

THE NEW PALGRAVE

# Game theory

Edited by

Steven N. Durlauf and  
Lawrence E. Blume



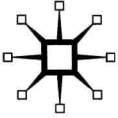
# Game Theory

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All articles first published in *The New Palgrave Dictionary of Economics*, 2<sup>nd</sup> Edition Edited by Steven N. Durlauf and Lawrence E. Blume in eight volumes, 2008

with the exception of Incentive Compatibility which first published in *The New Palgrave: A Dictionary of Economics* Edited by John Eatwell, Murray Milgate and Peter Newman in four volumes, 1987

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First published 2010 by  
PALGRAVE MACMILLAN

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Palgrave Macmillan in the US is a division of St Martin's Press LLC, 175 Fifth Avenue, New York, NY 10010.

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ISBN-13: 978-0-230-23889-3 hardback  
ISBN-13: 978-0-230-23890-9 paperback

This book is printed on paper suitable for recycling and made from fully managed and sustained forest sources. Logging, pulping and manufacturing processes are expected to conform to the environmental regulations of the country of origin.

A catalogue record for this book is available from the British Library.

A catalog record for this book is available from the Library of Congress.

Printed and bound in Great Britain by  
CPI Antony Rowe, Chippenham and Eastbourne

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# General Preface

All economists of a certain age remember the “little green books”. Many own a few. These are the offspring of *The New Palgrave: A Dictionary of Economics*; collections of reprints from *The New Palgrave* that were meant to deliver at least a sense of the *Dictionary* into the hands of those for whom access to the entire four volume, four million word set was inconvenient or difficult. *The New Palgrave Dictionary of Economics, Second Edition* largely resolves the accessibility problem through its online presence. But while the online search facility provides convenient access to specific topics in the now eight volume, six million word *Dictionary of Economics*, no interface has yet been devised that makes browsing from a large online source a pleasurable activity for a rainy afternoon. To our delight, *The New Palgrave’s* publisher shares our view of the joys of dictionary-surfing, and we are thus pleased to present a new series, the “little blue books”, to make some part of the *Dictionary* accessible in the hand or lap for teachers, students, and those who want to browse. While the volumes in this series contain only articles that appeared in the 2008 print edition, readers can, of course, refer to the online *Dictionary* and its expanding list of entries.

The selections in these volumes were chosen with several desiderata in mind: to touch on important problems, to emphasize material that may be of more general interest to economics beginners and yet still touch on the analytical core of modern economics, and to balance important theoretical concerns with key empirical debates. The 1987 Eatwell, Milgate and Newman *The New Palgrave: A Dictionary of Economics* was chiefly concerned with economic theory, both the history of its evolution and its contemporary state. The second edition has taken a different approach. While much progress has been made across the board in the 21 years between the first and second editions, it is particularly the flowering of empirical economics which distinguishes the present interval from the 61 year interval between Henry Higgs’ *Palgrave’s Dictionary of Political Economy* and *The New Palgrave*. It is fair to say that, in the long run, doctrine evolves more slowly than the database of facts, and so some of the selections in these volumes will age more quickly than others. This problem will be solved in the online *Dictionary* through an ongoing process of revisions and updates. While no such solution is available for these volumes, we have tried to choose topics which will give these books utility for some time to come.

Steven N. Durlauf  
Lawrence E. Blume

# Introduction

Game theory was well-represented in *The New Palgrave: A Dictionary of Economics*, with important entries by Robert Aumann, John Harsanyi, Martin Shubik and others. However, timing was such that *The New Palgrave* just missed the renaissance of non-cooperative game theory that took place in the early 1980s. 1982 saw the publication of Kreps and Wilson on sequential equilibrium and the ‘Gang-of-Four’ papers, Rubinstein’s alternating-offer bargaining model, and John Maynard Smith’s *Evolution and the Theory of Games*. While the ideas behind these key works had been percolating at the important workshops and conferences for some time, their publication set off an intellectual land rush into an area that had heretofore been explored only by a small and specialized community. By 1989 game theory had a new journal, *Games and Economic Behaviour*, and the publication of David Kreps’ *A Course in Microeconomic Theory* and then Mas-Colell, Whinston and Green’s *Microeconomic Theory* squeezed general equilibrium theory aside to make game theory an integral part of the graduate microeconomics core.

