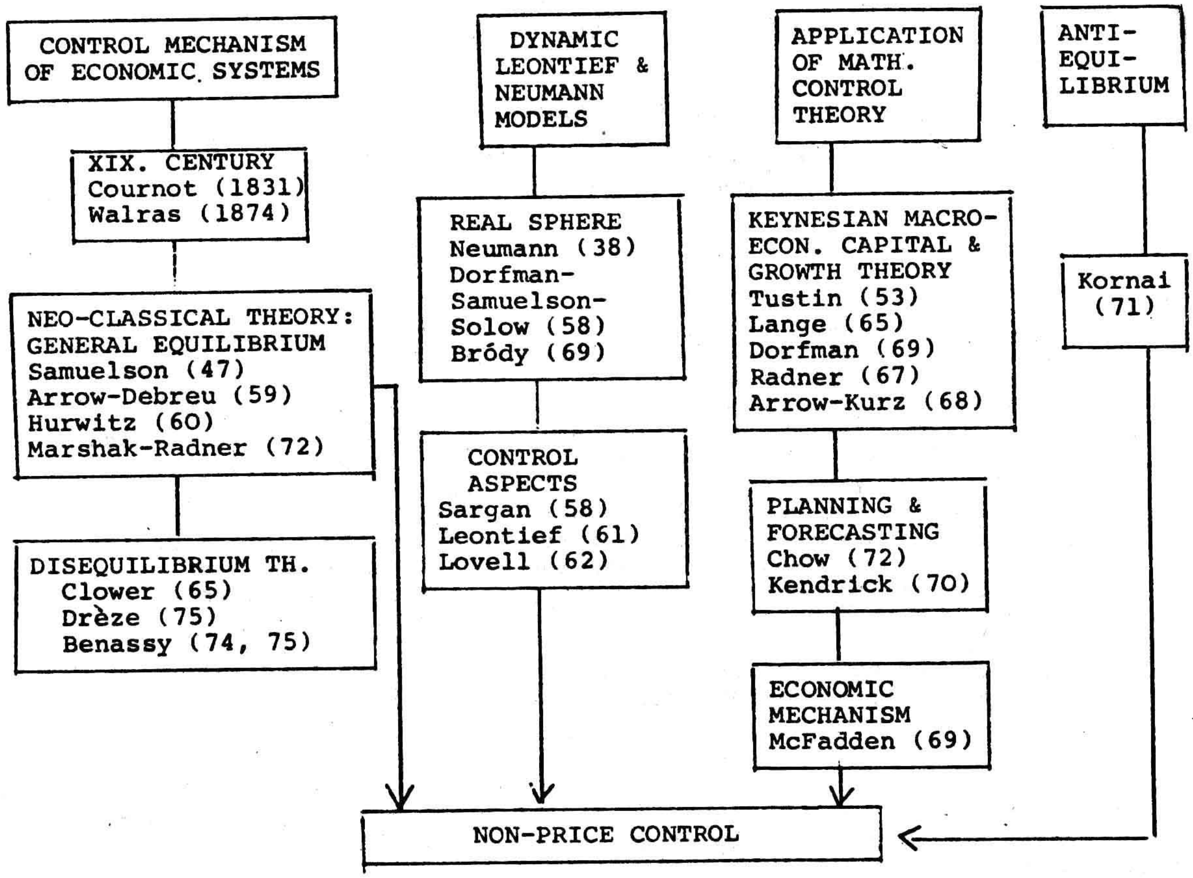

Fig. 1. Antecedents

Off-line vs on-line control. In many models of the real sphere (e.g. Walras, Neumann) control variables (e.g. prices) appear as byproducts which have no effect whatsoever on real processes. If we consider the tatônnement process of the Walrasian auctionnaire or the interpretation of the decomposition algorithms of linear programming as abstract control processes, then we observe that the time stops as long as these control processes run their course. The non-equilibrium or non-optimal prices and production programs which are consecutively produced in this process do not influence the real processes, they control off-line. In our models the control is on-line, the clocks are not stopped, real and control processes run simultaneously and influence each other whether they are on the equilibrium/optimum level or not.

Secular vs historically limited dynamics. We deal with systems differing in the institutional framework, behavioural characteristics, normal states but for each of them all these are fixed. This means that the dynamic framework of each model is a historically limited one. We ask only the question, how the systems operate under given conditions, and neglect the secular dynamics, i.e. how the institutions, behavioural rules, norms develop in the historical time, where learing, self-adjustment and self-organization also exert effect on the economic-social system.

