



MUNEM • YIZZE

Precalculus

Functions and Graphs

FIFTH EDITION

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M. A. MUNEM

J. P. YIZZE

MACOMB COLLEGE

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Precalculus: Functions and Graphs Fifth Edition

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Precalculus

Preface

The fifth edition of *Precalculus*, like its predecessors, prepares students to take calculus and courses in business, economics, and the physical and social sciences that presume a working knowledge of functions and graphs. In this revision we have paid particular attention, both in the textbook and in the accompanying study guide, to opportunities to sharpen students' critical thinking skills, particularly in the solution of word problems. We assume that students have taken two years of high school mathematics, including algebra and some plane geometry, or an equivalent college course in intermediate algebra.

Our intention has been to write a readily accessible textbook that conveys important concepts and skills in a concise and appealing manner. We have taken pains to ensure that the average student will be able to read and follow the development of each section and to solve the problems that follow it. Several features of the book contribute to its effectiveness.

Features

- 1. Development of Ideas** Topics are presented in brief sections that develop concepts logically. We frequently remind students of what they already know before we introduce new material.
- 2. Procedural Guides** Students are shown how to do basic algebraic operations in concise, step-by-step guides.
- 3. Examples** After each concept is clearly explained, it is illustrated with explicit examples that are solved showing all procedures.
- 4. Problem Sets** Each section within a chapter ends with an extensive set of problems that are graded in difficulty. Most of the *odd-numbered problems* are similar in scope to the worked-out examples in the text. Answers to *odd-numbered problems, including graphs*, are provided at the end of the book. The *even-numbered problems* are also similar to the worked-out examples; however, some are more challenging and probe for a deeper understanding of concepts.

5. **Review Problem Sets** Review problems cover the entire chapter, giving students an opportunity to check their mastery of all topics and reinforce their understanding before moving on. The answers to all odd-numbered review problems are also provided at the end of the book.
6. **Chapter Tests** New to this edition is a brief test at the end of each chapter that students can use for practice, review, and to determine their readiness for a class test. All test answers are provided.
7. **Word Problems and Applications** Word problems and strategies for solving them are explained in numerous examples. There are many new word problems in examples and in problem sets. Applications cover a wide variety of fields—engineering, geometry, business, economics, medicine, navigation, and the physical and social sciences. There are sections devoted entirely to applications in chapters 4, 5, and 6.
8. **Algebra of Calculus** Emphasis has been given to the types of examples and problems involving algebra that arise in the study of calculus.
9. **Use of Calculators** In this edition, we continue to assign the calculator to its rightful place—a tool that is useful at times and unnecessary at other times. The number of problems and examples that call for the use of a calculator has been increased; all are indicated with the symbol \boxed{c} . In chapters 4, 5, and 6, we continue to de-emphasize the use of logarithmic and trigonometric tables in favor of the use of scientific calculators.
10. **Flexibility of use** Many colleges have used the book successfully without following exactly its order of sections or chapters. This flexibility of use allows teachers to design their syllabus solely to suit the objectives and length of the course.

Improvements

We have rewritten and reorganized the presentation of some topics and added new material to reflect the needs and interests of students and the goals of the course. Some of the changes are:

1. The *Cartesian coordinate system* and the *equation of the line* are now included in Chapter 1, as are the *union and intersection of two sets*. The chapter provides a review of algebra for students who need it.
2. Chapter 2 now develops the general concepts of graphing more extensively, including a separate section on graphing techniques. Direct and inverse variation are also included in the chapter in conjunction with functions.
3. In Chapter 3, guidelines are provided in section 3.7 to help students proceed in a systematic manner when sketching the graphs of rational functions. The concept of *oblique asymptote* has also been added to this section. New material on locating irrational zeros of polynomial functions by using the *method of successive enlargements* has been added to section 3.5. Complex numbers are now incorporated with complex zeros of polynomials and covered in section 3.6.

4. Applications of exponential and logarithmic functions are now integrated in section 4.4 of Chapter 4. New applications that show how to calculate *car and mortgage payments* have been added to this section.
5. Chapter 5 on trigonometric functions has been rewritten and reorganized. Applications to right triangle trigonometry are now covered in sections 5.4 and 5.8. Also, the inverse trigonometric functions are now covered in section 5.8.
6. *Computer generated graphs* are presented as an important tool for approximating irrational zeros (Section 3.6) and for dealing with graphs of certain trigonometric functions (Section 5.7).
7. Examples involving *nonlinear inequalities and systems of nonlinear inequalities* have been added to section 7.6.
8. More *plane geometry* is included in this edition. A brief review of plane geometry in Appendix III provides reference for students studying ideas involving geometry located throughout the book, such as sections 1.6 and 6.5.
9. Appendix IV presents a procedure for finding an inverse of a square matrix.