The game theoretic explosion of the 1980s and 1990s is documented in this volume. The four entries titled ‘Epistemic game theory: . . .’ and the four entries ‘Learning and evolution in games: . . .’ survey the epist and evolutionary foundations of non-cooperative game theory. Important applications covered in this volume include bargaining and mechanism design. Cooperative game theory is represented by ‘Games in coalitional form’ and ‘The Shapley value’. Game theory never has been simply a subfield of economic analysis. ‘Game theory and biology’ surveys the application of evolutionary game theory to biological phenomena. Among the most exciting game theoretic developments of the last several years is the growing interaction between computer scientists and game theory. ‘Computer science and game theory’ and ‘Graphical games’ sample this growing body of work.

The 1950s were a revolutionary decade for the theory of decision making by single individuals. Similarly, the last twenty five years have seen a revolution in the understanding of rational decision making by interacting agents. In his afterword to the 60th anniversary edition of *The Theory of Games and Economic Behaviour*, Ariel Rubinstein writes about the theory of interacting decision making that, ‘it is my impression that the well of game theory is relatively dry.’ The *Dictionary* entries reprinted in this volume are proof of how full that well has been over the past two decades. Game theory has opened economics to the possibilities of modelling directly the interaction of individual economic actors in social situations. In doing so it has dramatically enriched the neoclassical view of markets. The well of decision theory ran

dry in the 1960s only to refill under the influence of behavioural studies of choice. If the well of game theory is now empty, this too is in all likelihood only a hiatus, as our understanding of agent interaction is refreshed with insights from biology, sociology and computer science.

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## **bargaining**

In its simplest definition, 'bargaining' is a socio-economic phenomenon involving two parties, who can cooperate towards the creation of a commonly desirable surplus, over whose distribution the parties are in conflict.

The nature of the cooperation in the agreement and the relative positions of the two parties in the status quo before agreement takes place will influence the way in which the created surplus is divided. Many social, political and economic interactions of relevance fit this definition: a buyer and a seller trying to transact a good for money, a firm and a union sitting at the negotiation table to sign a labour contract, a couple deciding how to split the intra-household chores, two unfriendly countries trying to reach a lasting peace agreement, or out-of-court negotiations between two litigating parties.

In all these cases three basic ingredients are present: (a) the status quo, or the disagreement point, that is, the arrangement that is expected to prevail if an agreement is not reached; (b) the presence of mutual gains from cooperation; and (c) the multiplicity of possible cooperative arrangements, which split the resulting surplus in different ways.

If the situation involves more than two parties, matters are different, as set out in von Neumann and Morgenstern (1944). Indeed, in addition to the possibilities already identified of either disagreement or agreement among all parties, it is conceivable that an agreement be reached among only some of the parties. In multilateral settings, we are therefore led to distinguish pure bargaining problems, in which partial agreements of this kind are not possible because subcoalitions have no more power than individuals alone, from coalitional bargaining problems (or simply coalitional problems), in which partial agreements become a real issue in formulating threats and predicting outcomes. An example of a pure bargaining problem would be a round of talks among countries in order to reach an international trade treaty in which each country has veto power, whereas an example of a coalitional bargaining problem would be voting in legislatures. In this article we concentrate on pure bargaining problems, leaving the description of coalitional problems to other articles in the dictionary. We are likewise not concerned with the vast informal literature on bargaining, which conducts case studies and tries to teach bargaining skills for the 'real world' (for this purpose, the reader is referred to Raiffa, 1982).

### **Approaches to bargaining before game theory**

Before the adoption of game theoretic techniques, economists deemed bargaining problems (also called bilateral monopolies at the time) indeterminate. This was certainly the position adopted by important economic theorists, including Edgeworth (1881) and Hicks (1932). More specifically, it was believed that the solution to a

bargaining problem must satisfy both individual rationality and collective rationality properties: the former means that neither party should end up worse than in the status quo and the latter refers to Pareto efficiency. Typically, the set of individually rational and Pareto-efficient agreements is very large in a bargaining problem, and these theorists were inclined to believe that theoretical arguments could go no further than this in obtaining a prediction. To be able to obtain such a prediction, one would have to rely on extra-economic variables, such as the bargaining power and abilities of either party, their state of mind in negotiations, their religious beliefs, the weather and so on.

