B·P

College Algebra

MERLIN M. OHMER
The University of Southwestern Louisiana

CLAYTON V. AUCOIN
Clemson University

BLAISDELL PUBLISHING COMPANY
A Division of Ginn and Company
WALTHAM, MASSACHUSETTS · TORONTO · LONDON

Some material included in this book was published previously in *Elementary Contemporary Algebra* by Merlin M. Ohmer, Clayton V. Aucoin, and Marion J. Cortez, copyright © 1965 by Blaisdell Publishing Company, and in *Elementary Contemporary Mathematics* by Merlin M. Ohmer, Clayton V. Aucoin, and Marion J. Cortez, copyright © 1964 by Blaisdell Publishing Company.

Library of Congress Catalog Card Number: 66-14620
Printed in the United States of America

COLLEGE ALGEBRA

A Blaisdell Book in Pure and Applied Mathematics

CONSULTING EDITOR
SEYMOUR SCHUSTER, *University of Minnesota*

Preface

PRIOR TO THE REVOLUTION in school mathematics, the typical college algebra textbook included all of the topics of high school algebra in addition to many topics not included in the standard high school curriculum. At that time the standard college algebra course was part of the curriculum of most entering college freshmen. This course provided a review of high school algebra and an introduction to additional topics which are desirable and useful in the study of analytic geometry and calculus.

Today, many students enter college with two years or more of high school algebra. These students usually omit the college algebra course and begin their college studies with analytic geometry and calculus. Those topics previously included in the college algebra text, which were not included in the high school algebra courses, are now included in the analytic geometry and calculus course. It would be inconsistent with the philosophy of the more up-to-date programs to include, in a college algebra course, all of the topics of high school algebra and, in addition, those additional topics which would have to be repeated in the analytic geometry and calculus course. Furthermore, in a one-semester college algebra course, it is impossible to cover adequately the two-year high school algebra course, at a higher level of abstraction than the typical high school course, if the additional topics are included. For this reason we have written this text as a high-level review of the standard two-year high school algebra course. We believe that the main purpose of a good college algebra course is to provide the student with an adequate foundation in those topics which are absolutely essential to a systematic study of analytic geometry and calculus; therefore, we have omitted some of the traditional topics of the traditional college algebra text.

College Algebra may be taught, in one semester, according to *any one* of the following outlines:

1. Chapters 1 through 7, or
2. Chapters 1 through 8, or
3. Chapters 1 through 9, or
4. Chapters 1 through 7, Chapters 10 and 11, or
5. Chapters 1 through 8, Chapters 10 and 11, or
6. Chapters 1 through 11.

The material has been taught in most of the above combinations at the University of Southwestern Louisiana and at Clemson University.

The end of the proof of each theorem is indicated by the symbol "◇." Although it is preferable to include the proofs, the instructor who wishes to omit them may do so without disrupting the continuity. As each theorem is motivated by examples preceding and following it, the text flows well without the proofs.

We are indebted to many persons, especially to our colleagues who urged us to write *College Algebra.* In particular, we express our gratitude to Dr. Seymour Schuster of the University of Minnesota for his valuable suggestions, comments, and criticisms. He followed and read the manuscript as it was developed and reread it after it was completed. We are grateful also to Dr. Z. L. Loflin, Chairman of the mathematics department at the University of Southwestern Louisiana, and to Miss Jessie May Hoag, also of the mathematics department, for their encouragement; to Mrs. Louise Fulmer of Clemson University for her valuable comments; and to Mr. M. J. Cortez of Allemand School, to Miss Diana Kay Regan, and Miss Mary Catherine Dugas for typing most of the manuscript and preparing the answers to the exercises.

Finally, we thank our wives and children for their patience, understanding, and encouragement during the preparation of the manuscript.

