Finite Groups 2003

EDITORS

CHAT YIN HO
PETER SIN
PHAM HUU TIEP
ALEXANDRE TURULL

de Gruyter

Finite Groups 2003

Proceedings of the Gainesville Conference
on Finite Groups

March 6 – 12, 2003

Editors

C. Y. Ho
P. Sin
P. H. Tiep
A. Turull

Walter de Gruyter · Berlin · New York

Editors

Chat Yin Ho
Department of Mathematics
University of Florida
PO Box 118105
358 Little Hall
Gainesville, FL 32611-8105
USA
e-mail: cyh@math.ufl.edu

Peter Sin
Department of Mathematics
University of Florida
PO Box 118105
358 Little Hall
Gainesville, FL 32611-8105
USA
e-mail: sin@math.ufl.edu

Pham Huu Tiep
Department of Mathematics
University of Florida
PO Box 118105
358 Little Hall
Gainesville, FL 32611-8105
USA
e-mail: tiep@math.ufl.edu

Alexandre Turull
Department of Mathematics
University of Florida
PO Box 118105
358 Little Hall
Gainesville, FL 32611-8105
USA
e-mail: turull@math.ufl.edu

Mathematics Subject Classification 2000:
20-06; 05Bxx, 17Bxx, 20Cxx, 20Dxx, 20Exx, 20Fxx, 20Gxx, 20Jxx

Keywords:
buildings, classification of finite simple groups, cohomology of groups, finite geometries, finite *p*-groups, Lie algebras and superalgebras, representation theory of finite and algebraic groups

∞ Printed on acid-free paper which falls within the guidelines of the ANSI to ensure permanence and durability.

Library of Congress Cataloging-in-Publication Data

Finite Groups 2003 (2003 : Gainesville, Fla.)
Finite Groups 2003 : proceedings of the Gainesville conference on finite groups, March 6–12, 2003 / edited by Chat Yin Ho ... [et al.].
p. cm.
ISBN 3-11-017447-2 (cloth : alk. paper)
1. Finite groups – Congresses. I. Ho, Chat-Yin, 1946–
II. Title.
QA174.F56 2004
512'.23–dc22 2004021279

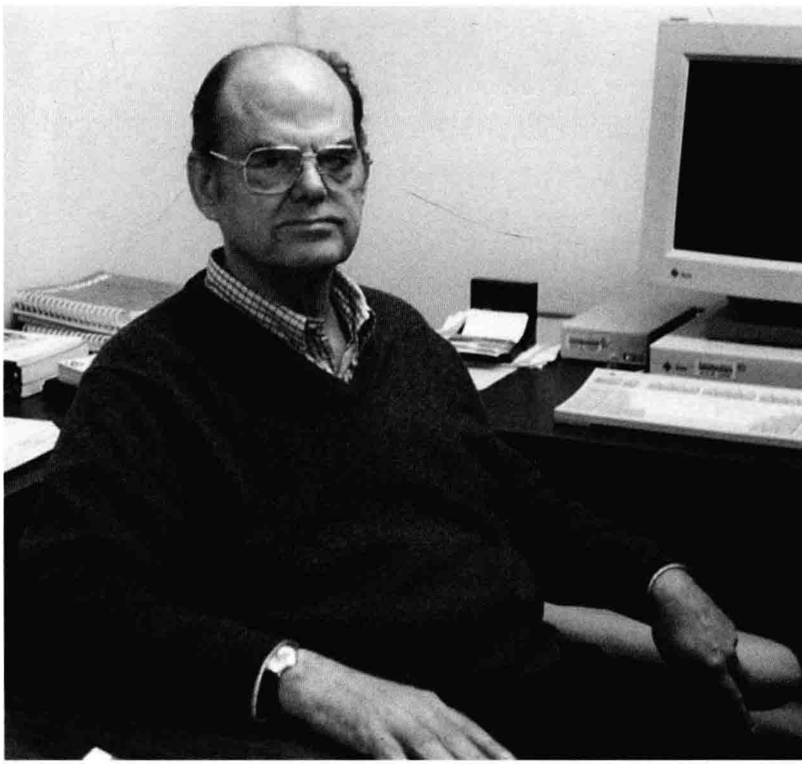
ISBN 3-11-017447-2

Bibliographic information published by Die Deutsche Bibliothek

Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the Internet at <http://dnb.ddb.de>.

Printed in Germany.
Cover design: Thomas Bonnie, Hamburg.
Typeset using the authors' $\TeX$ files: I. Zimmermann, Freiburg.
Printing and binding: Hubert & Co. GmbH & Co. KG, Göttingen.