Student Aids

1. The *Student Guide* has been thoroughly revised. It now includes worked-out solutions to every other odd-numbered problem in the textbook, as well as study objectives, semiprogrammed problems for each chapter, and additional word problems. There are two practice tests for each chapter—a multiple-choice test and a problem-solving test. A cumulative review problem set for each chapter, including representative problems from all prior chapters, has been added to refresh skills acquired earlier in the course. The answers and solutions to the cumulative reviews and the chapter tests are included in the *Student Guide*.
2. *Computer Software*—Users of this textbook may obtain from the publisher a microcomputer diskette designed for either the Apple II or the IBM PC families of computers. The programs include a series of routines—synthetic and polynomial division, general triangles, conic sections, and the bisection method—involving algebra and trigonometry. The IBM software contains additional programs in matrix operations.

Instructor Aids

Solution Manual—The step-by-step solution to each problem in the textbook is available in this manual *for instructors*. A glance at the worked-out solutions will help in selecting the appropriate problems to assign.

Testbank—A packet of 81 tests, nine for each chapter of the textbook, is available. These tests have been written at graded levels of difficulty, with *four* of the nine tests for each chapter made up of multiple-choice questions and *five* of standard problem-solving questions. Answers to all test questions are included.

Acknowledgments:

In preparing this edition, we have drawn from our own experience in teaching from *Precalculus*, as well as the feedback provided by our students. The suggestions obtained from instructors using the fourth edition were of great help. We wish to thank all of these people and, in particular, to express our gratitude to the following: James Bright, *Clayton Junior College*; Louis Hoelzle, *Bucks County Community College*; Joan Levine, *Kean College of New Jersey*; Thomas Sharp, *West Georgia College*; Alexandra Tauson, *Community College of Allegheny County*; Howard Taylor, *West Georgia College*; Henry Tjoelker, *California State University of Sacramento*; and Ann Wagner, *Towson State University*. The following teachers reviewed page proofs of the fifth edition and made suggestions for improvements, for which we are grateful: Thomas Farmer, *Miami University, Ohio*; Merle Friel, *Humboldt State University*; Robert Gebhart, *Community College of Morris*; Karl Gentry, *University of North Carolina, Greensboro*; Henry Gore, *Morehouse College*; Thomas J. Sharp, *West Georgia College*; and Ben Zirkle, *Virginia Western Community College*.

Special thanks are due to our many colleagues at Macomb College who taught from previous editions of the book and shared their experiences with us. We especially wish to thank Wayne Hille of *Wayne State University*, who solved all the problems in the book, and our colleague Steve Fasbinder, who assisted in the proof-reading of the book and the student guide. We are also grateful, as always, to the staff of Worth Publishers.

M. A. Munem
J. P. Yizze

Contents

CHAPTER	1	Fundamentals of Algebra	1
	1.1	Number Sets	1
	1.2	Algebraic Expressions	13
	1.3	Equations	25
	1.4	Inequalities	36
	1.5	Rectangular Coordinate System	46
	1.6	Equations and Slopes of Lines	55
		Review Problem Set	66
		Chapter Test	69
CHAPTER	2	Functions and Graphs	70
	2.1	Functions	70
	2.2	Graphs of Functions	81
	2.3	Graphing Techniques	92
	2.4	Algebra of Functions and Composition of Functions	101
	2.5	Inverse Functions	108
		Review Problem Set	116
		Chapter Test	118
CHAPTER	3	Polynomial and Rational Functions	120
	3.1	Quadratic Functions	120
	3.2	Graphs of Polynomial Functions	129
	3.3	Division of Polynomials	136

