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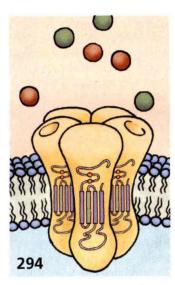
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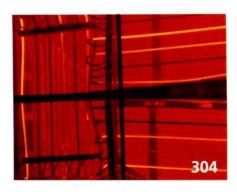
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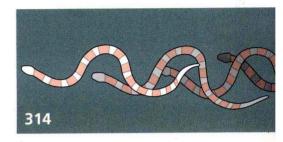


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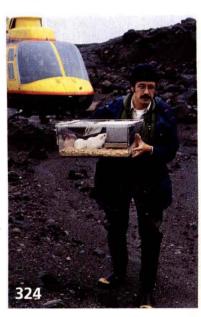
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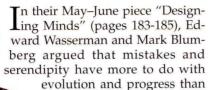
Ressearch proceeded near Mount St. Helens despite many obstacles Douglas W. Larson



THE COVER

In this 1983 photograph, water-quality technician Gail Flory of the U.S. Army Corps of Engineers (on the cover, left) and hydrologist Steven Sumioka of the U.S. Geological Survey, pause while collecting water samples amidst log debris that littered Spirit Lake after Mount St. Helens erupted. In "Science After the Volcano Blew" (pages 324–333), limnologist Douglas W. Larson details multiple challenges he and colleagues encountered while conducting research in that blast zone. As important, Larson displays those difficulties in a series of vivid photographs, most of which he made with a used 35-millimeter Contaflex Super camera he had purchased years before. Says Larson: "The story of Spirit Lake and the effort to study it are inherently visual. Mere words alone were incapable of telling this compelling story. People may argue over the authenticity of charts and tables of scientific data, but the photos are proof of what actually happened. I was often bowled over by what I was seeing."

Science—the Unintended Adventure



does intentional design. As Henry Petroski's editor for some 17 years, I was not new to this thinking; I do have the notion that To Engineer is Human. Still, Wasserman and Blumberg offered a different angle and sharpened

And sure enough, I see it everywhere in the issue you

my focus.

hold in your hands. This time, though, the topical nuance seems to drill in on unintended consequences—none more so than Robert Hargraves and Ralph Moir's "Liquid Fluoride Thorium Reactors" (pages 304-313). It seems that uranium reactors were the technology of choice during World War II and thereafter in part because they were good at producing a byproduct people were looking for from them: fissile material to make bombs. Electricity was a happy offshoot of excess heat. Nearly 60 years later one of our most serious international problems is nuclear proliferation, because uranium reactors make as a byproduct material that can be turned into ... bombs. Turns out that it's not nearly so easy to get those materials from thorium reactors, and the world has lots of thorium. So far, though, India is the only country with an operating thorium reactor, and it remains to be seen if more will be developed. Oh well.

The next candidate at least isn't ironic. When Merck and the Kitasato Institute jointly came up with a preventive and treatment for heartworm in pets and livestock, it was a great boon. Philip A. Rea, Vivian Zhang and Yelena S. Baras tell the story in "Ivermectin and River Blindness" (pages 294-303). Commercialized by Merck, ivermectin went on to achieve sales of \$1 billion per year. But when Merck investigators looked into whether the drug might have human applications, a funny thing happened. A wildly successful pilot test program in West Africa found that ivermectin worked wonders for people suffering from onchocerciasis, a parasitic nematode infection. Only thing was, the patients couldn't afford even \$1 per dose, let alone the \$3 going rate. It was clear that Mectizan (ivermectin for humans) would never be profitable. In the end, Merck did the right thing, using its profits from the veterinary product to fund donation of the Mectizan for humans. You see, Fido really is man's best friend.

Of course, surprises in the lab come as no surprise to anyone who's spent much time in one, and Douglas Larson's lab has been particularly productive in that regard, in large part because his lab has so often been at large. Doug should be no stranger to long-time readers. The limnologist's prose has graced these pages several times, and the current offering, "Science After the Volcano Blew" (pages 324–333), ties many of those stories together. You'll find Larson and his team processing samples outdoors in ash storms, stuck in log jams, applying camouflage that sticks out like a sore thumb, bailing sinking boats while awaiting purchase orders, calling in helicopter medical evacuations and, despite the often long odds, showing ingenuity and doing solid science. Doug proves that the Navy has no corner on adventure; the Army Corps of Engineers also has it aplenty.—David Schoonmaker

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LETTERS TO THE EDITORS

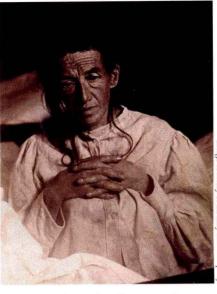
Inspiring Art and Science

To the Editors:

I read Ralf Dahm's article "Finding Alzheimer's Disease" (March-April) with interest for two reasons. First, as a cognitive neuroscientist I have studied the effect of early-stage Alzheimer's disease on memory and on the appreciation of music and art. In addition, I am a member of a community chorus here in Pennsylvania, the Susquehanna Valley Chorale, which recently commissioned and premiered a choral work on the topic of the ailment, called Alzheimer's Stories.

