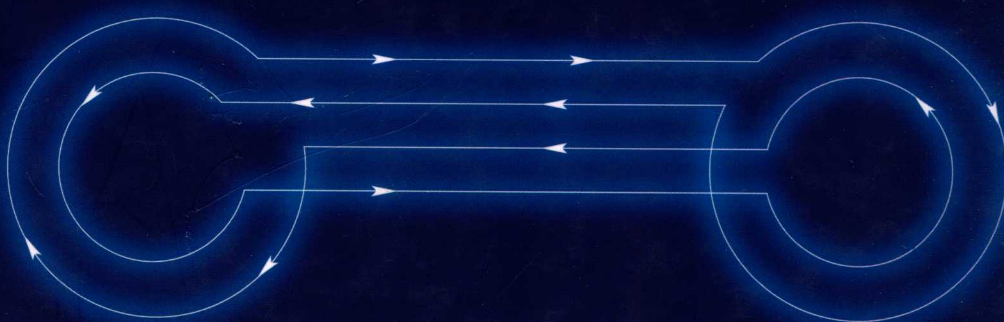
Introduction to Classical and Modern Analysis and Their Application to Group Representation Theory

Debabrata Basu



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Introduction to Classical and Modern Analysis and Their Application to Group Representation Theory

Dedicated to My Departed Guru
Late Professor Sudhansu Datta Majumdar
Who Taught Me the Fundamentals of
Classical Analysis and Representation Theory

Foreword

This book, by Debabrata Basu, is novel and valuable in several different ways. In this work, the author offers a quality introduction and review of important analytical methods and tools in Part I, and in Part II applies them to an important set of Lie groups presenting them in a modern analytical language that also makes close contact with the language of the well known coherent states. The author effectively uses numerous examples to guide the reader to an ever increasing appreciation of the analytical methods he discusses. As such, this first part already serves as a useful summary of basic fundamental analytical notions. The second part dealing with several examples of locally compact groups offers the reader a smooth introduction into this basic and very important territory. I am pleased to congratulate the author for his significant contribution to this general subject matter.

John Klauder
Gainesville, Florida
January 15, 2009

Preface

During the past few decades analytical methods are being increasingly applied to group representation theory which primarily developed as a branch of algebra in the hands of Frobinius and Schür. Although the application of analytical methods is now the standard approach in Lie groups there is as yet no standard textbook dealing with classical and modern analysis as applied especially to locally compact groups.

It is expected that this gap will be bridged by this book which is essentially an amplification of the lectures of the author to M.Sc. students of the Physics Department of Indian Institute of Technology, Kharagpur. For clarity many standard topics in this book have been treated in a way which substantially differs from traditional treatment and is in a more teachable form.

The author himself does not understand the sophistry of pure mathematics and those who look for elegance and rigour will be sorely disappointed. The book does not provide the most general topological definition of Lie groups, not that the author is unwilling to learn it, but that it is deemed inessential in a preliminary course which this book intends to cover. In a sense, following Ivan Karamazov, discussions are all conducted "as stupidly as possible..." because "the stupider, the more to the point. The stupider, the clearer. Stupidity is brief and artless but intelligence shifts and shuffles and hides itself. Intelligence is a knave, while stupidity is straightforward and honest."

Even a casual reader browsing through this book will not fail to notice the indebtedness of the author to the Russian masters of functional analysis and representation theory. Of course, this does not come anywhere near their magnum opus in depth, breadth of coverage and originality; it is only a modest endeavour to make accessible to the graduate students the fundamentals of the subject created by them.

The first eight chapters of this book may be covered in any traditional graduate course in mathematical physics. In particular later parts of Chapter 8 supplies the mathematical framework of the octet model of Gell-Mann and Neéman which is the foundation of the present day quark model, an inseparable part of standard model. The remaining three chapters deal with infinite dimensional representations of the simplest locally compact groups, namely, SL(2, R), SL(2, C) and the Heisenberg–Weyl group. They are becoming increasingly important in several areas of quantum optics and quantum gravity.

The references at the end of each chapter are those that have been consulted by the author and are a reflection of personal taste rather than anything else.

The author would like to thank Professor J. R. Klauder for not only writing the foreword but also for constructive criticism as well as numerous conceptual and technical corrections, to say nothing of his excellent review submitted to the publishers. The author has immensely benefited from collaboration with Prof. P. Majumdar of Saha Institute of Nuclear Physics, Kolkata in the SERC Winter School held at Benares in several sections of Chapters 8. Finally the author would like to express his heartfelt affection and admiration for the students of Indian Institute of Technology, Kharagpur, who are the main inspiration of this endeavour.

Debabrata Basu

Acknowledgment

Professor Debabrata Basu passed away in November 2009 after a prolonged illness, when the book was still with the publisher. Much of it was written in between his stays at the hospital and the final manuscript was proofread by two of his most favorite students Wrick Sengupta and Dr KV Shajesh. The family of Professor Basu is deeply indebted to Mr Sengupta and Dr Shajesh for having done extremely detailed proofreading of the manuscript. Words are not enough to thank them for their contribution in ensuring that the book sees that the light of the day.

Contents

PART I

Analysis

The first two chapters are of an introductory nature and provide a brief survey of the process of analysis. They touch upon only the most essential points leading to the residue theorem and its applications to the evaluation of definite integrals. The power and scope of this apparently simple theorem has been amply demonstrated in later chapters where it has been employed in the problem of analytic continuation of the hypergeometric series (Chapter 4) on the one hand and in the Clebsch–Gordan problem of $SL(2, R)$ (Chapter 9) on the other.