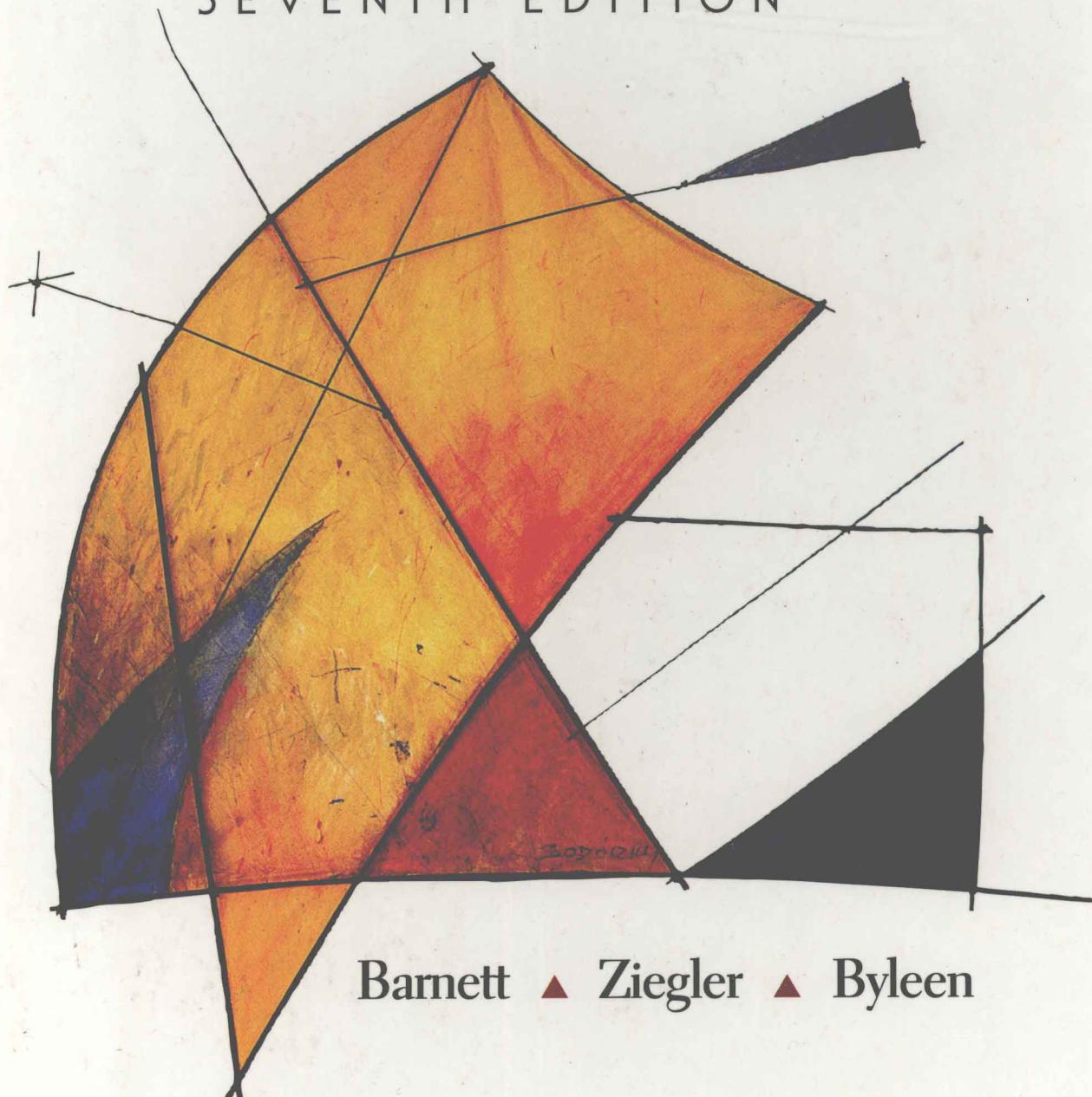


Analytic Trigonometry

with APPLICATIONS

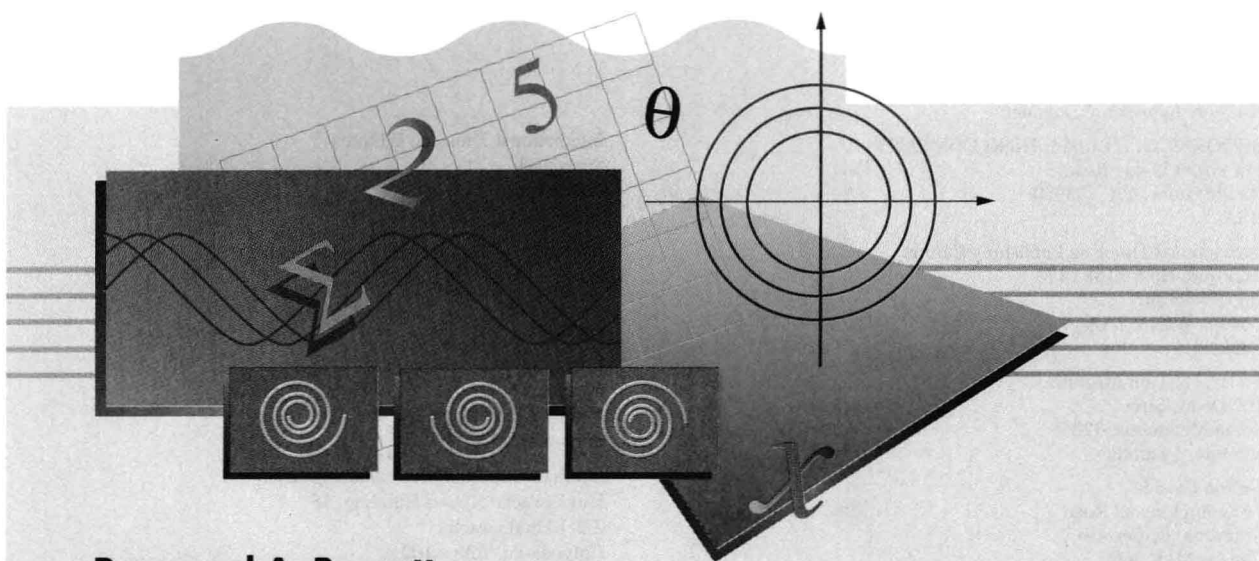
SEVENTH EDITION



Barnett ▲ Ziegler ▲ Byleen

ANALYTIC TRIGONOMETRY

with Applications SEVENTH EDITION



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Dedicated to the memory of Don Dellen

Preface

The seventh edition of *Analytic Trigonometry with Applications* has benefited from the generous response of the many users of the earlier editions. Prerequisites for the book are $1\frac{1}{2}$ –2 years of high school algebra and 1 year of high school geometry or their equivalents. Great care has been taken to produce a book that students can actually read, understand, and enjoy.

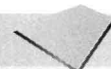
■ Important Features Retained from the Sixth Edition

- The focus of this book is on student comprehension. An **informal style** is used for exposition, definitions, and theorems. Precision, however, is not compromised.
- To gain reader interest quickly, the text moves directly into trigonometric concepts and applications. **Review material** from prerequisite courses is either integrated in certain developments (particularly in Chapters 4 and 5) or can be found in the appendixes. This material can be reviewed as needed by the student or taught in class by an instructor.
- Concept development proceeds from the **concrete to the abstract**. **Trigonometric functions** are defined first in terms of angle domains using degree and radian measure side-by-side, and then in terms of real number domains. All of this is done early in the book and is reinforced throughout. By the end of the course, students should be comfortable with all three modes.
- Almost every concept is illustrated by an **example** followed by a **matching problem** (with answers given near the end of each section) to encourage an active rather than passive involvement in the learning process.
- There are enough relevant and interesting **applications** from diverse fields to convince even the most skeptical student that trigonometry is really useful. Many developments are motivated by interesting and relevant real-world applications. These applications are distributed uniformly throughout the book.