The librettist Herschel Garfein and the composer Robert Cohen used the interview between Dr. Alzheimer and Auguste D. (right), which Dr. Dahm reproduced in his article, as material for the piece's first movement. The dialogue is sung first by soloists. Later the chorus takes up the plaintive refrain of Auguste's "Ich hab mich verloren" (I have lost myself). The setting of this text is hymnlike at first and is then repeated over and over in simpler form later in the movement as the orchestra sounds increasingly dissonant, reminiscent of the disorganization of the mind that accompanies the disease.

I found it interesting that the factual record, seen through the lens of what



we know about the disease today, can be affecting enough to inspire contemporary art. Such art can bring further understanding of this condition to the performers, who become the voice of the patient, and to the audience.

The work's lyrics and audio files of the chorale's performance are online at http://www.robertscohen.com. Video of the performance can be found at http://www.youtube.com.

Andrea Halpern **Bucknell University**

Why Do We Forget?

To the Editors:

Richard Woo's principle point that the coronal streamers dominated discussion of the solar wind, as described in his article "Revealing the True Solar Corona" (May-June 2010), is no doubt correct. However, the spherical nature of the solar corona was asserted and well demonstrated in a 1937 Astrophysical Journal article by Brian O'Brien and colleagues. This article also states that the prominence of the coronal rays is a "well-known property of the eye."

Why did this article not guide later discussions of the solar wind? In the wealth of scientific publications today, only careful bibliographical research would find such an article. How many such gems lie forgotten in our libraries?

Malcolm P. Savedoff University of Rochester

Skip the Memes, Please

To the Editors:

Olli Arjamaa and Timo Vuorisalo wrote a very interesting and informative article, "Gene-Culture Coevolution and Human Diet" (March-April). But rather than confine themselves to easily verifiable historical types of explanation-

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narratives, analyses or interpretations implied from concrete instances and events—they introduce an abstraction: "meme." (I almost said "distraction.")

Like the German philosopher Gottfried Leibniz's "monads," a meme can be anything. How does it aid understanding to explain the familiar with the less familiar? How does it aid understanding to invoke remote causation, where easily accessible historical explanations do the job well enough? Crème brûlée is just crème brûlée; to describe it as a meme seems gratuitous. Had I thought the article a mere meme, I might have avoided reading it. If I had, I'm rather glad to say, I would have been the poorer for it.

José Alfredo Bach San Marcos, TX

Uncertainty Is Allowed

To the Editors:

Although I am very pleased with the scientific perspective shared by Ed-

ILLUSTRATION CREDITS

Computing Science

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Science Observer

Page 290

Barbara Aulicino

Ivermectin and River Blindness

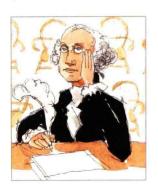
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Barbara Aulicino

Liquid Fluoride Thorium ReactorsFigures 2–6 Tom Dunne

Wiggling Through the World

Figures 2, 3 (*center, right*), 5, 6 (*bottom*), 7 (*left*), 9, 11–13 Tom Dunne



ward Wasserman and Mark Blumberg in "Designing Minds" (May-June), I am also left somewhat bewildered. Wasserman and Blumberg's postulation that "design" is a "mentalist" concept is indeed the consequent conclusion from their analysis. I admire Wasserman's and Blumberg's view that, technically speaking, the anti-religionist Richard Dawkins is out on a scientific limb when he ascribes "purpose" to nearly anything, as in "... and this is why the moth has eyes on its wings." There is still a presumption of "reason" in this kind of thinking and I'm pleased that Wasserman and Blumberg have weeded it out.

Conversely, I am appalled. Although certainly "design" is a "mentalist" concept, it is rather clear that people are capable of "mentality." They design. They imagine and attempt to implement. Sometimes, in their attempts, they fail. Sometimes, unthinkingly, they come across or "discover"—hence, the "Fosbury Flop" (or perhaps Fosbury thought about it ahead of time, engaging in "the mentalist concept" and then implemented his imaginings and therefore designed them).

It appears to me that Wasserman and Blumberg are in the end guilty of doing the very thing they decry in William Paley and in Dawkins's book The Blind Watchmaker. They cook up a simplistic example (a straw man) in the Fosbury Flop and knock it down. Please understand: I am entirely open to the concept that this universe is a massive illusion with nothing more than chaos at its helm. But I am at a loss to explain the apparent "order" that I see every day. I wonder when I will hear from any evolutionary scientist what I often heard from my own father (a lifelong researcher in chemical engineering): "We don't know."