Finite Groups 2003



John Thompson in his Little Hall office
Photo: Jane Dominguez, Courtesy CLAS News & Publications.

To John Thompson on the occasion of his 70th birthday

Preface

During the Academic Year 2002–2003, a number of events took place on both sides of the Atlantic to celebrate the 70th Birthday of our illustrious colleague and friend Professor John G. Thompson. A conference was held in Cambridge, England, in Thompson's honor. A special issue of the Journal of Algebra was edited to mark the occasion.

Here in Gainesville, the occasion was marked by the University of Florida mainly with two events. The Thompson Assistant Professorship was launched, and the whole Academic Year 2002–2003 was a Special Year in Algebra. One of the highlights of the Special Year in Algebra was the conference Finite Groups 2003 of which the present volume is the Proceedings.

We are grateful to the Mathematics Department, the College of Liberal Arts and Sciences, and the Office of Research and Graduate Programs at the University of Florida, as well as the National Science Foundation, and the National Security Agency for their support of the Special Year in Algebra in general, and the conference Finite Groups 2003 in particular.

Finite Groups 2003 took place in Gainesville from March 6, 2003 to March 12, 2003. As befits a conference in honor of John Thompson, talks on many different topics were given. A list of speakers and their topics is included below. Many of the speakers emphasized the relationship of their work to the work of John Thompson. At the conference banquet, Professor Broué presented John Thompson with a copy of the special issue of the Journal of Algebra edited in honor of his 70th Birthday.

The present Proceedings is a collection of articles in honor of John Thompson. They do not represent a record of the talks as given, but, instead, they are articles prepared specifically for these Proceedings. Each article has undergone a strict refereeing process, and we are sorry that, because of the scope and the size of the proceedings, not all interesting articles submitted could be accepted. Most of the articles here present original research results with their proofs. A few are survey articles written by leading researchers. It was the intention of all participants at the conference to honor John Thompson. We hope these Proceedings do honor him, for we, as indeed all group theorists do, owe him a great debt of gratitude for helping make our subject as interesting as it is today.

Gainesville, August 2004 *C. Y. Ho, P. Sin, P. H. Tiep and A. Turull*

List of talks

M. Aschbacher (California Institute of Technology): A question of Farjoun

C. Bachoc (Université Bordeaux): Codes and designs in Grassmannian spaces

A. Bereczky (Sultan Qaboos University): Standard bases for unipotent elements in classical groups

R. Boltje (University of California, Santa Cruz): Characterizations of Adams operations on certain representation rings

M. Broué (Institut Henri Poincaré): Representations of reductive groups in transverse characteristic

W. K. Chan (Wesleyan University): On almost strong approximation for algebraic groups defined over number fields

U. Dempwolff (Universität Kaiserslautern): On automorphisms of symmetric designs

R. Dipper (Universität Stuttgart): The rational Schur algebra

G. Ebert (University of Delaware): Odd order flag-transitive affine planes

D. Frohardt (Wayne State University): Genus zero actions

M. Geck (Université Claude Bernard-Lyon I): On the computation of some Schur indices for cuspidal unipotent characters

S. Glasby (Central Washington University): Modules over non-algebraically closed fields induced from a normal subgroup of prime index