- The book includes **more than 2,000 carefully selected and graded problems**. The problems in most exercise sets are divided into A, B, and C groupings. The A problems are easy and routine, the B problems are more challenging but still emphasize mechanics, and the C problems are a mixture of difficult mechanics and theory. In short, the text is designed so that an average

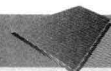
or struggling student will be able to experience success and a very capable student will be challenged.

- **Answers** to most of the odd-numbered problems and all chapter and cumulative review exercises are included at the end of the book. Most of the even-numbered answers can be found in the *Instructor's Resource Manual*.
- **Cautions** alerting students to potential problem areas have been inserted where appropriate (see, for example, Sections 1.1, 2.3, and 4.2).
- **Dashed “think boxes”** are used to indicate steps that are usually performed mentally after a concept or procedure is understood (see, for example, Sections 1.1, 2.4, and 6.5).
- **Chapter reviews** are included at the end of each chapter, and **cumulative reviews** are included after Chapters 3, 5, and 7.
- **Formulas and symbols** (keyed to sections in which they are first introduced) and the **metric system** are summarized inside the front and back covers of the book for convenient reference.
- The content of the text **satisfies the requirements for many succeeding courses**, including calculus, analytic geometry, physics, and applied mathematics courses.
- **Pedagogical use of color:** Color is used not only to make the text more attractive, but more importantly it is used functionally to improve communications. For example, color is used in:
 1. Commentary that accompanies a solution process (see, for example, Sections 1.3, 4.2, and 5.3).
 2. Graphing to visually separate the various parts of a graph (especially in Chapters 3 and 7).
 3. Boxed highlighted material to distinguish among Assumptions/Definitions, Theorems, and Strategies/Processes (see, for example, Sections 1.3, 4.2, and 6.5).