Peter Capell Carnegie Mellon University

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The Widening Gyrus

Charles T. Ambrose

N OLD QUESTION: What accounts for highly intelligent and greatly gifted individuals? Three centuries ago, phrenologists thought that geniuses could be distinguished from criminals by the shapeliness and bumpiness of their skulls. When Charles Darwin's head was considered by a German psychological society, one member declared that Darwin "had the bump of reverence developed enough for ten priests." In 19th-century Europe and America, a number of distinguished academics bequeathed their brains for anatomical study—Gauss, Broca, Gall, Pavlov, Osler and others. Some probably sought to leave behind posthumous confirmation of their own genius. Originally, the anatomists compared gross weight and volume of brains and made what they could of the various lobes and surface convolutions. With the advent of microscopic anatomy, they were able to look for histological differences, yet by 1928 researchers were concluding that nothing in these early studies provided "a basis from which mental abilities [might] be inferred." However, neurophysiologists of that era were beginning to identify specific areas of the brain responsible for general motor function and sensory activity. In recent decades, neurohistologists have developed cytoarchitectonics, enabling them to count neurons and support cells-oligodendrocytes, astrocytes and glial cells-in different areas of the brain. And during the past decade, brain imaging with positron

Concert pianists could be model organisms for studying the physiological basis of intellectual greatness

emission tomography and functional magnetic resonance imaging (fMRI) has allowed noninvasive localization of various functions and responses. Even with these new tools the essential question-whence talent?-remains an enigma.

The most revered modern genius was Albert Einstein, who died at age 76 in 1955 at Princeton. His brain had a circuitous fate; for several years it rested in a jar of formaldehyde in a Kansas closet, was later carried to Berkeley and is now preserved in Hamilton, Ontario. At McMaster University, Einstein's brain was compared with brains of an age-matched male group. It was within normal limits except for the parietal lobes, which were 1 centimeter (15 percent) wider than that of the control group. This region of the brain is responsible for visual-spatial cognition and mathematical thinking, and according to Sandra F. Witelson, presumably achieved its distinctive form early in Einstein's life, based on the understanding at that time of cerebral development. Each of Einstein's posterior parietal lobes consisted of one distinct compartment instead of the usual two separated by the Sylvian fissure. Earlier, at the University of California, Berkeley, Marian Diamond

and coworkers had reported that the isocortex (the outer six layers of gray matter) of Einstein's left parietal lobe (Brodmann area 9) contained 77 percent more glial cells per neuron than brains of 11 normal males aged 47 to 80. This ratio "suggests a response by glial cells to greater neuronal metabolic need ... [and] might reflect the enhanced use of this tissue in the expression of his unusual conceptual powers."

Ranking surely with mathematical geniuses in intellectual stature are people with acute musical ability, such as eminent composers, conductors and concert performers. For great pianists, several functions are involved: hearing/perception/appreciation, memory and performance. Hearing involves the primary auditory cortex, which is confined largely to the anteriomedial region of Heschl's gyrus (anterior transverse gyrus of the temporal lobe). A study published in Nature Neuroscience in 2002 reported that this gyrus is 2.3 times as large and twice as active in the brains of professional musicians as in nonmusicians, suggesting plasticity in human brains expressed under conditions of intense musical training. The histologic nature of that increase has not yet been studied.

In the mid-19th century, Jean Pierre Flourens (1794–1867) maintained that cognitive functions are the integrated activity of the entire brain. And recently, Nobel laureate Eric Kandel and psychiatrist Larry Squire reiterated the point that "Memory is not a unitary faculty of the mind but is composed of multiple systems that have different logic and neuroanatomy." For example, memory involves two systems: short- and long-term memory, which are located in different regions of the brain. Longterm memory consists of two types: explicit/declarative/conscious recall of facts and events, and implicit/nondeclarative/unconscious recall of motor

Charles T. Ambrose earned his M.D. from the Johns Hopkins Medical School and pursued research in cellular immunology at Harvard Medical School and at INSERM in Paris for 14 years. He now teaches pathogenic microbiology at the University of Kentucky. He has published widely on the history of medicine. Address: Dept of Microbiology, College of Medicine, University of Kentucky, Lexington, KY 40536. E-mail: cambros@uky.edu



Lang Lang performs in his highly anticipated Carnegie Hall debut at 21 years old. Recent work is revealing how the acquisition of specific skills is represented by changes in the structure of the brain. What could we discover from studying the brain of this exquisitely skilled virtuoso?

skills, habits and so on. Both types have been localized by various authors in the isocortex of the "upstream" brain regions—specifically, in the frontal and parietal cortices and medial temporal lobes. Complicating the study of memory storage is the assumption that remembering "what" and "how to" are different matters that may be centered in different regions of the neocortex. At present it is unknown to what degree the association areas for specific memories are circumscribed. For example, we don't know whether the remembered experience and knowledge of a Chopin étude resides in the same region of the cortex as a Bach prelude.

Concert pianists not only memorize massive amounts of music, they also

engage complex motor skills, as both hands play different notes and chords, often at high speed and with different tempos and rhythms, while their feet work the piano's pedals. It is reasonable to suppose that certain areas of the motor cortex of concert pianists are very highly developed.