A precursor to the game theoretic study of bargaining, at least in its attempt to provide a more determinate prediction, is the analysis of Zeuthen (1930). This Danish economist formulated a principle whereby the solution to a bargaining problem was dictated by the two parties' risk attitudes (given the probability of breakdown of negotiations following the adoption of a tough position at the bargaining table). The reader is referred to Harsanyi (1987) for a version of Zeuthen's principle and its connection with Nash's bargaining theory. The remainder of this article deals with game theoretic approaches to bargaining.

### **The axiomatic theory of bargaining**

Nash (1950) and Nash (1953) are seminal papers that constitute the birth of the formal theory of bargaining. Two assumptions are central in Nash's theory. First, bargainers are assumed to be fully rational individuals, and the theory is intended to yield predictions based exclusively on data relevant to them (in particular, the agents are equally skilful in negotiations, and the other extraneous factors mentioned above do not play a role).

Second, a bargaining problem is represented as a pair  $(S, d)$  in the utility space, where  $S$  is a compact and convex subset of  $\mathbb{R}^2$  – the feasible set of utility pairs – and  $d \in \mathbb{R}^2$  is the disagreement utility point. Compactness follows from standard assumptions such as closed production sets and bounded factor endowments, and convexity is obtained if one uses expected utility and lotteries over outcomes are allowed. Also, the set  $S$  must include points that dominate the disagreement point, that is, there is a positive surplus to be enjoyed if agreement is reached and the question is how this surplus should be divided. As in most of game theory, by 'utility' we mean von Neumann–Morgenstern expected utility; there may be underlying uncertainty, perhaps related to the probability of breakdown of negotiations. We shall normalize the disagreement utilities to 0 (this is without loss of generality if one uses expected utility because any positive affine transformation of utility functions represents the same preferences over lotteries). The resulting bargaining problem is called a normalized problem.

With this second assumption, Nash is implying that all information relevant to the solution of the problem must be subsumed in the pair  $(S, d)$ . In other words, two bargaining situations that may include distinct details ought to be solved in the same

way if both reduce to the same pair  $(S, d)$  in utility terms. In spite of this, it is sometimes convenient to distinguish between feasible utility pairs (points in  $S$ ) and feasible outcomes in physical terms (such as the portions of a pie to be created after agreement).

Following the two papers by Nash (1950; 1953), bargaining theory is divided into two branches, the so-called axiomatic and strategic theories. The axiomatic theory, born with Nash (1950), which most authors identify with a normative approach to bargaining, proposes a number of properties that a solution to any bargaining problem should have, and proceeds to identify the solution that agrees with those principles. Meanwhile, the strategic theory, initiated in Nash (1953), is its positive counterpart: the usual approach here is the exact specification of the details of negotiation (timing of moves, information available, commitment devices, outside options and threats) and the identification of the behaviour that would occur in those negotiation protocols. Thus, while the axiomatic theory stresses how bargaining *should* be resolved between rational parties according to some desirable principles, the strategic theory describes how bargaining *could* evolve in a non-cooperative extensive form in the presence of common knowledge of rationality. Interestingly, the two theories connect and complement one another.

#### *The Nash bargaining solution*

The first contribution to axiomatic bargaining theory was made by John Nash in his path-breaking paper published in 1950. Nash wrote it as a term paper in an international trade course that he was taking as an undergraduate at Carnegie, at the age of 17. At the request of his Carnegie economics professor, Nash mailed his term paper to John von Neumann, who had just published his monumental book with Oskar Morgenstern. John von Neumann may not have paid enough attention to a paper sent by an undergraduate at a different university, and nothing happened with the paper until Nash arrived in Princeton to begin studying for his Ph.D. in mathematics.