Control by norms vs optimal control. In our models control by norm is the basic approach. This includes the assumptions that for some state and/or control variables there exist norms (or normal paths, if variable in time) as the accepted or customary value of a variable, around which the actual value is taken on the effect of the operating control. The behaviour of the participants are assumed to show up some regularity, but it is not required that this behaviour should be an optimizing one. This is why we do not rely on the more modern and fashionable theory of optimal processes.

Descriptive theory vs normative theory. Our purpose is to describe the functioning of the economy (or more modestly: some aspects thereof) and analyse models which reflect possible representations of this functioning, rather then tell, how the economic agents and institutions should behave, what rules are desirable to follow. This is also connected with another dichotomy: we deal with economic mechanism and not with finding a satisfactory (or optimal) economic policy, to which the most effort in the application of control theory to economics has been directed in the past.

Centralization vs Decentralization

We found the usual dichotomy between decentralized and centralized control inadequate to the classification of systems. Therefore a more detailed classification scheme has been elaborated by Kornai and Martos.

Elementary processes. The part of a complex control process which is not further divided into parallel or subsequent subprocesses is an elementary process. From the point of view of the signal formation an elementary process may be

- isolated, if the signal is formed by the control unit of a single real organization
- interactive, if it is formed jointly by control units of several real organizations
- centralized, if it is formed by one or more control organizations. (A common name for interactive or centralized processes is: coordinated.)

From the point of view of signal transfer the elementary process may be

- non-communicative, if the signal does not leave the organization where it originated
- transactionally communicative, if the signal is transmitted between a pair of real organizations and refers to an actual or potential real-transaction (usually transfer of goods or services) between this pair. (E.g. price offers, bills, orders, etc.)
- non-transactionally communicative in any other case (especially if a control organization is involved).

Combining signal formation with transfer some combinations are incompatible as Table 1. shows.

TABLE 1 Classification of Elementary Control Processes

Signal formation / Signal transfer	Isolated	Coordinated: Interactive	Coordinated: Centralized
Non-communicative	+	∅	+
Transactionally communicative	+	+	∅
Non-transactionally communicative	+	+	+

Complex control processes are composed of elementary processes possibly belonging to different classes of the above classification. Thus it is not generally possible to tell if a complex process is more or less centralized than another. Still we formed 5 stages of typical forms in growing order of centralization.

The term vegetative in these definitions refers to systems consisting of elementary processes which are all isolated and either non-communicative or transactionally communicative. The five stages are then:

I. Vegetative, non-communicative

II. Vegetative, communicative (transactionally)

III. Interactive

IV. Partially centralized (Policentric)

V. Monocentric

Points of View of the Analysis

Our models will be analysed according to the following criteria:

- Stability Analysis
- Viability Analysis (usually non-negativity of some variables)
- Structural Analysis
- Comparative Analysis (Similarity, Equivalence)

Notation

n = No. of sectors or products

$q(t)$ = output stock vector

$V(t)$ = input stock matrix

$r(t)$ = production vector

$Y(t)$ = commodity transfer matrix

$c(t)$ = consumer's demand vector (observable, uncontrollable)

$A(t)$ = input coefficient matrix, $\rho(A) \leqq 1$. A is indecomposable

$p(t)$ = price vector

$\underline{1}$ = $[1,1,\dots,1]'$, the summation vector.

MODELS WITH CONSTANT NORMS: STOCK, PRICE, PROFIT

This chapter deals with linear control models where the real sphere is an open or closed Leontief economy, the control is based on stock signals or intermediate signals derived from them, and the controller is of the integrating or integrating plus proportional type. Some of the models work with continuous time and without lag, some with discrete time and uniform lags.

Models of Kornai and Martos (1971, 1973)

The first attempts to establish models of vegetative control resulted in two models. Here we give simplified versions of them, a more complete analysis can be found in Kornai and Martos (1973). An open Leontief model $[\rho(A) < 1]$ is controlled here by two types of control.

The real process

$$\dot{q} = r - Y\underline{1} - c \quad \text{(Output stock balance)} \qquad (1)$$

$$\dot{V} = Y - A\langle r\rangle \quad \text{(Input stock balance),} \qquad (2)$$

where $\langle r\rangle$ is the diagonal matrix formed from vector r.