	3.4	Rational Zeros of Polynomial Functions	145
	3.5	Irrational Zeros of Polynomial Functions	152
	3.6	Complex Zeros of Polynomial Functions	160
	3.7	Rational Functions	165
		Review Problem Set	174
		Chapter Test	176
CHAPTER	4	Exponential and Logarithmic Functions	178
	4.1	Exponential Functions	178
	4.2	Logarithmic Functions	187
	4.3	Properties of Logarithms	197
	4.4	Applications and Models Involving Exponential and Logarithmic Functions	204
		Review Problem Set	217
		Chapter Test	218
CHAPTER	5	Trigonometric Functions	220
	5.1	Preliminary Trigonometric Concepts	220
	5.2	Trigonometric Functions	234
	5.3	Properties of Trigonometric Functions	241
	5.4	Evaluation of Trigonometric Functions—Right Triangles	247
	5.5	Basic Identities	259
	5.6	Graphs of the Sine and Cosine Functions	266
	5.7	Graphs of Other Trigonometric Functions	281
	5.8	Inverse Trigonometric Functions	290
		Review Problem Set	300
		Chapter Test	302
CHAPTER	6	Analytic Trigonometry	304
	6.1	Trigonometric Identities	304
	6.2	Sum, Difference, and Related Trigonometric Formulas	308

	6.3	Multiple-Angle Formulas	319
	6.4	Trigonometric Equations	326
	6.5	Law of Sines and Law of Cosines	333
	6.6	Polar Coordinates	342
	6.7	Trigonometric Forms of Complex Numbers	348
	6.8	Plane Vectors and Their Applications	357
		Review Problem Set	366
		Chapter Test	369
CHAPTER	7	Systems of Equations and Inequalities	370
	7.1	Linear Systems of Equations	370
	7.2	Matrices and Row Reduction	378
	7.3	Determinants	392
	7.4	Partial Fractions	400
	7.5	Systems Containing Nonlinear Equations	406
	7.6	Systems of Inequalities and Linear Programming	409
		Review Problem Set	417
		Chapter Test	419
CHAPTER	8	Analytic Geometry and the Conics	420
	8.1	Circles and Ellipses	421
	8.2	Parabolas	431
	8.3	Hyperbolas	438
	8.4	Translations and Rotations of Axes	447
	8.5	Polar Forms of Conics and Parametric Equations	455
		Review Problem Set	462
		Chapter Test	464
CHAPTER	9	Sequences, Series, and Mathematical Induction	465
	9.1	Sequences	465
	9.2	Summation Notation and Series	470
	9.3	Mathematical Induction and the Binomial Theorem	478

		Review Problem Set	488
		Chapter Test	489
APPENDIX	I	Tables of Logarithms	490
		Table IA Common Logarithms	494
		Table IB Natural Logarithms	496
APPENDIX	II	Tables of Trigonometric Functions	498
		Table IIA Trigonometric Functions—Degree Measure	501
		Table IIB Trigonometric Functions—Real Number or Radian Measure	507
APPENDIX	III	Plane Geometry Review	511
APPENDIX	IV	Procedure for Finding the Inverse of a Matrix	517
		Answers to Selected Problems	A-1
		Index	521

CHAPTER 1

Fundamentals of Algebra

This chapter reviews basic concepts and methods of algebra that are needed for the study of functions and their graphs. The topics covered include sets of real numbers, the number line, exponents, algebraic expressions, equations, inequalities, the rectangular coordinate system, and straight lines.

1.1 NUMBER SETS


We often use the term **set** to refer to a collection of numbers. Any one of the numbers in a set is called an **element** or a **member** of the set. Most sets considered in this book are sets of real numbers. The set of **real numbers** can be thought of as the collection of all numbers that can be expressed as decimal numbers. The set of real numbers is denoted by the symbol \mathbb{R} . A set A is called a **subset** of set B , and is written $A \subseteq B$, if every element of A is also an element of B . Table 1 lists some special subsets of \mathbb{R} .

Table 1

Symbol	Subset of \mathbb{R}	Description
\mathbb{N}	Natural numbers or positive integers	Counting numbers; that is, the set $\{1, 2, 3, \dots\}$
\mathbb{I}	Integers	Positive integers, negative integers, and zero, that is, the set $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$
\mathbb{Q}	Rational numbers	All numbers that <i>can</i> be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. For example, $\frac{-3}{4}$, $5\frac{1}{3} = \frac{16}{3}$, and $0.37 = \frac{37}{100}$ are rational numbers
\mathbb{J}	Irrational numbers	Real numbers that are not rational numbers; $\sqrt{2}$, $\sqrt[3]{5}$, and π are examples

By using long division, we can express a rational number as a decimal. For instance, if we divide 2 by 5, we obtain $\frac{2}{5} = 0.4$, a **terminating decimal**. If we divide 2 by 3, we obtain $\frac{2}{3} = 0.666\cdots$, a **nonterminating repeating decimal**. A repeating decimal, such as $0.666\cdots$, is often written as $0.\overline{6}$, where the overbar indicates the digit or digits that repeat; hence $\frac{2}{3} = 0.\overline{6}$.