The localization of motor function began when Flourens's French countryman, Marc Dax (1771–1837), reported aphasia in right-handed patients suffering a stroke with right hemiplegia—that is, disability due to some left hemisphere injury. Several decades later this observation was corroborated by Paul Broca (1824–1880), and the motor speech area in the left cerebral hemisphere was eponymized

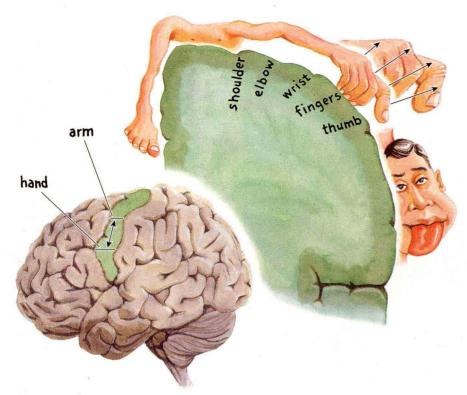
as Broca's area (left inferior frontal gyrus). The localization in the brain of sensory and motor functions engaged early neurophysiologists, but of late they have focused more on localization of memory. Compared with memory and hearing (meaning the entire complex response to auditory stimulation), motor functions may be more localized and possibly easier to investigate. They have been mapped out in discrete projection areas on the cerebral hemisphere's surface, as depicted by the famous motor homunculus devised by the neurosurgeon Wilder Penfield (see page 272).

The process by which fine motor skills are honed may have parallels with how memory storage is increased, hence the preceding digression. The mechanisms proposed for enhanced memory have centered on one of two alternative explanations: existing synapses may change as a result of changes in local gene expression, yielding new proteins and possibly increasing the number of vesicles near the presynaptic membrane, and/or new synapses may be generated (synaptogenesis). In memory studies little attention has thus far been given to a third possibility: neurogenesis. In key motor areas, increased synapses and increased synaptic efficiency may account in part for piano virtuosity, but I suggest that neurogenesis should also be considered.

Plasticity in the Adult Brain

The dogma of neurophysiology up until the 1970s was that generation of new neurons was limited to the period of embryogenesis for most of the brain, with a few exceptions, including the granule cells of the olfactory bulb and hippocampus. After birth, regeneration of neurons was believed to be limited to the peripheral nerves. As it became clear that all cortical areas of the brain exhibit plasticity (changes over time), modulations in function and activity were ascribed to synaptic changes. However, recent studies have suggested that neurogenesis may also be in play. Following are nine reports that describe simple enlargement of the brain or an increase in the volume of specific cortical areas after various stimuli. In the first four reports neurogenesis was not considered by the researchers; in the latter five reports, it plainly occurred.

In the early 1980s, William Greenough and colleagues trained adult rats



This adaptation of Wilder Penfield's motor homunculus pictorially represents the amount of primary motor cortex dedicated to individual body parts. Intense training has been shown to increase cortical representation of the trained body part, for example, in the region associated with the fingers of a skilled musician, shown figuratively here.

on multiple maze patterns and found that afterward, "visual cortex pyramidal neuron dendritic fields were larger" than in controls. Rats trained on complex motor tasks also showed greater "cerebellar cortex thickness" than control rats given "far more physical activity" using devices such as treadmills. According to the authors, the results "strongly implicate changes in the number of synapses in the memory process." They did not pursue a conclusive histological investigation.

In the 1990s, Gregg Recanzone and coworkers trained monkeys "to discriminate between two vibrating stimuli applied to one finger" and after several thousand trials found that "the cortical representation of the trained finger became more than twice as large as the corresponding areas for other fingers."

In 1995, Thomas Elbert used neuroimaging to study right-handed string musicians and found that "the cortical representation of the fingers of the left hand ... was larger than that in controls."

In 2004, Bogdon Draganski and colleagues employed fMRI scans of young volunteers who had mastered over a three-month period "a classic three-ball cascade juggling routine"

and found expansion in gray matter in the mid temporal area and left posterior intraparietal sulcus. Notably, the expansion decreased three months later.

In the following five reports, postnatal neurogenesis was explicitly addressed.

Studies by Fernando Nottebohn in birds may have relevance to neurogenesis in the human brain. In the early 1980s, Nottebohn's lab reported in several papers that in the female canary forebrain (hyperstriatum), the volume of two vocal control nuclei (functional collections of neurons and associated cells) increased markedly during the peak of the singing season, then declined, then increased again during the successive singing season. The song repertoire of the canaries changes each year. The birds were injected with ³Hthymidine, which becomes incorporated in the DNA of replicating cells. In birds that received the label, it was shown that the volume increase in one particular area was accompanied by labeled glial, endothelial and migrating neuronal precursor cells—all of which were interpreted as signs of neurogenesis.

The earliest neurological studies using radioactive labeling were done in

1962 by Joseph Altman, who injected ³H-thymidine into lesions created in the lateral geniculate body of adult rats. He found labeled glial cells, neuroblasts, and a few neurons in or near the lesion area. The presence of labeled neuroblasts was judged to support "a process of neurogenesis" in the area of repair.