MERLIN M. OHMER
CLAYTON V. AUCOIN

Lafayette, Louisiana

Table of Symbols

U	*the universal set*
$\varnothing$	*the empty, or null, set*
$\tilde{A}$, $U \setminus A$	*the complement of A*
$A \subset B$	*A is a subset of B*
$A \not\subset B$	*A is not a subset of B*
$a \in A$	*a is an element of A*
$a \notin A$	*a is not an element of A*
$A \approx B$	*A is equivalent to B,*
	A is in one-to-one correspondence with B
$A \not\approx B$	*A is not equivalent to B*
	A is not in one-to-one correspondence with B
$A \cup B$	*A union B, the union of A and B*
$A \cap B$	*A intersection B, the intersection of A and B*
$A \times B$	*A cross B, the Cartesian product of A and B*
$a \, \mathsf{R} \, b$	*a is related to b*
$a \, \not{\mathsf{R}} \, b$	*a is not related to b*
C_0	*the set of counting numbers,* $\{0, 1, 2, 3 \ldots\}$
I^+	*the set of positive integers,* $\{1, 2, 3, \ldots\}$
I^-	*the set of negative integers,* $\{^-1, {}^-2, {}^-3, \ldots\}$
I	*the set of integers* $\{\ldots, {}^-3, {}^-2, {}^-1, 0, 1, 2, 3, \ldots\}$
R_a	*the set of rational numbers*
R	*the set of real numbers*
C	*the set of complex numbers*
$(F, +, \times)$	*a field with binary operators* $+$ *and* $\times$
$(R_a, +, \times, <)$	*the rational number system with order relation* $<$
$(R, +, \times, <)$	*the real number system with order relation* $<$
$\sim p$	*not p (negation)*

$p \vee q$	*p or q (disjunction)*
$p \wedge q$	*p and q (conjunction)*
$p \rightarrow q$	*if p, then q (or p only if q) (conditional)*
$p \leftrightarrows q$	*p if and only if q (biconditional)*
$a < b$	*a is less than b*
$a \leq b$	*a is less than or equal to b*
$\|a\|$	*the absolute value of a*
$b\|a$	*b divides a*
gcd	*the greatest common divisor*
lcm	*the least common multiple*
$\sqrt{a}, a^{\frac{1}{2}}$	*the principal (nonnegative) square root of the non-negative real number a*
$\sqrt[m]{a}, a^{\frac{1}{m}}$	*the principal mth root of a*
(a, b)	*the ordered pair whose first member is a and whose second member is b*
$f\colon A \rightarrow B$	*function from A to B*
$p(x)$	*polynomial in x*
i	*an imaginary number such that* $i^2 = {}^-1$
$a + bi$	*a complex number* $(a \in R, b \in R)$
$(G, \odot)$	*a group with binary operator* $\odot$
a'	*the inverse of the element a of a group*
$\begin{pmatrix} 123 \\ 312 \end{pmatrix}$	*a permutation on three symbols*

COLLEGE ALGEBRA

Contents

CHAPTER 3. *Linear Equations and Linear Inequalities in One Variable*

CHAPTER 4. *Functions and Graphs*

CHAPTER 5. *Systems of Linear Equations and Inequalities*

CHAPTER 6. *Quadratic Equations and Factoring*

CHAPTER 1

Sets

1.1 Introduction

The purpose of this section is to present a brief introduction to some of the fundamental terminology in the subject of mathematics. In any discussion, we must *agree* on the definitions of the terms used, or else we must leave certain terms *undefined.* You have probably witnessed heated arguments between friends. Usually such an argument ends where it begins; each person leaves still convinced of his own point of view. Actually, the debaters may have been in almost complete argreement. This apparent paradox has a simple explanation—the two persons did not agree beforehand on the definitions of the words used. Each attached his own special meanings to the words. For example, a United States official may have a definition of the word *democracy* far different from that of a USSR official. Your experience tells you that the two officials probably cannot agree on an issue involving democracy. To avoid ambiguity and contradictions in mathematics, we must agree, in advance, on the definitions of all technical words used or else we must state which words are undefined. The reason for not defining certain mathematical words is that it is impossible to define all mathematical words without permitting our definitions to form circular chains. As simple as this fact may seem, many great mathematicians and philosophers of the past did not realize this. If you look up the definition of an unfamiliar word in a standard dictionary, you discover that this unfamiliar word is defined in terms of other words. If you look up these other words, you find that they are defined in terms of other words. Eventually one of the new words is the original word whose definition you were seeking. For example, we might find the following:

