

ACCEPTANCE SAMPLING IN QUALITY CONTROL

EDWARD G. SCHILLING

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NOTE FROM THE SERIES EDITOR

The use of acceptance sampling has grown tremendously since the Dodge and Romig *Sampling Inspection Tables* were first widely distributed in 1944. Throughout this period many people have contributed methods and insight to the subject. One of these contributors is the author of this book, which might better be identified as a compendium of acceptance sampling methods. The American Society for Quality Control has recognized Dr. Schilling's contributions by awarding him the Brumbaugh Award three times, first in 1973 and again in 1978 and 1979. This award is given each year to the author of that paper published in either the *Journal of Quality Technology* or *Quality Progress* which an American Society for Quality Control committee judges has made the largest single contribution to the development of industrial applications of quality control.

Dr. Schilling has been employed both as an educator and as an industrial statistician. This broad experience qualifies him to write this treatise as few others are qualified. The beginner will find much of interest in this work, while the experienced person will also find many interesting items because of its encyclopædic coverage.

I am very pleased with the completeness and clarity exhibited in this book, and it is with great pleasure that I recommend it to others for their use.

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FOREWORD

As the field of quality control enters the 1980s, it is having new responsibilities thrust upon it. The public is demanding products free from defects, and often making these demands in costly court cases. Management is demanding that all departments contribute to technical innovation and cost reduction while still continuing to justify its own costs. The quality control specialist is caught like others in this squeeze between perfect performance and minimum cost. He or she needs all the help that fellow professionals can give, and Edward Schilling's book is a worthy contribution. Written by one of the foremost professionals in the field, it is comprehensive and lucid. It will take its place as a valuable reference source in the quality control specialist's library.

My own first contact with a draft of the book came when I was teaching a quality control course to industrial engineers. Over the semester I found myself turning to this new source for examples, for better explanations of standard concepts, and for the many charts, graphs, and tables, which are often difficult to track down from references. Acceptance sampling is not the whole of statistical quality control, much less the whole of quality control. But Dr. Schilling has stuck to his title and produced a text of second-level depth in this one area, resisting the temptation to include the other parts of quality control to make a "self-contained work." The added depth in this approach makes the text a pleasure for a teacher to own and will make it a pleasure for students to use. This is one book that any student should take into the world where knowledge is applied to the solution of problems.

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PREFACE

The methods of statistical acceptance sampling in business and industry are many and varied. They range from the simple to the profound, from the practical to the infeasible and the naive. This book is intended to present some of the techniques of acceptance quality control that are best known and most practical — in a style that provides sufficient detail for the novice, while including enough theoretical background and reference material to satisfy the more discriminating and knowledgeable reader. The demands of such a goal have made it necessary to omit many worthwhile approaches; however, it is hoped the student of acceptance sampling will find sufficient material herein to form a basis for further explorations of the literature and methods of the field.

While the prime goal is the straightforward presentation of methods for practical application in industry, sufficient theoretical material is included to allow the book to be used as a college level text for courses in acceptance sampling at a junior, senior, or graduate level. Proofs of the material presented for classroom use will be found in the references cited. It is assumed, however, that the reader has some familiarity with statistical quality control procedures at least at the level of Irving W. Burr's *Statistical Quality Control Methods* (Marcel Dekker, Inc., New York, 1976). Thus, an acceptance sampling course is a natural sequel to a survey course at the level suggested.

The text begins with a fundamental discussion of the probability theory necessary for an understanding of the procedures of acceptance sampling. Individual sampling plans are then presented in increasing complexity for use in the inspection of single lots. There follows a discussion of schemes which may be applied to the more common situation of a stream of lots from a steady supplier. Finally, specific applications are treated in

the areas of compliance sampling and reliability. The last chapter is concerned with the administration of acceptance control and, as such, is intended as a guide to the user of what sampling plan to use (and when). Readers having some familiarity with acceptance sampling may wish to read the last chapter first, to put into context the methods presented.

This book views acceptance quality control as an integral and necessary part of a total quality control system. As such, it stands with statistical process quality control as a bulwark against poor quality product, whose foundations are rooted deep in mathematics but whose ramparts are held only by the integrity and competence of its champions in the heat of confrontation.

It is fitting that this book on acceptance sampling should begin with the name of Harold F. Dodge. His contributions have been chronicled and are represented in the Dodge Memorial Issue of the *Journal of Quality Technology* (Vol. 9, No. 3, July 1977). Professor Dodge, as a member of that small band of quality control pioneers at the Bell Telephone Laboratories of the Western Electric Company, is considered by some to be the father of acceptance sampling as a statistical science. Certainly, he nurtured it, lived with it, and followed its development from infancy, through adolescence, and on into maturity. In no small way he did the same for the author's interest in the field, as his professor and his friend.

Edward G. Schilling

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Books are not made — they grow. It is impossible to acknowledge all the help and support which has come from friends and associates in the development and construction of the present volume. A few may be singled out not only for their individual contributions, but also as a sample of those yet unnamed. In particular, I wish to thank Carl Mentch for suggesting the possibility of such an undertaking in September of 1965 and for his unflagging encouragement and help since that time. My thanks also go to Mrs. Lucille I. Johnson, whose technical and editorial assistance helped to bring concept into reality. I must also mention Dr. Lloyd S. Nelson for his continued interest and suggestions, and Dan J. Sommers and Professor Emil Jebe for their constructive comments and theoretical insight. Certainly, my appreciation goes to Dr. Donald P. Petarra, Dr. James R. Donnalley and Dr. Pieter J. von Herrmann of the General Electric Lighting Research and Technical Services Operation for their encouragement and support throughout.

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Finally, these debts of gratitude are in terms of time and talent. How much more the debt to my wife, Jean, and to my daughters, Elizabeth and Kathryn, who are as much a part of this book as the author himself.

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