The control process

$$\dot{r} = \dot{Y}\underline{1} + \dot{c} + \gamma^2(q^* - q) - 2\beta\gamma\dot{q} \quad \text{(Production control)} \qquad (3)$$

$$\dot{Y} = A\langle\dot{r}\rangle + \dot{A}\langle r\rangle + \gamma^2(V^* - V) - 2\beta\gamma\dot{V} \quad \text{(Control of transfers)} \qquad (4)$$

where * arterisk refers to the normal value of a variable. For $\beta = 0$ the scheme represents an integrating control, while for $\beta \neq 0$ we have an additional proportional term.

Stability analysis. The pure integral control is Ljapunov-stable but not assymptotically stable, the integral plus proportional control is asymptotically stable for $\beta, \gamma > 0$.

Viability analysis. $q > 0$, $V > 0$ and $r > 0$ if (sufficient conditions)

$$0 < \beta < 1$$

$$\gamma > \frac{1}{\sqrt{1-\beta^2}}\,\Phi\,(r^0, Y^0, q^0, V^0)$$

$$c_i(t) > \frac{1}{\sqrt{1-\beta^2}}\left|r_i^0 - \sum_j A_{ij}^0 r_j^0 - c_i^0\right|$$

for $i=1,2,\dots,n$ and $t \geqq 0$,

where the superscript ^{0} refers to the initial values at $t=0$ and Φ is a complicated function of the initial values.

Structural analysis. The system is vegetative and non-communicative.

The main lesson drawn from this analysis is the existence of a vegetative and non-communicative control scheme which can be constructed in such a way that the system is stable and viable.

Models of Dancs, Hunyadi and Sivák (1973)

They studied discrete variants of the Kornai-Martos models replacing simply the differential operators by difference operators and hereby introducing one unit time lag in the control process.

The analysis showed that the discrete variant of the integral control became unstable, while the stability domain for the values of β, γ in the discrete variant of the integral plus proportional control is more restricted than in the continuous case. (See Fig. 2.)

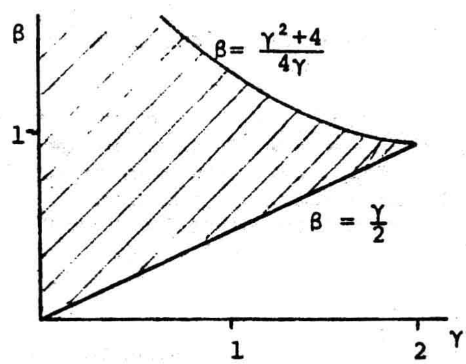

Fig. 2. Stability domain for the discrete variant of the integral plus proportional control

Models of Bródy (1973)

A more systematic analysis of a simpler model was given by András Bródy. He works with a closed Leontief real-sphere [$\rho(A) = 1$] and with constant input matrix. Moreover he disregards the transfer of commodities and makes no distinction between input and output stocks; the vector v will denote total stocks.

Hence he deals with the real process

$$\dot{v} = (E-A)r \quad (5)$$

in the continuous case, and correspondingly with

$$\Delta v = (E-A)r \quad (6)$$

in the discrete case.

To both he fits four different control mechanisms, which are summarized in Table 2.

Further analysis of the asymptotically stable cases by Bródy showed that from the point of view of the transient behaviour the control from the change of profit has advantages (and disadvantages) as compared to the control from the change of stocks. In his opinion neither of these control mechanisms is completely satisfactory.

A Price-Communication Model of Martos (1976)

I tried to combine the ideas of the Kornai-Martos model with the price formation and profitability calculation process introduced by Bródy. This resulted in a model which took over the real processes and the control of the transfer of goods from the former [Equations (1), (2) and (4)] while replacing equation (3), the production control, by the following three-step process:

$$\dot{p} = \mu^2(q^*-q)-2\lambda\mu\dot{q} \quad \text{(Price setting)} \quad (7)$$

$$g = (E-A')p \quad \text{(Calculation of sectoral value added)} \quad (8)$$

$$\dot{r} = Y\dot{\underline{l}}+\dot{c}+\pi\dot{g} \quad \text{(Production decision)} \quad (9)$$

This control process has a vegetative structure with price communication and was proven to be asymptotically stable and viable under conditions formally similar to the stability and viability conditions of the Kornai-Martos model, with the additional conditions:

$$\mu > \underline{\mu} > 0$$

$$\pi \geqq 1$$

$$\lambda^2\pi > \frac{1-\sqrt{1-\rho^2}}{2(1-\rho^2)},$$

where $\underline{\mu}$ is a constant depending on the initial values and on λ, but independent of time.

