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ALGEBRAIC GEOMETRY

ARCATA 1974

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PREFACE

This volume contains the proceedings of the Summer Institute in Algebraic Geometry held in July and August 1974 in Arcata, California. Some details of the organization and activities of the institute will be found in the report following.

In preparing this volume, we have attempted to adhere to the same principles which guided the organization of the summer institute: we hope to make algebraic geometry accessible to a wider audience, and we hope to dispel some of the confusion and mystery which such a rapidly changing and complex field has inspired in the uninitiate.

To this end, we have included the texts of almost all the expository lectures series, and those seminar talks which were of a sufficiently broad nature to serve as introductions to their respective areas. This volume should therefore provide orientation to the newcomer or the specialist exploring new fields, by surveying the present state of the art, and giving references for further study.

Two papers originally intended for publication here have outgrown this volume. E. Brieskorn's much appreciated lectures on *Special singularities–resolution, deformation and monodromy* will be published later as a separate monograph. P. Deligne's lectures *Inputs of étale cohomology* are being written up jointly with J.-F. Boutot, and will appear in a companion volume to SGA5. While we have lost these two papers, we are fortunate to be able to include a survey article, not presented at Arcata, *Classification and embedding of surfaces* by E. Bombieri and D. Husemoller.

ROBIN HARTSHORNE
BERKELEY, CALIFORNIA
FEBRUARY 21, 1975

REPORT ON THE SUMMER INSTITUTE

The American Mathematical Society held its twenty-first Summer Research Institute at Humboldt State University, Arcata, California, from July 29 to August 16, 1974. "Algebraic Geometry" was selected as the topic. Members of the Committee on Summer Institutes at the time were Louis Auslander, Richard E. Bellman, William Browder (chairman), Louis Nirenberg, Walter Rudin, and John T. Tate. The institute was supported by a grant from the National Science Foundation. The Organizing Committee for the institute consisted of Michael Artin, Phillip A. Griffiths, Robin Hartshorne, Heisuke Hironaka, Nicholas Katz, and David Mumford (chairman).

The Institute marked the 10th anniversary of the first Summer Institute to be devoted exclusively to algebraic geometry: the 1964 Woods Hole Institute; and the 20th anniversary of the 1954 Boulder Summer Institute in several complex variables and algebraic geometry. The subject has grown immensely in this period, and thanks to the efforts of the fathers of this modern period of growth—Zariski, Weil and Grothendieck—seems to have attained a certain maturity. All the basic foundational work needed to put the subject on a modern axiomatic footing with sufficient generality to encompass characteristic p and mixed characteristic cases as well as the traditional complex case seems to have been done. A large proportion of the work of the Italian school has been relearned and assimilated. Our hope is that with such ample preparation, our generation will be fortunate enough to penetrate deeper into the nature and structure of algebraic varieties.

Program

Although the central subject of the institute was algebraic geometry, because of the broad overlap of this field with many neighboring fields, the program expanded to express the interests of those present in arithmetic problems, analytic geometry, commutative algebra, K-theory (and to a lesser extent, algebraic groups) as well. The committee attempted to plan about half the program in advance, by soliciting specific lectures on specific topics, and to let the remaining half develop spontaneously at the meeting. In particular, they invited ten mathematicians to present series of expository talks on particular areas of algebraic geometry and encouraged them to put together a survey of these areas including not only their own particular specialty but the area as a whole. As far as the committee could tell, all the speakers involved seemed to enjoy the challenge of communicating the central ideas of these areas in a relatively short space of time and did an excellent job. The committee also planned five seminars in advance, appointed chairmen for these and asked them to approach the first few speakers in advance. At Arcata, ten more seminars were organized spontaneously, and although many seminars had to meet simultaneously, most people were able to attend all of the seminars in which they had a very strong interest.