The decimal representation of an irrational number *never* forms a repeating pattern. Whenever a decimal representation of a real number is desired, we often use a calculator to determine an approximation to the number of required digits.

Example 1  Change each number to a decimal form, then round off the result to five decimal places.

- (a) $\frac{156}{199}$ (b) $\sqrt{237}$ (c) $(5.7)^3$ (d) $\sqrt[3]{25}$

Solution The solutions are given in Table 2. Note that in parts (c) and (d) we utilize a y^x or a comparable key along with the fact that $\sqrt[3]{25} = 25^{1/3}$. ■

Table 2

Given Number	Calculator Value	Rounded Off to Five Decimal Places
(a) $\frac{156}{199}$	0.788944724	0.78894
(b) $\sqrt{237}$	15.39480432	15.39480
(c) $(5.7)^3$	185.1930000	185.19300
(d) $\sqrt[3]{25}$	2.924017738	2.92402

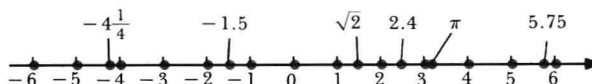
Often the symbol \approx is used to indicate an approximation. For instance, $\sqrt{237} \approx 15.395$ and $(5.7)^3 \approx 185.19$. However, in this text, we usually use the *equal sign* to represent the approximate decimal value of a real number. Thus, we write $\sqrt{237} = 15.395$ and $(5.7)^3 = 185.19$.

The Real Number Line and Absolute Value

The set of real numbers can be represented *geometrically* as the set of all points on the **number line** or **coordinate axis**. By repeating the scale unit used to construct the line and by moving from left to right, starting at the point representing 0, called the **origin**, we can associate the set of positive integers $\mathbb{N} = \{1, 2, 3, 4, \dots\}$ with equispaced points on the line. Moving from right to left, starting at 0, we can associate the set of negative integers $\{-1, -2, -3, -4, \dots\}$ with equispaced points on the line (Figure 1). The remaining real numbers can be “located” or “plotted” on

the real line by using *decimal representations* as illustrated in Figure 1. The number that is associated with a point on the number line is called the **coordinate** of the point. Note that the number is not the point, nor is the point the number. The point *represents* the number. It is customary, however, to use the words *real number* and *point* interchangeably. Thus we often speak of the *point* $\frac{3}{4}$ rather than the *point corresponding to the real number* $\frac{3}{4}$.

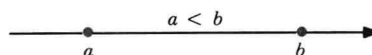
Figure 1



The real numbers to the right of zero are the **positive** numbers, and the real numbers to the left of zero are the **negative** numbers. The positive numbers and zero together are called the **nonnegative** numbers.

The number line enables us to establish **order relationships** between real numbers. If the point with coordinate a lies to the left of the point with coordinate b (Figure 2), we say that a is **less than** b (or equivalently, that b is **greater than** a) and we write $a < b$ (or $b > a$). More formally, we have the following definition.

Figure 2



Definition 1 Order

Assume that a and b are real numbers. Then $a < b$ (or $b > a$) means that $b - a$ is a positive number.

For instance, $-5 < 2$ since $2 - (-5) = 7$, which is positive. A statement of the form $a < b$ (or $b > a$) is called an **inequality**. The inequality symbol \leq means **less than or equal to**; the symbol \geq means **greater than or equal to**.

The notation $a < x < b$, called a **compound inequality**, means

$$a < x \text{ and simultaneously } x < b \quad (\text{Figure 3})$$

The set of all real numbers x that satisfy $a < x < b$ is called an **open interval** with **endpoints** a and b . Note that points a and b do *not* belong to this set. The classifications of other types of intervals, as well as the notation sometimes used to denote intervals, are shown in Table 3. The *parenthesis* symbol denotes the *exclusion* of an endpoint and the *bracket* indicates the *inclusion* of an endpoint.