set – group
group – assemblage
assemblage – collection
collection – set

Once we have selected the relatively small number of undefined words in the mathematical system under investigation, we strive to have the definitions obey the following properties:

1. Any definition of a new word must be expressed in terms of the undefined words and/or the previously defined words and common nontechnical English words.
2. Any definition must be consistent with itself and with other definitions.
3. Any definition must be meaningful.
4. Any definition must be expressed in such a manner that it includes all desired cases and excludes all undesired cases.

Just as we begin with certain undefined words, we also begin with certain statements which we assume to be true. These assumed statements are called *postulates*. Other names for the assumed statements are *axioms* and *assumptions*. Among other things, the postulates describe the undefined words. For example, the words *point, line*, and *plane* are left undefined in Euclidean geometry. However, they are described in the postulates. Two of these postulates are: (1) two different points determine exactly one line; (2) three noncollinear points determine a unique plane.

Just as we define the new words in terms of the undefined words and/or the previously defined words, we prove new statements, called *theorems*, from the postulates, definitions, and previously proved theorems. The reasoning process which we employ in proving theorems is controlled by the laws of logic or rules of inference. The laws of logic control the combination of given sentences (called the *hypotheses*) into one or more new sentences (called the *conclusion*).

Since our use of the word *sentence* is different from the English usage, we begin with its definition.

DEFINITION 1. A *sentence* is any declarative statement which is either true or false, but not both true and false.

Thus we see that a sentence must be meaningful and unambiguous. The following are examples of sentences.

Example 1. My mother has blue eyes.

Example 2. All dogs are quadrupeds.

Example 3. Some dogs are quadrupeds.

Example 4. No dogs are quadrupeds.

Example 5. Benjamin Franklin wore glasses.

Example 6. $14 + 2 = 16$.

Example 7. $6 + 3 = 3 + 6$.

Example 8. 5 is equal to 6.

Example 9. All ninth grade algebra students study mathematics and some ninth grade students study English.

Example 10. If John comes home, then his wife will cook dinner.

Although we could refuse to admit that Examples 1, 5, and 10 are sentences on the grounds that one cannot determine, for example, whether the sentence in Example 10 is true until he knows who *John* is, we prefer to agree that any name appearing in a sentence specifies a particular person, object, and so on. In everyday conversation when one of your acquaintances tells you that Jack Jones is ill, you know that he is referring to a particular Jack Jones, in spite of the fact that, in reality, there are numerous persons named *Jack Jones*. Actually, although one would ordinarily say that the sentence of Example 6 is true, it is similar to Examples 1, 5, and 10. When we write "$14 + 2 = 16$," we mean "fourteen plus two is equal to sixteen." The symbols "14," "2," and "16" are similar to the names *John, Ben*, et cetera. They symbolize or represent the numbers just as the name *John* symbolizes or represents the person. You probably have learned that the symbol "14" may represent some number other than fourteen.* However, when we write "$14 + 2 = 16$," you know that we are referring to particular numbers. Thus we agree to visualize a sentence as stated by a specific person at a specific time and place. In this way any possible ambiguities are clarified by the context.

The similar statement "$x + 2 = 16$" is *not* a sentence because the symbol x does not specify a particular number. Until we know that the symbol x specifies a particular number, we cannot determine whether "$x + 2 = 16$" is true or not. For example, if $x = 14$, we can say that

* For example, in base 5 the symbol "14" represents the number *nine*.