ASSUMPTIONS/DEFINITIONS



STRATEGIES/PROCESSES



■ New Features in the Seventh Edition

The impact of **mathematics reform** movements cannot be ignored. Curriculum and textbook changes resulting from mathematics reform include increased use of technology, cooperative learning, exploration and discovery, writing, and multiple interpretations of mathematical concepts and results (numeric, graphic, verbal, and symbolic). The following new text features were added to accommodate these reform changes.



- A **graphing calculator or utility** is not required to use this book; nevertheless, the use of a graphing calculator or utility will add significantly to the understanding of certain topics. For instructors who wish to require the use of a graphing utility with this book, there is more than enough material to justify this requirement. The expanded graphing utility material is interspersed in context throughout the book (see, for example, Section 3.1, Problems 21–26, and Section 5.4). All material requiring the use of a graphing calculator is identified by the calculator icon shown in the margin, and can be omitted without loss of continuity.
- Each section contains from one to four **Explore–Discuss** boxes that anticipate certain developments (for example, Explore–Discuss 1, Section 1.2), expand on developments (for example, Explore–Discuss 1, Section 1.1), or tie several developments together (for example, Explore–Discuss 2, Section 1.1). The Explore–Discuss material can be used for written student responses, in-class discussions, or as group activities. A comprehensive **Chapter Group Activity**, which ties together and extends several chapter concepts, has been added to the end of each chapter. Many problems and applications in the exercise sets can also be used for group activities (for example, Problems 27–30 in Exercise 1.4, Problems 45–50 in Exercise 3.3, and Problem 71 in Exercise 5.1). Any Explore–Discuss box or Chapter Group Activity may be omitted if time is of concern. Sketched solutions for the Explore–Discuss material and the Chapter Group Activities can be found in the *Instructor's Resource Manual*.
- There is now a little less emphasis on drill and **more emphasis on concept development, understanding, and communication**. Problems in exercise sets that require a **written response** (from a sentence to a paragraph) have the problem numeral printed in red for easy identification.
- More applications in exercise sets are **multistep discovery** problems that require **multiple interpretations** (for example, Problems 27 and 28 in Exercise 1.2, Problems 45–50 in Exercise 3.3, and Problem 71 in Exercise 5.1).
- More **historical remarks** have been added and others have been expanded (see, for example, Sections 1.1 and 2.4).

■ Ancillaries for Students

- A **student solutions manual** is available for student purchase. The manual includes detailed solutions to all odd-numbered problems and all chapter and cumulative review exercises.

- A **graphing calculator supplement** that coordinates the use of a graphing calculator with appropriate text topics is available for student purchase.
- A perforated **Quick Reference Card** is included in the text. This card can be removed, and places key equations and graphs at the student's fingertips.

■ Ancillaries for Instructors

- An ***Instructor's Resource Manual*** includes most of the answers that are not included in the text. It also includes outlines of solutions to the Explore–Discuss material and Chapter Group Activities.
- A **student solutions manual**, containing worked out solutions to all chapter review and cumulative review exercises and all odd-numbered problems in the book, is available without charge to any instructor adopting the book.
- **Thomson World Class Learning Testing Tools.** This integrated testing and tutorial software package features algorithmic test generation, on-line testing, class management capabilities, and tutorials.
- **Boxer Trigonometry.** This comprehensive tutorial on CD–ROM teaches the fundamentals of trigonometry in a creative and interactive learning environment.
- **Transparencies.** Reproductions of selected diagrams from the text are available for classroom presentation.
- **Test items.** A bound test bank is available free to adopters.

■ Error Check

Because of the careful independent checking and proofing by three competent college mathematics instructors, the authors and publisher believe this book to be substantially error-free. If any errors remain, the authors would be grateful if corrections were sent to: Reprints Coordinator, Brooks/Cole Publishing Company, 511 Forest Lodge Road, Pacific Grove, CA 93950-5098.