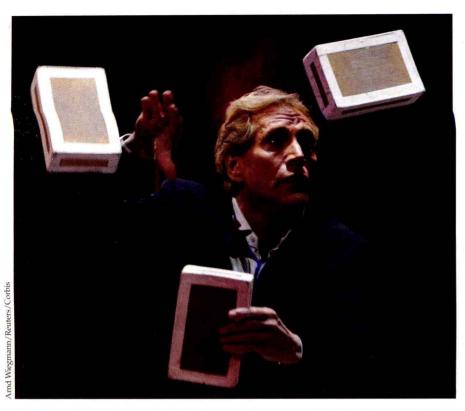
In 1999, Elizabeth Gould and coworkers injected bromodeoxyuridine (BrdU, a synthetic nucleotide analog that, like ³H-thymidine, gets incorporated into DNA in replicating cells) into adult macaques and after one week found labeled mature neurons in the prefrontal, inferior temporal and parietal cortices, indicating that neurons "are added to primate neocortex in adulthood." The authors did not regard the prior studies by Altman and Nottebohn as definitive or as establishing neurogenesis.

Peter Ericksson and colleagues reported in 1998 on the injection of five terminally ill patients with BrdU. They found that after their deaths (from several weeks to two years later), cells in the hippocampal dentate gyrus were labeled with both BrdU and a neuron-specific marker. They interpreted their findings to indicate the "genesis" of new neurons from "dividing progenitor cells" in the gyrus.

Finally, work by Marian Diamond and colleagues indicated that the continued growth of human brain after birth is influenced by nutritional factors and environmental influences. They found that rats given playthings and treadmills ("an enriched environment") develop more glial cells per neuron in the occipital cortex. Other changes noted were "neuronal stroma size, neuronal density, length of dendritic branches, dendritic spine density, length of synapses and glial cell counts."

The Brains of Concert Pianists

It is now recognized clinically that the human brain has remarkable plasticity and potential for restoring lost function. Following localized injuries to the brain, neurological deficits can often be ameliorated by special training. In other words, changes in the brain can be electively produced. Compensating for lost function is one thing, but what would we expect to find in healthy people who perform mentally demanding, fine movements (for example, typists and musicians)? Do they develop discernable and consis-



In a 2004 study, volunteers trained for three months to master a basic juggling routine. Functional MRI confirmed the expansion of gray matter in two localized areas of their brains. It seems quite possible that areas of the motor cortex in concert pianists—products of long, intense training—would be strikingly and characteristically overdeveloped.

tent morphological changes in certain motor areas of their brains?

Pianists were considered good candidates for studies of this sort early on. In the 1920s, Rudolf Klose examined the brain of a young piano prodigy, Goswin Sökeland (1872–1900), but reported (in five densely packed German pages) only the gross morphology—"der Gyrus supramarginalis ist ganz enorm entwickelt," and so on. Today a detailed brain study of highly proficient pianists would examine neuronal topography and other details of fine structure and might disclose additional distinctive features in that ganz enorm gyrus.

One current consensus of neurogenesis is that new neurons may result from the transformation of stem cells and their migration into relevant sites. The physiological basis for these changes has yet to be defined but may involve adjacent accessory cells releasing chemical factors and/or endothelial cells stimulating the growth of new circulatory vessels (angiogenesis).

Circulation in the Brain

The neuropathologist Alfred Meyer referred in a footnote to a work from 50

years before by B. K. Hindze of Moscow, "who had shown that in brains of persons of outstanding ability the arterial supply is more elaborate than in brains from persons of mediocre ability." But this study was "too small to permit definitive conclusions." In 1974 a report described blood flow in the brains of patients with chronic schizophrenia, and another report examined blood flow in normal persons in the hemisphere associated with speech and reading. Both studies used the Xenon-133 method with 32 detectors placed alongside the patient's head. In the latter report the authors stated, "The blood flow of the brain is ultimately regulated by the metabolic activity of the neuronal tissue." A 2008 paper by Fred Wolf and Frank Kirchhoff measured blood flow by fMRI and asserted that "astrocyte activity affects local blood flow." Not reported in these studies is whether increased blood flow occurred via existing capillaries or newly formed ones-the latter arising perhaps in a process analogous to tumor angiogenesis. A reciprocal consideration is whether increased blood flow in an area might stimulate further neuronal or astrocyte development there, just as, in certain solid tumors, increased blood flow allows proliferation of malignant cells.

Michael Chopp and colleagues have recently examined agents that promote neurogenesis and angiogenesis during recovery from strokes induced in animals: "Matrix metalloproteinases expressed in the periinfarct vasculature are chemotactic for neuroblasts migrating from the subventricular zone." Here angiogenesis was monitored by MRI.

Coda

Concert pianists represent a human model of highly integrated motor activity. The primary motor area for the hands and arms is in the precentral gyrus of the frontal lobe. If an fMRI of this area in trained pianists reveals increased blood flow, we may question whether it is due to neocapillaries induced earlier by an angiogenic peptide—similar to the macrophagederived angiogenesis factor or tumor necrosis factor alpha. Angiogenesis has been studied extensively in certain brain tumors (glioblastomas) and recently in cases of stroke-both pathological conditions. This essay suggests a nonpathological function for angiogenesis in the healthy, stimulated brain.