EQUIVALENCE OF SYSTEMS AND A COMPARATIVE ANALYSIS

By developing an equivalence concept and corresponding criteria on one

TABLE 2 Eight Control Mechanisms of Bródy

Control signal	Continuous time: Control equations	Continuous time: Stability condition	Discrete time: Stability condition	Structural characterization
Level of stocks	$\dot{r} = \gamma(v^*-v)$	unstable	unstable	vegetative non-communicative
Change of stocks	$\dot{r} = -\gamma\dot{v}$	asymptotically stable for $\gamma>0$	asympt. stable for $\gamma>0$	
Level of profit (Goodwin, 1952)	$\dot{p} = -\gamma\dot{v}$ $\dot{r} = (E-A')p$	Ljapunov stable	unstable	vegetative with price communication
Change of profit	$\dot{p} = -\gamma\dot{v}$ $\dot{r} = (E-A')\dot{p}$	asympt. stable for $\gamma>0$	asympt. stable for a range of γ	

hand and a more abstract control scheme (both in terms of Laplace transforms) a common generalization of the Kornai-Martos and the Martos models emerged: From this three more equivalent (partially coordinated) control structures could be identified with the following characteristics:

(i) Coordinated price setting, isolated production control

(ii) Isolated price setting, centralized production control

(iii) Mixed (polycentric) system.

These results were presented to the 2nd (Vienna, 1977) Conference on Dynamic Modelling and Control, and published in the proceedings volume edited by Janssen, Pau and Straszak (1979). [See Martos (1979).] Hence I abstain from going into more details here.

MODELS WITH NORMAL PATHS: STOCK AND ORDER

The Stock Signal Model of Kornai and Simonovits (1975a , 1977)

This far the norms were assumed to be constants, from now on they are allowed to depend on time, but, of course, exogeneously given. Although some of the authors' results hold for any admissible normal path, for sharper theorems they assume, that the normal path is a Neumann path, and here I confine myself to this case. They work with discrete time: $t=0,1,2,\ldots$.

The normal Neumann path is:

$$r^* = r_o\lambda_o^t$$

$$Y^* = Y_o\lambda_o^t$$

$$q^* = q_o\lambda_o^t = <g>r_o\lambda_o^t$$

$$S^* = S_o\lambda_o^t = F<r_o>\lambda_o^t$$

where S = the slack input stock

λ_o = the Neumann growth rate

F = the constant matrix of input stock per production norms

g = the constant vector of output stock per production norms.

The slack input stock S is defined by the equation

$$S = V-B<r>, \qquad (10)$$

where B is the input-coefficient matrix of the technologically necessary input stocks. S is thus the difference between actual input stocks and the input stocks which are needed for the actual level r of production.

The real processes are represented by a closed Leontief model:

$$\Delta q = r-Y\underline{1} \quad \text{(Input stock balance)} \qquad (11)$$

$$\Delta V = Y-A<r> \quad \text{(Output stock balance)} \qquad (12)$$

It has been proved, that there is a unique non-negative Neumann path which satisfies equations (10), (11) and (12), A, B, F and g being given.

A proportional control process is applied:

$$r = r^* -<d>(q-q^*) \qquad (13)$$

$$Y = Y^* - C(V-V^*) \qquad (14)$$

where $d = [d_j]$, $C = [C_{ij}]$ are the control parameters.

Stability analysis

(a) The system is asymptotically stable if

$$0 < d_j < 1$$

$$0 < C_{ij} < 1$$

for all $i,j = 1,2,\ldots,n$.

(b) Supposing $d_j = C_{ij} = \delta$, the system is asymptotically stable if and only if

$$0 < \delta < \frac{2}{1+\sqrt{\rho}}$$

Viability analysis. Let x^o denote the vector of $2n+2n^2$ initial values: q^o, Y^o, r^o, V^o. We use the vector norm: $||a|| = \max_i |a_i|$. A viable initial neighbourhood is defined by the inequality

$$||x^o|| < \min_{i,j} \left\{ \frac{r_j^*}{d_j}, \frac{Y_{ij}^*}{C_{ij}}, q_j^*, \frac{S_{ij}^*}{1+B_{ij}d_j} \right\} \qquad (15)$$