Lecture Series

Special singularities–resolution, deformation, and monodromy by E. Brieskorn (5 lectures)

**Some transcendental aspects of algebraic geometry* by M. Cornalba and P. Griffiths (3 lectures each)

Inputs of étale cohomology by P. Deligne (6 lectures)

**Serre problem and homological methods in commutative algebra* by D. Eisenbud (3 lectures)

**Equivalence relations on algebraic cycles, and subvarieties of small codimension* by R. Hartshorne (4 lectures)

**Triangulation of algebraic sets* by H. Hironaka (1 lecture)

**Introduction to resolution of singularities* by J. Lipman (3 lectures)

**Eigenvalues of Frobenius acting on algebraic varieties over finite fields* by B. Mazur (2 lectures)

Problems on l-adic representations by J.-P. Serre (3 lectures)

**Theory of moduli* by C. S. Seshadri (4 lectures)

Seminar Series

(1) Proof of the Weil conjectures and the hard Lefschetz theorem (Chairman, M. Artin)

Beginning of the proof of Weil conjectures by J. Milne

Continuation of proof: Lefschetz pencils by M. Artin

Kazdan and Margulis theorem by M. Artin

Chebotarev density theorem and proof of Deligne's Main Lemma by S. Bloch

* { *Deligne's proof of hard Lefschetz theorem.* I by W. Messing

Hard Lefschetz theorem. II: *group theoretic reductions* by W. Messing

Hard Lefschetz theorem. III: *the Hadamard-de la Vallée Poussin argument* by W. Messing }

(2) Varieties of low codimension (Chairman, R. Hartshorne)

Local cohomological dimension of algebraic varieties (*Ogus thesis*) by L. Szpiro

Conditions for embedding varieties in projective space (*work of Holme*) by R. Speiser

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* denotes a paper in this volume

**Homotopy groups of projective varieties (work of Larsen)* by W. Barth
Gorenstein ideals in codimension 3 by G. Evans
Rings of invariants are Cohen-Macaulay (work of Hochster and Roberts) by A. Ogus
**Unique factorization in complete local rings, etc.* by J. Lipman
Vector bundles on projective spaces by W. Barth

(3) Classification questions and special varieties (Chairman, D. Mumford)
Introduction to Enriques' classification by D. Mumford
Hilbert modular surfaces. I by F. Hirzebruch
Hilbert modular surfaces. II by F. Hirzebruch
Kodaira dimension and classification in higher dimensions by K. Ueno
**Matsusaka's big theorem* by D. Lieberman
A finiteness theorem for curves over function fields (Parshin and Arakelov) by F. Oort
Surfaces of general type by A. Van de Ven

(4) Topics in analytic algebraic geometry (Chairman, P. Griffiths)
Toledo-Tong proof of Riemann-Roch by J. King (3 lectures)
Schmid's difficult theorem by J. Carlson (4 lectures)

(5) Toroidal embeddings (Chairman, P. Wagreich)
Varieties with torus actions by T. Oda
**Varieties with commutative group action* by P. Wagreich
Varieties with algebraic vector fields by J. Carrell

(6) deRham and crystalline cohomology (Chairman, N. Katz)
**Differentials of the first, second and third kinds* by W. Messing
Gauss-Manin connection by D. Lieberman (2 hours)
Differential equations on algebraic varieties ... by A. Ogus (2 hours)
**Report on crystalline cohomology: What's true!* by L. Illusie (2 hours)
**The slopes of Frobenius.* I, II by P. Berthelot (2 hours)
Report on flat duality by J. Milne
Report on Mazur-Messing by S. Bloch

(7) Singularities, equisingularity (Chairman, H. Hironaka)
**Introduction to equisingularity problems* by B. Teissier
Polar curves of plane curve singularities by M. Merle
**Topological use of polar curves* by Lê Dũng Tráng
Singularities with group actions by P. Wagreich
**A survey of knot theoretic invariants of singularities* by A. Durfee
Equisingular deformations by J. Wahl
Subanalytic chains, integration and intersections by P. Dolbeault

Rigid singularities and nonsmoothable singularities by H. Pinkham
More on resolution of singularities by H. Hironaka

(8) Recent work on compactification of moduli (Chairman, M. Rapoport)
Compactification of tube domains by D. Mumford
Regularity of automorphic forms at the cusps by Y.-S. Tai
A 'good' compactification of the Siegel moduli space by Y. Namikawa
Seshadri and Oda's compactification of the Picard variety of a singular curve by M. Rapoport
Stable vector bundles on a degenerating family of curves by D. Gieseker

(9) Arithmetic (Chairmen, J.-P. Serre and J. Tate)
Modular forms of weight 1 by J. Tate
**p-adic l-functions via moduli (Hurwitz case)* by N. Katz
**p-adic l-functions via moduli (Siegel case)* by K. Ribet
The Mordell conjecture for the modular curves $X_0(N)$ and $X_1(N)$ over Q by B. Mazur
Problems on l-adic representations by J.-P. Serre
Representations and nonabelian class field theory by P. Deligne
Formal groups and their division points by J.-M. Fontaine