According to Nash (1950), a solution to bargaining problems is simply a function that assigns to each normalized utility possibility set  $S$  one of its feasible points (recall that the normalization of the disagreement utilities has already been performed). The interpretation is that the solution dictates a specific agreement to each possible bargaining situation. Examples of solutions are: (a) the disagreement solution, which assigns to each normalized bargaining problem the point  $(0,0)$ , a rather pessimistic solution; and (b) the dictatorial solution with bargainer 1 as the dictator, which assigns the point in the Pareto frontier of the utility possibility set in which agent 2 receives 0 utility. Surely, neither of these solutions looks very appealing: while the former is not Pareto efficient because it does not exploit the gains from cooperation associated with an agreement, the latter violates the most basic fairness principle by being so asymmetric.

Nash (1950) proceeds by proposing four desirable properties that a solution to bargaining problems should have.

1. *Scale invariance or independence of equivalent utility representations.* Since the bargaining problem is formulated in von Neumann–Morgenstern utilities, if utility functions are re-scaled but they represent the same preferences, the solution should be re-scaled in the same fashion. That is, no fundamental change in the recommended agreement will happen following a re-normalization of utility functions; the solution will simply re-scale utilities accordingly.
2. *Symmetry.* If a bargaining problem is symmetric with respect to the 45 degree line, the solution must pick a point on it: in a bargaining situation in which each of the threats made by one bargainer can be countered by the other with exactly the same threat, the two should be equally treated by the solution. This axiom is sometimes called ‘equal treatment of equals’ and it ensures that the solution yields ‘fair’ outcomes.
3. *Pareto efficiency.* The solution should pick a point of the Pareto frontier. As elsewhere in welfare economics, efficiency is the basic ingredient of a normative approach to bargaining; negotiations should yield an efficient outcome in which all gains from cooperation are exploited.
4. *Independence of irrelevant alternatives (IIA).* Suppose a solution picks a point from a given normalized bargaining problem. Consider now a new normalized problem, a subset of the original, but containing the point selected earlier by the solution. Then, the solution must still assign the same point. That is, the solution should be independent of ‘irrelevant’ alternatives: as in a constrained optimization programme, the deleted alternatives are deemed irrelevant because they were not chosen when they were present, so their absence should not alter the recommended agreement.

With the aid of these four axioms, Nash (1950) proves the following result:

**Theorem 1.** There is a unique solution to bargaining problems that satisfies properties (1–4): it is the one that assigns to each normalized bargaining problem the point that maximizes the product of utilities of the two bargainers.

Today we refer to this solution as the ‘Nash solution’. Although some of the axioms have been the centre of some controversy – especially his fourth, IIA, axiom – the Nash solution has remained as the fundamental piece of this theory, and its use in applications is pervasive.

Some features of the Nash solution ought to be emphasized. First, the theory can be extended to the multilateral case, in which there are  $n \geq 3$  parties present in bargaining: in a multilateral problem, it continues to be true that the unique solution that satisfies (1–4) is the one prescribing that agreement in which the product of utilities is maximized. See Lensberg (1988) for an important alternative axiomatization.

Second, the theory is independent of the details of the negotiation-specific protocols, since it is formulated directly in the space of utilities. In particular, it can be applied to problems where the utilities are derived from only one good or issue, as well as those where utility comes from multiple goods or issues.

Third, perhaps surprisingly because risk is not explicitly part of Nash's story, it is worth noting that the Nash solution punishes risk aversion. All other things equal, it will award a lower portion of the surplus to a risk-averse agent. This captures an old intuition in previous literature that risk aversion is detrimental to a bargainer: afraid of the bargaining breakdown, the more risk-averse a person is, the more he will concede in the final agreement. For example, suppose agents are bargaining over how to split a surplus of size 1. Let the utility functions be as follows:  $u_1(x_1) = x_1^\alpha$  for  $0 < \alpha \leq 1$ , and  $u_2(x_2) = x_2$ , where  $x_1$  and  $x_2$  are the non-negative shares of the surplus, which add up to 1. The reader can calculate that the Pareto frontier of the utility possibility set corresponds to the agreements satisfying the equation  $u_1^{1/\alpha} + u_2 = 1$ . Therefore, the Nash solution awards the utility vector  $(u_1^*, u_2^*) = ((\frac{\alpha}{\alpha+1})^\alpha, \frac{1}{\alpha+1})$ , corresponding to shares of the surplus  $(x_1, x_2) = (\frac{\alpha}{\alpha+1}, \frac{1}{\alpha+1})$ . Note how the smaller  $\alpha$  is, the more risk-averse bargainer 1 is.