Searching for a talent-linked angiogenic peptide (or peptides) in neurogenesis would be difficult using master pianists as the model. Concert performers are rare and revered and might resent neurochemical probing of their brains. Furthermore, the development of their enhanced motor skills takes place over many years of practice. This long interval might make identifying a putative angiogenic peptide difficult if it were present and active only with the inception of the new capillaries. On the other hand, it seems plausible, based on work such as Nottebohn's birdsong studies mentioned above, that maintaining a high level of pianistic skill requires continuous stimulation by an angiogenic factor for preservation of an enhanced local capillary bed.

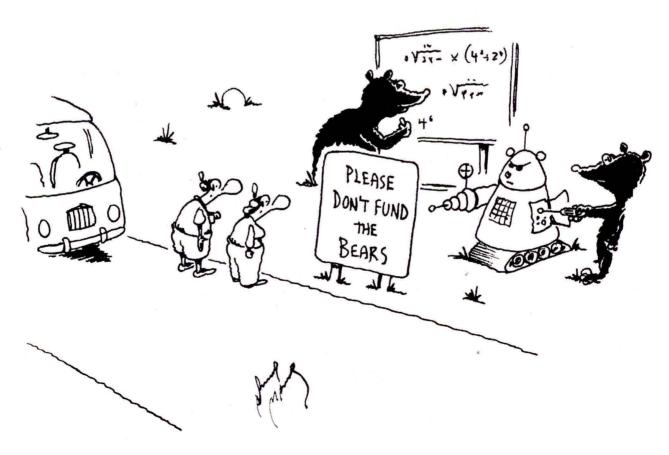
My fascination with this subject developed after attending a private performance by Dr. Paul Bachner, chair of pathology at the University of Kentucky and a gifted concert pianist. It would be interesting, if intrusive and perhaps unwelcomed, to determine if his pianistic skill is linked to high levels of a cerebral angiogenic peptide.

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E Pluribus Confusion

Barry Cipra

very decade the United States body politic undergoes a spasm of a decidedly mathematical nature. It will do so again in the next couple of years. That's when the 2010 census will be used to determine the face of the next Congress, in a computational process called *apportionment*. The computation boils down to figuring out how many seats each state receives in the House of Representatives. But that glosses over a host of subtleties—and with it, a history of political rancor.

It wouldn't seem that hard: You take the census state by state, you settle on the size of your House, and you assign each state a number of seats in proportion to their percentage of the total population. The solution is expressible in a formula that a grade-schooler can grasp: s = Sp/P, where s is the number of seats a state with population p should receive in a House with S seats altogether for a nation with P people in all. Under current law, the formula is even simpler: Since 1964, the size of the House of Representatives has been fixed at 435, so s = 435p/P.

The problem, of course, is that *Sp/P* is rarely a whole number. Apportionment is a matter of turning fractions into integers. You might think the Constitution would have specified a procedure for doing so. It doesn't. The closest it comes to a formula for anything actually turns integers into fractions: Our founding fathers called for the census to count each slave as three-fifths of a person. The Constitution left it up to

Barry Cipra is a freelance mathematics writer based in Northfield, Minnesota. He has been a contributing correspondent for Science magazine and a regular writer for SIAM News, the monthly newsletter of the Society for Industrial and Applied Mathematics. He wrote the first five volumes of What's Happening in the Mathematical Sciences (American Mathematical Society, 1993-2002), and is the author of Misteaks... and How to Find Them Before the Teacher Does: A Calculus Supplement (A.K. Peters, Ltd., 2000). He received a Ph.D. in mathematics from the University of Maryland, College Park. E-mail: bcipra@rconnect.com

There's more than one way to turn census data into congressional seats

Congress itself to decide not only how big to be, but how to dole out the seats. Its only stipulations were that each state get at least one seat, and that there not be more than one representative for each 30,000 people. If you like formulas, that's $1 \le s \le \max(p/30000, 1)$.

The one-per-30,000 ceiling was a serious constraint early on. Indeed, it entered into the argument over the initial size of Congress. Nowadays it's not an issue, with the smallest state, Wyoming, having nearly a half-million residents.

Whole Numbers

Most people, when asked for a reasonable way to turn fractions into integers (and you may wish to stop reading for a moment to see what comes to mind), propose something along the following lines. First, note that each fraction Sp/P—called the state's quota—lies between two consecutive integers, so start by rounding to the nearest integer, with whatever rule you like for handling the unlikely possibility that the fractional part of Sp/P is exactly one-half. If the sum of all these rounded numbers turns out to be exactly S, you're done. If it's less than S, find the states with the largest quotas that got rounded down, and give them an extra seat until the total gets up to S; similarly, if it's more than S, find the states with the smallest quotas that got rounded up, and take seats away from them (subject to the constitutional guarantee that you don't reduce a 1 to a 0).

If you don't like the notion of taking seats away after initially allotting them, an equivalent method is to round everyone down at first, then give an additional seat each to states with the largest fractional parts until you get to *S*. Whichever rounding route you take, this approach is named after Alexander Hamilton, who offered it as the rationale behind the first apportionment bill passed by Congress in 1792 and sent to President Washington for his signature.