Figure 3

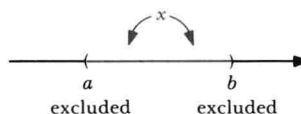




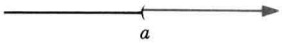
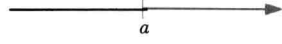
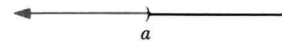
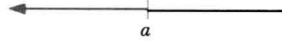



Table 3

Terminology	Interval Notation	Inequality Notation	Number Line Representation
Bounded intervals:			
Open interval	(a, b)	$a < x < b$	
Closed interval	$[a, b]$	$a \leq x \leq b$	
Half-open interval	$[a, b)$	$a \leq x < b$	
Half-open interval	$(a, b]$	$a < x \leq b$	
Unbounded intervals:			
	(a, ∞)	$a < x$	
	$[a, \infty)$	$a \leq x$	
	$(-\infty, a)$	$x < a$	
	$(-\infty, a]$	$x \leq a$	
Real number line	$(-\infty, \infty)$	—	

Note that ∞ (“infinity”) and $-\infty$ are convenient symbols; they are *not* real numbers.

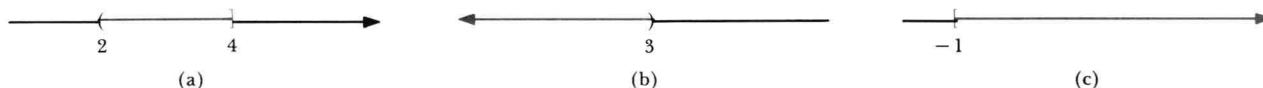
Note also that the parenthesis, $($, on the number line designates a coordinate that is *excluded* from the solution set. Conversely, the bracket, $[$, designates a coordinate that is *included* in the solution set.

Example 2 Express each set in interval notation and represent the set on the number line.

- (a) $2 < x \leq 4$ (b) $x < 3$ (c) $x \geq -1$

Solution (a) $(2, 4]$ (Figure 4a)
 (b) $(-\infty, 3)$ (Figure 4b)
 (c) $[-1, \infty)$ (Figure 4c)

Figure 4

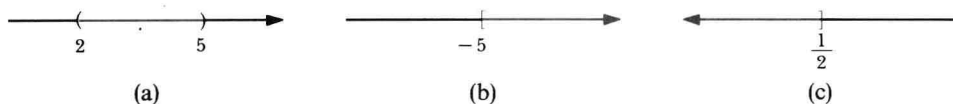


Example 3 Express each interval in inequality notation and represent the set on the number line.

- (a) $(2, 5)$ (b) $[-5, \infty)$ (c) $(-\infty, \frac{1}{2}]$

- Solution (a) $(2, 5)$ includes all x such that $2 < x < 5$ (Figure 5a).
 (b) $[-5, \infty)$ includes all x such that $-5 \leq x$ (Figure 5b).
 (c) $(-\infty, \frac{1}{2}]$ includes all x such that $x \leq \frac{1}{2}$ (Figure 5c).

Figure 5

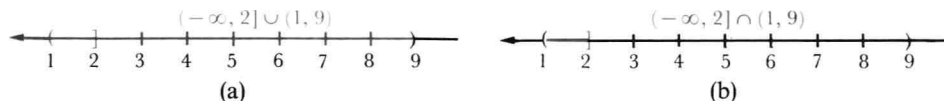


At times, two given number sets are combined to form a third set by using the *set operations of union and intersection*. If A and B are sets, then the set consisting of all elements that belong to A or to B (or to both) is called the **union** of A and B and is written as $A \cup B$. The set of all elements that are *both* in A and in B is called the **intersection** of A and B and is written as $A \cap B$. For example, if we are given intervals $(-\infty, 2]$ and $(1, 9)$, then

$$(-\infty, 2] \cup (1, 9) = (-\infty, 9) \quad (\text{Figure 6a})$$

and
$$(-\infty, 2] \cap (1, 9) = (1, 2] \quad (\text{Figure 6b})$$

Figure 6



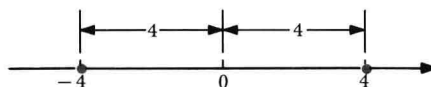
If x represents a number in set $(-\infty, -3) \cup [5, \infty)$, then x satisfies either $x < -3$ or $x \geq 5$.

Absolute Value

If x is a real number, then $|x|$ represents the (nonnegative) number of units of distance on the number line between the point with coordinate x and the point with coordinate 0. The nonnegative number $|x|$ is called the **absolute value** of x . For instance, both the point with coordinate -4 and the point with coordinate 4 are 4 units from the origin (Figure 7); that is,

$$|-4| = |4| = 4$$

Figure 7



More formally, we have the following definition.