(10) K-theory (Chairmen, H. Bass and S. Gersten)
Report on SGA6 by L. Illusie
**Riemann-Roch theorem for varieties with singularities* by W. Fulton
Finite generation of K_i of curves over finite fields (Quillen) by H. Bass
Algebraic cycles by S. Bloch
Higher K-groups of finite fields by E. Friedlander
Vector bundles on affine surfaces by M. P. Murthy
Higher regulators and zeta functions by A. Borel

(11) Special examples of intermediate Jacobians, etc. (Chairmen, H. Clemens and A. Landman)
Prym varieties associated to plane curves of odd degree by H. Clemens
Intersection of two quadrics in $\mathbf{P}_{2n+1}$ by A. Landman
Cubics containing a linear space of codimension two by H. Clemens
Prym varieties associated to cubic threefolds (algebraic theory) by J. Murre
Intersection of two quadrics as a moduli space of vector bundles by M. S. Narasimhan
A survey of moduli of vector bundles on curves by M. S. Narasimhan

(12) Algebraic groups (Chairman, A. Fauntleroy)
**Survey of representation theory.* I, II by A. Borel

Mumford's conjecture by C. S. Seshadri
Picard groups of algebraic groups by B. Iversen

(13) Weierstrass points (Chairman, R. Lax)
The number of Weierstrass points of a line bundle by J. Hubbard
Existence of Weierstrass points and deformations of curves by H. Pinkham
Arbarello's thesis (Weierstrass points and moduli of curves) by R. Lax
Weierstrass points of forms by A. Iarrobino

(14) Complex manifolds (Chairman, K. Ueno)
Surfaces of class VII_0 by M. Inoue
On deformations of quintic surfaces by E. Horikawa

(15) Deformation of complex analytic spaces (Chairman, A. Douady)
Deformation of complex analytic spaces by A. Douady (3 lectures)

Summary

The broad areas emphasized by this summer institute can be summarized as follows:

Étale cohomology and the Weil conjectures. Because of Deligne's recent and spectacular proof of the Weil conjectures for the absolute values of the Frobenius acting on the étale cohomology of a variety over a finite field, a great deal of interest focused on seeing not only the details of his proof, but on learning thoroughly the techniques of étale cohomology needed for this proof. Deligne talked for the first seven days, each day on the étale cohomology in general; this was followed by Artin's seminar which discussed his proof both of the Weil conjectures and the hard Lefschetz theorem.

Singularities. Many lectures concerned singularities from various points of view. Lipman discussed various approaches to proving resolution of singularities; Brieskorn discussed the topology of singularities and the tie-in with the work of the Thom school on unfoldings as well as many special types of singularities. Hironaka gave one morning talk explaining his beautiful proof of the triangulability of varieties; Hironaka led a seminar covering many more topics such as equisingularity, rigidity of singularities, and singularities with group action.

Cycles, commutative algebra, K-theory. This is a broad area united to some extent by the theme of wanting to understand the geometry of cycles of codimension greater than one, but very diverse in its methods. Hartshorne explained recent progress on the structure of the Chow ring and desingularizing cycles mod rational equivalence; Eisenbud lectured on developments in commutative algebra, e.g. on Serre's conjecture, on the intersection problem and on the structure of resolutions. Hartshorne led a seminar centered on varieties of small codimension but covering related topics too; Bass led a seminar in *K*-theory proper, which at several points tied in with the Chow ring.

Analytic geometry. Analytic and algebraic geometry are, of course, inseparable as was amply demonstrated by the survey talks of Cornalba and Griffiths. Griffiths led a seminar exploring particularly the recent Toledo-Tong proof of Riemann-Roch and Schmid's deep work on degeneration of Hodge structures. Ueno and Douady both ran seminars in the last week concerning classification of complex manifolds and deformations of complex analytic spaces respectively.

Moduli. Although closely tied at points to the theory of deformations of singularities and to the theory of variation of Hodge structure, the theory of moduli proper was the topic of Seshadri's survey talks. Mumford ran a seminar on classification questions related to moduli. Clemens ran a seminar on cubic three-folds, their intermediate Jacobians and rationality, and on the moduli space of vector bundles. Rapoport ran a seminar on the compactification of various moduli spaces, which tied in with the seminar of Wagreich on torus actions and toroidal embeddings. Lax ran a seminar on Weierstrass points of curves which ties in very closely with moduli problems for curves.