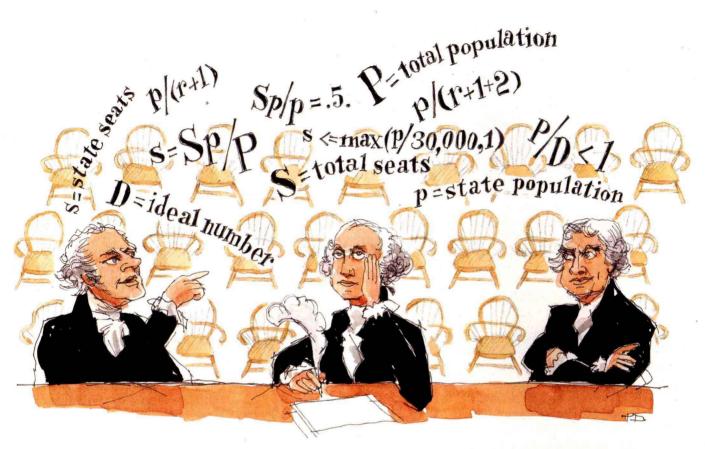
It was the first bill ever vetoed.

The veto came at the urging of Thomas Jefferson, who had his own idea for apportioning seats. Jefferson's approach was to pick an ideal number D for the size of a district—he liked the number 30,000—then divide it into each state's population p, and round the result p/D down to the nearest integer to get s. If the total number of seats misses the target, Jefferson noted, you can go back, modify D either up or down, and try again.

You might worry that Jefferson's method might never find a value of *D* that produces a House of size *S*—or, worse, that two different values of *D* would apportion the *S* seats differently. But neither worry is necessary. There is always a range of values for *D* that produces an apportionment with *S* seats, and you get the exact same apportionment for anything in the range.

One way to see this is to imagine starting with a value of D so unrealistically large that every state's *quotient*, p/D, is less than 1, so nobody gets a seat. Then start reducing D. As you do so, each state's quotient increases, and every so often some state's quotient will reach and then exceed an integer value. If you ignore the unlikely possibility that this happens to two or more states simultaneously (or if you invent a tiebreaking rule to handle such a scenario), you can think of the House as growing in size from 0 all the way up to P, with seats being doled out one at a time.

If, once all the seats are doled out, some states are unrepresented, it may be necessary to take the last few doled-



out seats back and reassign them to the unrepresented states, in order to satisfy the constitutional requirement. Were Jefferson's method currently in use, something like this would have happened in the last apportionment: Based on the 2000 census, neither Wyoming nor Vermont would have received a seat under a "pure" Jeffersonian method; the last two seats would have gone to Minnesota and Pennsylvania.

Making Lists

Such issues notwithstanding, the notion that seats in Congress can be doled out one at a time until the House is full has a certain conceptual appeal. In fact, if you didn't come up with Hamilton's method earlier, it's likely you came up with something along the following lines (and again, you may wish to see what comes to mind before reading further): Line the states up from biggest to smallest and give each one a seat to start with, thus taking care of the constitutional requirement right off the bat. At each step thereafter, keep a list, in decreasing order, of the ratios p/r, r being the current number of seats a state has been assigned. Now give the next seat to the state at the top of the list, decreasing its ratio to p/(r+1), and reinsert the state into the lineup to keep the list in decreasing order. This makes intuitive sense, since p/r is a measure of district size: It is the number of people represented by each of the seats.

If this is what you thought of, then you're in league with John Quincy Adams. Jefferson's method was used for the first five apportionments, but by the 1830s politicians from New England began grousing about being cheated out of seats. Jefferson's method, it turns out, favors large states (of which his home state of Virginia was right at the top in 1790 and 1800) in a peculiar way: It has a penchant for assigning large states more seats than their quota suggests. In 1820, for example, New York and Pennsylvania had quotas of 32.503 and 24.917, respectively, yet wound up with 34 and 26 seats (out of 213 altogether). In 1830, New York was again granted an "extra" seat, receiving 40 despite a quota of 38.593.

Adams proposed a method that has the opposite effect. His approach is identical to Jefferson's in that it divides a test number D into each state's population p, but instead of rounding down, Adams's method rounded up each quotient p/D. When all the math is said and done, it turns out this approach doles out seats in the fashion just described—in other words, according to the largest ratio p/r.

Indeed, the "pure" Jefferson method can also be described this way, except that the ordered list consists of ratios *p*/(*r*+1)—that is, states are prioritized according to how they would stand *after* another seat were added. A nice way to think of this algorithmically is to let the Adams approach run past 435, assigning all the way to 485 seats, and then ignore the first 50 assignments (thus taking away the first 50 seats, which went one to each state).

It's instructive to see how the methods of Adams and Jefferson would have doled out the 105 seats of the first Congress, based on the 1790 census for the 15 states then comprising the United States. The populations are given in the second figure (page 278), along with their Hamiltonian quotas and the apportionment tallies for the two methods. The doling out of the seats appears in the third figure (page 279). The Adams apportionment is rows one through seven in the third figure; the Jefferson apportionment is rows two through eight. Note that Virginia and Pennsylvania appear twice in the last row, whereas Vermont and Delaware appear not at all.