G. Glauberman (University of Chicago): An extension of Thompson's replacement theorem by algebraic group methods

D. Gluck (Wayne State University): The final stage of the proof of the $k(GV)$-conjecture

J. González (Universidad del País Vasco): The power structure of potent p-groups

R. Gow (National University of Ireland): The arithmetic of Steinberg lattices of a Chevalley group

R. Gramlich (Technische Universität Darmstadt): Phan-type theorems

R. Guralnick (University of Southern California): Conjugacy classes and the non-coprime $k(GV)$-problem

D. Hemmer (University of Georgia): Specht module filtrations for representations of the symmetric group

M. Herzog (Tel-Aviv University): The existence of normal subgroups in finite groups

G. Hiss (RWTH Aachen): Some observations on products of characters of finite classical groups

A. Jaikin-Zapirain (Universidad Autónoma de Madrid): The number of finite p-groups with bounded number of generators

M. Kallaher (Washington State University): On semiovals

T. Keller (Southwest Texas State University): On the $k(GV)$-conjecture

R. Kessar (Ohio State University): Shintani descent and perfect isometries for blocks of finite general linear groups

A. Kleshchev (University of Oregon): W-algebras

B. Klopsch (Universität Düsseldorf): Enumerating highly non-soluble groups

I. Korchagina (Rutgers University): Groups of simultaneously even- and p-type

P. Lescot (Université de Picardie): Variations on Thompson's J-subgroup

M. Lewis (Kent State University): Character degree graphs with 5 vertices

K. Lux (University of Arizona): A database for basic algebras of simple groups

R. Lyons (Rutgers University): Second generation proof

G. Malle (Universität Kassel): Families of characters for complex reflection groups

D. McNeilly (University of Alberta): Realizability of Weil representations

A. Moretó (Universidad del País Vasco): Heights of characters and defect groups

G. Navarro (Universitat de València): Problems on characters and Sylow subgroups

S. Onofrei (Kansas State University): A point-line characterization of the building of type E_7

C. Pillen (University of South Alabama): Extensions for finite Chevalley groups, Frobenius kernels, and algebraic groups

A. Prince (Heriot-Watt University): Oval configurations of involutions in the symmetric group

D. Riley (Western Ontario University): Beyond the restricted Burnside problem

G. Robinson (University of Birmingham): Upper bounds for numbers of characters

M. Ronan (University of Illinois at Chicago): Classification of buildings

J. Sangroniz (Universidad del País Vasco): Characters of algebra groups

N. Sastry (Indian Statistical Institute): On the conjugacy classes of finite Suzuki and Ree groups

M. Sawabe (Naruto University of Education): On a p-local geometry associated with a p-subgroup complex

C. Scoppola (Università degli Studi di L'Aquila): Space and loop Lie algebras

L. Scott Jr. (University of Virginia): Some new examples in 1-cohomology

G. Seitz (University of Oregon): Unipotent centralizers and saturation

J. Shareshian (Washington Universtity): Topology of intervals in subgroup lattices

S. Shpectorov (Bowling Green State University): Classification of amalgams

S. Smith (University of Illinois at Chicago) : Quasithin groups – in retrospect

R. Solomon (Ohio State University): The signalizer method

B. Srinivasan (University of Illinois at Chicago): Remarks on Dade's conjecture for $\mathrm{GL}(n, q)$

K. Uno (Osaka University): Some variations of conjectures on character degrees

L. Wilson (University of Florida): Large powerful subgroups of p-groups

T. Wolf (Ohio University): Regular and large orbits of induced modules

A. E. Zalesski (University of East Anglia): Minimal polynomials of group elements in linear representations of finite groups

J. Zhang (Peking University): On p-local rank of finite groups

Table of contents

On a question of Farjoun

*Michael Aschbacher**

Let G be a group, H a subgroup of G, $\mathrm{Hom}(H, G)$ the set of all group homomorphisms from H into G, and $\mathrm{End}(G) = \mathrm{Hom}(G, G)$.

Following E. Farjoun, we say the embedding of H in G is *closed* if each member of $\mathrm{Hom}(H, G)$ extends uniquely to a member of $\mathrm{End}(G)$.

Example 1. G is closed in itself for each group G.

Example 2. Let H and G be simple. Then H is closed in G iff $\mathrm{Aut}(G)$ is transitive on subgroups of G isomorphic to H, $\mathrm{Aut}_{\mathrm{Aut}(G)}(H) = \mathrm{Aut}(H)$, and $C_{\mathrm{Aut}(G)}(H) = 1$. In particular if G is the alternating group of degree n, and H is the stabilizer of a point, then H is closed in G for each $n > 5$ with $n \neq 7$.

As part of his investigation of idempotent augmented functors, Farjoun posed the following question:

Question 1. If H is a finite closed nilpotent subgroup of a group G, is $G = H$?

For technical reasons it is sometimes easier to work with a slightly weaker condition. Define H to be *nearly closed* in G if each member of $\mathrm{End}(H)$ extends uniquely to a member of $\mathrm{End}(G)$.

Question 2. If H is a finite nearly closed nilpotent subgroup of a group G, is $G = H$?

In this paper we introduce some machinery useful in investigating Question 2 and present some partial results which show the Question has a positive answer in various special cases. For example we reduce Question 2 to the case where H is a p-group for some prime p, we show Question 2 has an affirmative answer when G is finite and the nilpotence class of H is at most 3, and we show Question 2 has an affirmative answer when G is finite for other interesting classes of p-groups. Here are some specifics:

Let $\mathcal{C}$ be a class of groups such that if $G \in \mathcal{C}$ and $H \leq G$ then $H \in \mathcal{C}$. Define a group P to be $\mathcal{C}$-*rigid* or *rigid* with respect to $\mathcal{C}$ if whenever P is nearly closed in $G \in \mathcal{C}$ then $P = G$. Our first result says:

Theorem 1. *Let $\mathcal{C}$ be a class of groups and P_1 and P_2 groups which are $\mathcal{C}$-rigid. Then $P_1 \times P_2$ is $\mathcal{C}$-rigid.*

*This work was partially supported by NSF-0203417.