■ Acknowledgments

In addition to the authors, the publication of a book requires the effort and skills of many people. We would like to extend particular thanks to several very competent people: Brooks/Cole mathematics editors Elizabeth Rammel and Margot Hanis, for their skill, dependability, and support; production supervisor Phyllis Niklas, for her considerable expertise in preparing and guiding the book to publication; Fred Safier of City College of San Francisco, for his careful checking of the exercise sets and his masterly preparation of the solutions manual that accompanies this text; and Gholamhossein Hamedani, Robert Mullins, and Caroline Woods, for their careful checking and proofing of the entire manuscript, including examples and exercise sets.

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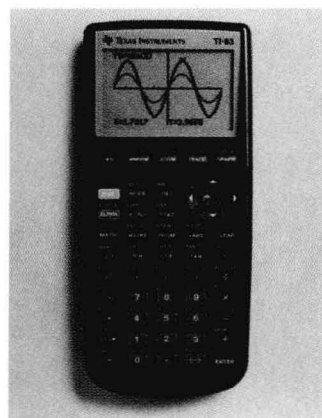
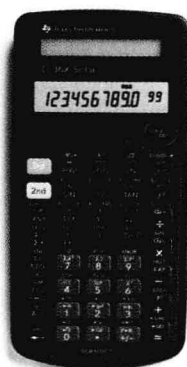
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Grand Rapids Community College

*Raymond A. Barnett
Michael R. Ziegler
Karl E. Byleen*

A Note on Calculators

Use of calculators is emphasized throughout this book. Many brands and types of scientific calculators are available and can be found starting at about \$10. Graphing calculators are more expensive; but, in addition to having most of the capabilities of a scientific calculator, they have very powerful graphing capabilities. Your instructor should help you decide on the type and model best suited to this course and the emphasis on calculator use he or she desires.

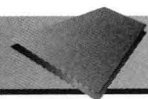


Scientific Calculator (left) and Graphing Calculator (right)

Courtesy Texas Instruments Incorporated

Whichever calculator you use, it is essential that you read the user's manual for your calculator. A large variety of calculators are on the market, and each is slightly different from the others. Therefore, take the time to read the manual. The first time through do not try to read and understand everything the calculator can do—that will tend to overwhelm and confuse you. Read only those sections pertaining to the operations you are or will be using; then return to the manual as necessary when you encounter new operations.

It is important to remember that a calculator is not a substitute for thinking. It can save you a great deal of time in certain types of problems, but you still must understand basic concepts so that you can interpret results obtained through the use of a calculator.



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Sections marked with the calculator icon require the use of a graphing utility. Inclusion of this material will enrich the course, but its omission will not affect the continuity of the course.



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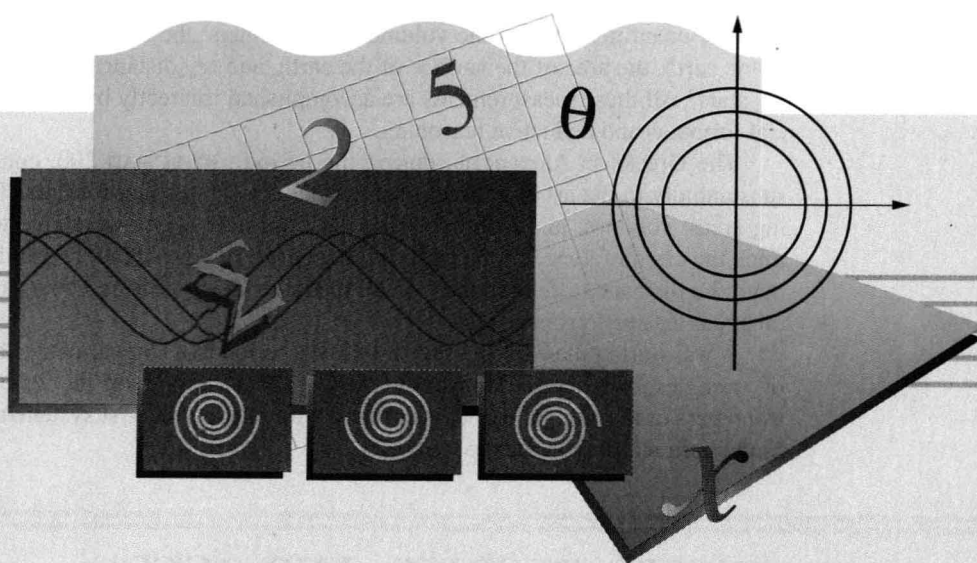
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RIGHT TRIANGLE RATIOS

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