Adams's method tends to short-change the large states. In 1830, it would have given New York (quota 38.593) only 37 seats and Pennsylvania (quota 27.117) only 26. In general, Jefferson's method never gives less than the quota rounded down, and Adams's method never gives more

		apportionment populations	quota for House size of 105	Adams	Jefferson
VA	Virginia	630,560	18.310	18	19
MA	Massachusetts	475,327	13.803	14	14
PA	Pennsylvania	432,879	12.570	12	13
NC	North Carolina	353,523	10.266	10	10
NY	New York	331,589	9.629	10	10
MD	Maryland	278,514	8.088	8	8
СТ	Connecticut	236,841	6.877	7	7
sc	South Carolina	206,236	5.989	6	6
NJ	New Jersey	179,570	5.214	5	5
NH	New Hampshire	141,822	4.118	4	4
VT	Vermont	85,533	2.484	3	2
GA	Georgia	70,835	2.057	2	2
KY	Kentucky	68,705	1.995	2	2
RI	Rhode Island	68,446	1.988	2	2
DE	Delaware	55,540	1.613	2	1

1700

In 1790 the House of Representatives had 105 seats and 15 states made up the Union, shown here in decending order of population. (Northern states are in shades of brown and southern states are in shades of blue.) The quota for each state's seats was not a whole number. The apportionment method of Thomas Jefferson and a later formula proposed by John Quincy Adams differ in the way they round the quota to whole figures, leading to politically significant differences in the composition of the House (*two right columns*).

than the quota rounded up—and for the most part, it's the large states that wander above or below what their quotas would suggest.

The Alabama Paradox

Needless to say, Adams's method was rejected in favor of Jefferson's. But when the 1840 Census rolled around, Congress split the difference, so to speak, and used a method proposed by Daniel Webster. Webster's method also uses a divisor D to produce quotients p/D, but instead of rounding everything up (Adams) or down (Jefferson), Webster rounds to the nearest integer—up if the fractional part is greater than 0.5 and down if it's less. As a doling-out algorithm, Webster's method amounts to keeping an ordered list of ratios $p/(r+\frac{1}{2})$.

Webster's method was only used once. With the 1850 census, Hamilton's method snuck back in, and it stayed on the books for the rest of the century—until it became too much of an embarrassment. Hamilton's method, it turns out, does strange things.

This idiosyncrasy became apparent with the 1880 census. The chief clerk of the Census Office noted something odd when he computed how Hamilton's method apportioned seats for various House sizes ranging from 275 to 350: In a House with 299 seats, Alabama, with a quota of 7.646, got rounded up, but in a House with 300 seats, Alabama's quota of 7.671 got rounded down!

What accounts for the "Alabama paradox" is the fact that at 299, Alabama's fraction, 0.646, barely beat out those of both Illinois (quota 18.640) and Texas (quota 9.640) for the "last" seat (meaning Alabama's fraction was the smallest one to get rounded up). But at 300, Illinois's quota went to 18.702, easily overtaking Alabama's 0.671, whereas Texas's quota squeezed by at 9.672. In effect, Illinois overtook Alabama for the 299th seat, and Texas grabbed the 300th.

Congress dealt with the Alabama paradox by settling on a House size 325, which just happened to be a number where the methods of Hamilton and Webster gave the same apportionment. Much the same took place with the 1890

census: Congress picked a size (356) for which Hamilton and Webster coincided. But in 1900, all hell broke loose.

When Hamilton's method was applied to the 1900 census for a House size ranging from 350 to 400, Maine's allotment bounced back and forth between three and four seats: three seats from 350 to 382, four from 383 to 385, back to three at 386, four again at 387 and 388, down again to three at 389 and 390, and back to four from 391 to 400. As Representative John E. Littlefield of Maine put it, "In Maine comes and out Maine goes.... God help the State of Maine when mathematics reach for her and undertake to strike her down."

Politics being politics, Congress opted for a House of size 386—but jettisoned Hamilton's method in favor of Webster's, which allowed Maine to keep its fourth seat. In 1910, Congress again went with Webster, allocating 433 seats among the 46 states then comprising the union, with a specification that Arizona and New Mexico be given a seat each, if and when they became states. Thus Congress reached its current size of 435.

It would seem that Webster had won the day. Then the mathematicians got involved.

Enter the Mathematicians

In 1911, Joseph Hill, a statistician in the Census Bureau, proposed a new method based on yet another sensible idea: Once you've allotted seats to the various states, check to see whether there is any pair of states for which transferring a seat from one to the other makes things more fair between the two of them. By "more fair" Hill meant that the ratio of the ratios p/s (for the first state in question) and p'/s' (for the second state) should get closer to 1. More precisely, the ratio of the larger ratio to the smaller ratio should get closer to 1. If it does, make the switch and look for another pair of states.

Hill's method was taken up by Harvard mathematician Edward Huntington, who carried out a spirited debate on the matter throughout the 1920s with Walter Willcox, a proponent of Webster's method. The politics of that decade were so contentious that Congress never did get around to reapportioning itself. It simply let stand the apportionment based on the 1910 census—a clear-cut case of ignoring the Constitution. (In 1930, Congress caught a break: The two methods happened to produce identical apportion-