As finite nilpotent groups are the direct product of their Sylow groups, Theorem 1 reduces Question 2 to the case where H is a p-group, so in the remainder of this introduction assume G is a group, p is a prime, and P is a finite p-subgroup of G. Moreover in the rest of the introduction we work in the class $\mathcal{C}$ of finite groups, so G is assumed to be finite.

The next result shows Question 2 has a positive answer when P is large:

Theorem 2. *If P is nearly closed in a finite group G and $P \in \mathrm{Syl}_p(G)$ then $G = P$.*

Theorem 3. *If P is metabelian or of nilpotence class at most 3, then P is rigid with respect to the class of finite groups.*

A. Mann has a slick proof that groups of class 2 are rigid in the class of *all* groups; indeed he shows that if P is closed in G then $Z_2(P) \leq Z_2(G)$.

The method used to prove Theorem 3 can be used to treat other classes of p-groups, but there are obstructions to a simple minded extension of the approach. See Section 9 for more discussion and a sketch of a proof that Question 2 has a positive answer for groups P of class at most 5 generated by 3 elements of order p.

One tool for investigating Question 2 involves the relationship between "splittings" of P and G, where a splitting of P is a decomposition of P as a split extension. In Section 2 we consider a "splitting category" and define a class of "splitting solvable" groups, which includes many interesting p-groups. Then we prove:

Theorem 4. *If P is splitting solvable then P is rigid with respect to the class of finite groups.*

Finally the near closure property does not inherit to subgroups or homomorphic images, so while the condition is very strong, it is not easy to exploit using traditional techniques from finite group theory. On the other hand various properties following from near closure do inherit. Thus in Section 10 we investigate classes of finite groups satisfying such properties. We would like to show the subnormal closure of P in each group G in the class is P-nilpotent (i.e. $G = O_{p'}(G)P$); such a result would give an affirmative answer to Question 2 when G is solvable, and give very strong information in the nonsolvable case. Unfortunately examples show this result is not true in general for the classes we've considered, even when G is solvable. Nevertheless we include an abbreviated discussion of this approach in one case, since that discussion does at least lead to our final theorem, and since in addition the result may be true if other properties are adjoined to obtain smaller classes.

Theorem 5. *If p is an odd prime, G is a finite solvable group, and P is a p-subgroup of G such that $C_G(P) = Z(G)$, then $O_{p'}(G)$ is the largest p'-signalizer for P.*

Recall a *p'-signalizer* for P is a P-invariant p'-subgroup of G.

The reader is directed to [2] for basic notation, terminology, and results involving finite groups.

The author would like to thank Yoav Segev for drawing his attention to Farjoun's question, for many helpful conversations about the problem, and for suggesting improvements to the proofs of a number of lemmas in this paper.

1. Monoids and semigroups

We will be concerned with the monoid $\mathrm{End}(G)$ of a finite group G, so we begin with some discussion of monoids.

In this section M is a finite semigroup; that is M is a set with an associative binary operation $\cdot$. Recall M is a *monoid* if M has an identity 1. If an identity exists, it is unique. A *zero* for M is an element $0 \in M$ such that for each $x \in M$, $0 \cdot x = 0 = x \cdot 0$. Again if M has a 0 then it is unique.

For $X \subseteq M$, write $[X]$ for the subsemigroup of M generated by X. Thus $[X]$ consists of all products $x_1 \dots x_n$ with $x_i \in X$ for all i. In particular for $x \in X$, $[x] = \{x^n : 0 < n \in \mathbb{Z}\}$.

An *idempotent* of M is an element $x \in M$ such that $x^2 = x$. An element $y \in M$ is said to be *split* if $y^n = y$ for some $n > 1$.

1.1. *Let $a \in M$, k the least positive integer such that $a^k = a^j$ for some $j > k$, and q the least positive integer such that $a^{k+q} = a^k$. Set $B = \{a^j : j \geq k\}$. Then:*

(1) *B is a cyclic subgroup of M of order q.*

(2) *B is the set of split elements in $[a]$.*

(3) *Let sq be the first positive multiple of q such that $k \leq sq$. Then a^{sq} is the identity of B and $B = \langle a^t \rangle$, for each $t \geq k$ with $(q, t) = 1$.*

Proof. As $a^k = a^{k+q}$, by induction on i, $a^{k+i} = a^{k+i+q}$ for each $i \geq 0$. Then by induction on r:

(i) $a^{m+rq} = a^m$ for each $m \geq k$ and $r \geq 0$.