Fourth, Zeuthen's principle turns out to be related to the Nash solution (see Harsanyi, 1987): in identifying the bargainer who must concede next, the Nash product of utilities of the two proposals plays a role. See Rubinstein, Safra and Thomson (1992) for a related novel interpretation of the Nash solution.

Fifth, the family of asymmetric Nash solutions has also been used in the literature as a way to capture unequal bargaining powers. If the bargaining power of player  $i$  is  $\beta_i \in [0, 1]$ ,  $\sum_i \beta_i = 1$ , the asymmetric Nash solution with weights  $(\beta_1, \beta_2)$  is defined as the function that assigns to each normalized bargaining problem the point where  $u_1^{\beta_1} u_2^{\beta_2}$  is maximized.

#### *The Kalai–Smorodinsky bargaining solution*

Several researchers have criticized some of Nash's axioms, IIA especially. To see why, think of the following example, which begins with the consideration of a symmetric right-angled triangle  $S$  with legs of length 1. Clearly, efficiency and symmetry alone determine that the solution must be the point  $(1/2, 1/2)$ . Next, chop off the top part of the triangle to get a problem  $T \subset S$ , in which all points where  $u_2 > 1/2$  have been deleted. By IIA, the Nash solution applied to the problem  $T$  is still the point  $(1/2, 1/2)$ .

Kalai and Smorodinsky (1975) propose to retain the first three axioms of Nash's, but drop IIA. Instead, they propose an individual monotonicity axiom. To understand it, let  $a_i(S)$  be the highest utility that agent  $i$  can achieve in the normalized problem  $S$ , and let us call it agent  $i$ 's aspiration level. Let  $a(S) = (a_1(S), a_2(S))$  be the utopia point, typically not feasible.

5. *Individual monotonicity.* If  $T \subset S$  are two normalized problems, and  $a_j(T) = a_j(S)$ , the solution must award  $i$  a utility in  $S$  at least as high as in  $T$ .

We can now state the Kalai–Smorodinsky theorem:

**Theorem 2.** There is a unique solution to bargaining problems that satisfies properties (1, 2, 3, 5): it is the one that assigns to each normalized bargaining problem the intersection point of the Pareto frontier and the straight line segment connecting 0 and the utopia point.

Note how the Kalai–Smorodinsky solution awards the point  $(2/3, 1/3)$  to the problem  $T$  of the beginning of this subsection. In general, while the Nash solution pays attention to local arguments (it picks out the point of the smooth Pareto frontier where the utility elasticity  $(du_2/u_2)/(du_1/u_1)$  is 1), the Kalai–Smorodinsky solution is mostly driven by ‘global’ considerations, such as the highest utility each bargainer can obtain in the problem.

#### *Other solutions*

Although the two major axiomatic solutions are Nash’s and Kalai–Smorodinsky’s, authors have derived a plethora of other solutions also axiomatically (see, for example, Thomson, 1994, for an excellent survey). Among them, one should perhaps mention the egalitarian solution, which picks out the point of the Pareto frontier where utilities are equal. This is based on very different principles, much more tied to ethics of a certain kind and less to the principles governing bargaining between two rational individuals. In particular, note how it is not invariant to equivalent utility representations, because of the strong interpersonal comparisons of utilities that it performs.

### **The strategic theory of bargaining**

Now we are interested in specifying the details of negotiations. Thus, while we may lose the generality of the axiomatic approach, our goal is to study reasonable procedures and identify rational behaviour in them. For this and the next section, some major references include Osborne and Rubinstein (1990) and Binmore, Osborne and Rubinstein (1992).