Number theory. As with analytic geometry, recent developments have tied number theory and algebraic geometry very intimately together. Serre gave general lectures on l-adic representations. Katz led a seminar on deRham and crystalline cohomology which tied together the analytic deRham approach to cohomology with the p-adic absolute values of Frobenius with crystalline sheaves (characterized by their ability to grow in a suitable medium and their rigidity!) acting as go between.

Location

The institute was held at Humboldt State University in Arcata, California; the university administration provided excellent support services. The participants lived almost entirely in residence halls overlooking the Jolly Giant Commons and immediately next to a beautiful redwood forest. Except for a bit of fog, the site was perfect and its remoteness from the distractions of civilization is believed to have promoted concentration on mathematics.

Two-hundred seventy mathematicians registered for the institute. Forty-nine were accompanied by their families and eighty-four participants were from foreign countries.

Saturday and Sunday excursions were organized to the Redwood National Park and Trinity Alps, where participants and their families had the opportunity to hike their choice of well-marked trails. Wine and cheese tasting parties were held on two evenings. On one occasion the group was transported to a picnic site on the banks of the Mad River. Some hardy individuals sharpened their appetites by a swim in the river before enjoying the steak cookout that was the highlight of the evening.

DAVID MUMFORD

CONTENTS

Preface v

Report on the Summer Institute ix

Part I—Lecture Series

Some transcendental aspects of algebraic geometry 3
MAURIZIO CORNALBA and PHILLIP A. GRIFFITHS

Some directions of recent progress in commutative algebra 111
DAVID EISENBUD

Equivalence relations on algebraic cycles and Subvarieties of small codimension 129
ROBIN HARTSHORNE

Triangulations of algebraic sets 165
HEISUKE HIRONAKA

Introduction to resolution of singularities 187
JOSEPH LIPMAN

Eigenvalues of Frobenius acting on algebraic varieties over finite fields 231
B. MAZUR

Theory of moduli 263
C. S. SESHADRI

Part II—Seminar Talks

Larsen's theorem on the homotopy groups of projective manifolds of small embedding codimension 307
WOLF BARTH

Slopes of Frobenius in crystalline cohomology 315
PIERRE BERTHELOT

Classification and embeddings of surfaces 329
ENRICO BOMBIERI and DALE HUSEMOLLER

Linear representations of semi-simple algebraic groups 421
ARMAND BOREL

Knot invariants of singularities 441
ALAN H. DURFEE

Riemann-Roch for singular varieties 449
WILLIAM FULTON
Report on crystalline cohomology 459
LUC ILLUSIE
p-adic L-functions via moduli of elliptic curves 479
NICHOLAS M. KATZ
Topological use of polar curves 507
LÊ DŨNG TRÁNG
Matsusaka's big theorem 513
D. LIEBERMAN and D. MUMFORD
Unique factorization in complete local rings 531
JOSEPH LIPMAN
Differentials of the first, second and third kinds 547
WILLIAM MESSING
Short sketch of Deligne's proof of the hard Lefschetz theorem 563
WILLIAM MESSING
p-adic interpolation via Hilbert modular forms 581
KENNETH A. RIBET
Introduction to equisingularity problems 593
B. TEISSIER
Algebraic varieties with group action 633
PHILIP WAGREICH

PART I: LECTURE SERIES

Proceedings of Symposia in Pure Mathematics
Volume 29, 1975

SOME TRANSCENDENTAL ASPECTS OF ALGEBRAIC GEOMETRY

Maurizio Cornalba and Phillip A. Griffiths[1]

TABLE OF CONTENTS

0.	Introductory remarks	3
1.	Generalities on complex manifolds	7
	Appendix to 1: Proof of the Hodge theorem	15
2.	Riemann-Roch and fixed point formulae	29
3.	Kähler metrics and Hodge theory	41
4.	Extensions of Hodge theory	50
5.	Hermitian differential geometry	63
	Appendix to 5: Proof of Crofton's formula and the Nevanlinna inequality	72
6.	Curvature and holomorphic mappings	75
7.	Proofs of some results on variation of Hodge structure	82
	Appendix to 7: On Schmid's second theorem	95

0. INTRODUCTORY REMARKS

The purpose of these seven lectures is to discuss some transcendental aspects of algebraic geometry. Historically, a great deal of the subject was initially developed by analytical and topological methods. This was probably due to the origins of much of algebraic geometry as a branch of complex function theory (Gauss, Abel, Jacobi, Riemann, Weierstrass,

AMS (MOS) subject classifications (1970). Primary 14C30, 14D05, 22E15, 30A42, 32C10, 32C25, 32C30, 32C35, 32G05, 32G20, 32H20, 32H25, 32J25, 32L10, 32M10, 32M15, 32G13, 35D05, 35D10, 35N15, 53B35, 53C30, 53C55, 53C65, 58C10, 58G05, 58G10; Secondary 14F05, 14F10, 14F25, 14H15, 14M15, 30A70, 14F35.