Hence:

(ii) $B = \{a^j : j \in J\}$, where $J = \{k + i : 0 \leq i < q\}$.

From (i) we see that $e := a^{rq}$ is the identity element of B, for each r such that $rq \geq k$, and hence, for each $b \in B$, $b^q = e$. It follows that for each $b \in B$, b^{q-1} is the inverse of b and B is a group. If $a^{j_1} = a^{j_2}$ for some $j_1 \leq j_2$ in J, then multiplying by $a^{(s+1)q-j_1}$, we get that $e = a^{(s+1)q} = a^{(s+1)q+j_2-j_1}$. Multiplying by a^k we get $a^k = a^{k+(s+1)q+j_2-j_1} = a^{k+j_2-j_1}$, because $a^{(s+1)q}$ is the identity element of B. But the equality $a^k = a^{k+j_2-j_1}$ contradicts the minimality of q, unless $j_1 = j_2$. It follows that $|B| = q$. Let $k \leq t \in \mathbb{Z}$ with $(t, q) = 1$, and set $c := a^t$. As $(t, q) = 1$:

(iii) $tJ = J \mod q$,

so $B = [c]$ by (i)–(iii). This establishes (1) and (3).

Finally if a^r is split then $a^r = a^t$ for some $t > r$, so $r \geq k$ by minimality of k. Thus (2) holds. □

For $a \in M$, let

$$\mathrm{sp}(a) = \{b \in [a] : b \text{ is split}\}.$$

By 1.1, $\mathrm{sp}(a)$ is a group, so it has a unique identity $\mathrm{id}(a)$.

1.2. *Let $a \in M$.*

(1) *$\mathrm{sp}(a)$ is a cyclic subgroup of M.*

(2) *$\mathrm{id}(a)$ is the unique idempotent in $[a]$.*

Proof. Part (1) follows from 1.1. Let $e = \mathrm{id}(a)$. Then $e^2 = e$ so e is an idempotent. Conversely if $f \in [a]$ is an idempotent then f is split, so $f \in \mathrm{sp}(a)$, and hence $e = f$ as groups have a unique idempotent. □

Define an equivalence relation $\sim$ on M by $a \sim b$ if $\mathrm{id}(a) = \mathrm{id}(b)$. Write $\tilde{a}$ for the equivalence class of a under $\sim$, and set

$$\mathrm{sp}(\tilde{a}) = \{b : a \sim b \text{ and } b \text{ is split}\}$$

1.3. *$\mathrm{sp}(\tilde{a})$ is a subgroup of M with identity $\mathrm{id}(a)$.*

Proof. Let $e = \mathrm{id}(a)$ and H the set of $x \in M$ such that $xe = ex = x$ and $yx = xy = e$ for some $y \in M$. It is easy to check that H is a subgroup of M. Then since M is finite, $e \in [h]$ and h is split for each $h \in H$, so $e = \mathrm{id}(h)$ by 1.2.2. Thus $H \subseteq \mathrm{sp}(\tilde{a})$, and of course $\mathrm{sp}(\tilde{a}) \subseteq H$, so the lemma holds. □

Define a relation $\leq$ on M by

$$b \leq a \text{ if } ab = ba = b.$$

For $a \in M$ let $M(\leq a) = \{b \in M : b \leq a\}$.

Write $\mathcal{I}$ for the set of idempotents in M.

1.4. (1) *$\leq$ is antisymmetric.*

(2) *$\leq$ is transitive.*

(3) *$a \leq a$ iff $a \in \mathcal{I}$.*

(4) *$\leq$ is a partial order on $\mathcal{I}$.*

(5) *If M has 0 or 1 then $0 \leq a \leq 1$ for all $a \in M$.*

(6) *For each $a \in \mathcal{I}$, $M(\leq a)$ is a subsemigroup of M with identity a.*

(7) *If $a \in \mathcal{I}$ then $M(\leq a) = aMa$ and $\mathrm{sp}(\tilde{a}) \subseteq M(\leq a)$.*

(8) *If $a, b \in \mathcal{I}$ and $b \leq a$ then $\mathrm{sp}(\tilde{b}) \subseteq M(\leq a)$.*

Proof. If $b \leq a$ and $a \leq b$ then $a = ab = b$, so (1) holds. Let $c \leq b \leq a$. Then

$$ac = a(bc) = (ab)c = bc = c,$$