#### *Nash’s demand game*

Nash (1953) introduces the first bargaining model expressed as a non-cooperative game. Nash’s demand game, as it is often called, captures in crude form the force of commitment in bargaining. Both bargainers must demand simultaneously a utility level. If the pair of utilities is feasible, it is implemented; otherwise, there is disagreement and both receive 0. This game admits a continuum of Nash equilibrium outcomes, including every point of the Pareto frontier, as well as disagreement. The first message that emerges from Nash’s demand game is the indeterminacy of equilibrium outcomes, commonplace in non-cooperative game theory. In the same paper, advancing ideas that would be developed a couple of decades later, Nash proposed a refinement of the Nash equilibrium concept based on the possibility of uncertainty around the true feasible set. The result was a selection of one Nash equilibrium outcome, which converges to the Nash solution agreement as uncertainty vanishes.

The model just described is referred to as Nash’s demand game with fixed threats: following an incompatible pair of demands, the outcome is the fixed disagreement point. Nash (1953) also analysed a variable threats model. In it, the stage of simultaneous demands is preceded by another stage, in which bargainers choose



threats. Given a pair of threats chosen in the first stage, the refinement argument is used to obtain the Nash solution of the induced problem in the ensuing subgame (where the threats determine an endogenous disagreement point). Solving the entire game is possible by backward induction, appealing to logic similar to that in von Neumann's minimax theorem; see Abreu and Pearce (2002) for a connection between the variable threats model and repeated games.

*The alternating offers bargaining procedure*

The following game elegantly describes a stylized protocol of negotiations over time. It was studied by Stahl (1972) under the assumption of an exogenous deadline (finite horizon game) and by Rubinstein (1982) in the absence of a deadline (infinite horizon game). Players 1 and 2 are bargaining over a surplus of size 1. The bargaining protocol is one of alternating offers. In period 0, player 1 begins by making a proposal, a division of the surplus, say  $(x, 1 - x)$ , where  $0 \leq x \leq 1$  represents the part of the surplus that she demands for herself. Player 2 can then either accept or reject this proposal. If he accepts, the proposal is implemented; if he rejects, a period must elapse for them to come back to the negotiation table, and at that time (period 1) the roles are reversed so that player 2 will make a new proposal  $(y, 1 - y)$ , where  $0 \leq y \leq 1$  is the fraction of surplus that he offers to player 1. Player 1 must then either accept the new proposal, in which case bargaining ends with  $(y, 1 - y)$  as the agreement, or reject it, in which case a period must elapse before player 1 makes a new proposal. In period 2, player 1 proposes  $(z, 1 - z)$ , to which player 2 must respond, and so on. The  $T$ -period finite horizon game imposes the disagreement outcome, with zero payoffs, after  $T$  proposals have been rejected. On the other hand, in the infinite horizon version, there is always a new proposal in the next period after a proposal is rejected.

Both players discount the future at a constant rate. Let  $\delta \in [0, 1)$  be the per period discount factor. To simplify, let us assume that utility is linear in shares of the surplus. Therefore, from a share  $x$  agreed in period  $t$ , a player derives a utility of  $\delta^{t-1}x$ . Note how utility is increasing in the share of the surplus (monotonicity) and decreasing in the delay with which the agreement takes place (impatience).

A strategy for a player is a complete contingent plan of action to play the game. That is, a strategy specifies a feasible action every time a player is called upon to act in the game. In a dynamic game, Nash equilibrium does little to restrict the set of predictions: for example, it can be shown that in the alternating offers games, any agreement  $(x, 1 - x)$  in any period  $t$ ,  $0 \leq t \leq T < \infty$ , can be supported by a Nash equilibrium; disagreement is also a Nash equilibrium outcome.

The prediction that game theory gives in a dynamic game of complete information is typically based on finding its subgame perfect equilibria. A subgame perfect equilibrium (SPE) in a two-player game is a pair of strategies, one for each player, such that the behaviour specified by them is a best response to each other at every point in time (not only at the beginning of the game). By stipulating that players must choose a best response to each other at every instance that they are supposed to act, SPE rules out incredible threats: that is, at an SPE players have an incentive to carry out the