[1] Research partially supported by National Science Foundation Grant GP 38886.

Poincaré, Picard, etc.). Another possible reason is that the intimate relationship between algebraic geometry and topology is more visible in the complex case (Lefschetz, and more recently the Hirzebruch-Riemann-Roch formula). Finally, the beautiful local and global methods of differential geometry are available in the complex case (Hodge, de Rham, Chern, Kodaira, etc.).

During the last 100 years, and especially in the last 40 years, much of the theory which was initially discovered by analytical methods has, quite properly, been put in a purely algebraic setting (the foundations, Torelli for curves, Riemann-Roch, etc.). Moreover, longstanding problems have been proved by algebraic methods (resolution of singularities). Finally, due primarily to the input from arithmetic, new and striking results over the complex numbers have been suggested and sometimes proved by algebra (Deligne's theory of mixed Hodge structures, the Tate conjectures, etc.). However, because of the fundamental nature of the complex numbers, there remains a transcendental aspect of algebraic geometry which is both essential to an understanding of the subject and quite beautiful for its own sake. In these lectures, we hope to focus on some facets of this theory.

Specifically, we shall concentrate on the following topics as illustrating transcendental algebraic geometry:

a) The _Hirzebruch-Riemann-Roch formula_ for compact, complex manifolds;

b) _Hodge theory_ for a single compact Kähler manifold, and the related _vanishing theorems_ for cohomology, theory of _mixed Hodge structures_, and _homotopy type_ of Kähler manifolds;

c) _variation of Hodge structure_ culminating in the recent work of Schmid; and

d) the global theory of transcendental holomorphic mappings (_Nevanlinna theory_) viewed as non-compact algebraic geometry.

The Hirzebruch-Riemann-Roch formula is of course well-known and has a purely algebraic proof in much stronger form. However, it is a basic result first proved by transcendental methods and has inspired a great deal of mathematics over the last 20 years. Moreover, there has recently been given an "elementary" proof by Toledo and Tong, one in which the local and global properties of the $\bar{\partial}$-operator are brought nicely into focus, and so this proof will be discussed in the second lecture. The complete argument will be presented in the analysis seminar.

Hodge theory for a compact Kähler manifold is again a subject which has been around for quite some time. However, we have deemed it worthwhile to sketch the theory in some detail, emphasizing Chern's conceptual explanation of the plethora of Kähler identities. As applications, we have given Le Potier's recent extension of the Kodaira-Nakano vanishing theorem to vector bundles, and a brief account of Deligne's theory of mixed Hodge structures and the homotopy type of Kähler varieties. Finally, in the belief that many, if not most, algebraic geometers are aware of the formal aspect of Hodge theory but have not been through the grubby analysis, we have (in the appendix to lecture one) given an account of the underlying analysis of the Laplacian which hopefully may appeal to tastes of the algebraists.

The main thrust of these lectures will be to discuss variation of Hodge structure leading up to the recent work of Schmid. Here the methods of complex analysis, Hermitian differential geometry, and Lie group theory blend together to maximally illustrate the flavor of transcendental algebraic geometry. The complete proofs of most of the main results will be covered between the lectures and the accompanying analysis seminar.

We shall also discuss some "non-compact" algebraic geometry. For example, Picard's theorem and its subsequent refinement, the beautiful value distribution theory of R. Nevanlinna, appear naturally as transcendental analogues of the fundamental theorem of algebra. The extension of this theory to transcendental holomorphic curves in $\mathbb{P}^n$ is based on the non-compact Plücker formulae of H. and J. Weyl. Although it is not yet established, one may adopt the philosophy that a global result concerning complex algebraic varieties is not properly understood unless one has analogous results for non-compact manifolds, and this is to some extent the viewpoint we shall take. It is on non-compact varieties that the full richness of the larger class of generally transcendental analytic functions and holomorphic mappings can be perhaps best exploited. An example of this is the consequence of a theorem of Grauert that all of the rational, even dimensional homology on a smooth, affine algebraic variety is representable