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C*-ALGEBRAS AND NUMERICAL ANALYSIS

Roland Hagen
Steffen Roch
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C*-ALGEBRAS AND NUMERICAL ANALYSIS

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Preface

This book is devoted to C^*-algebras as a tool in numerical analysis. Some readers might consider the use of C^*-algebras to study properties of approximation methods as unusual and exotic. We would like to encourage them to read and see for themselves the power of such techniques both for the investigation of very concrete discretization procedures and for establishing the theoretical foundation of numerical analysis. For a general overview of the fruitful interplay between C^*-techniques, concrete operator theory, and numerical analysis and, thus, of the contents of this book, we refer the reader to the Introduction.

The book is adressed to a wide audience. We hope that it proves to be of use both for the student who wants to see applications of functional analysis and to learn numerical analysis, and for the mathematician and the engineer interested in theoretical aspects of numerical analysis.

We wish to express our sincere appreciation to our friends and colleagues, Albrecht Böttcher, Torsten Ehrhardt, Peter Junghanns, and Marko Lindner, who read the bulk of the manuscript very carefully and not only made many corrections but also offered constructive criticism to improve the book substantially. Our students, Michael Ehrenberger and Florian Meyer, did an excellent job in drawing the figures and performing the test calculations. One of the authors (S. R.) was supported by a DFG Heisenberg grant while working on the manuscript. He is grateful to the German Research Foundation for this support, as well as to Bernd Kirstein and Wolfgang Wendland and their staffs for their hospitality during that time. Finally, we are pleased to express our gratitude to the publisher, Marcel Dekker, Inc., and to the mathematics series editor, Prof. Zuhair Nashed, for inviting us to write this monograph and for their careful work on the book.

Roland Hagen
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