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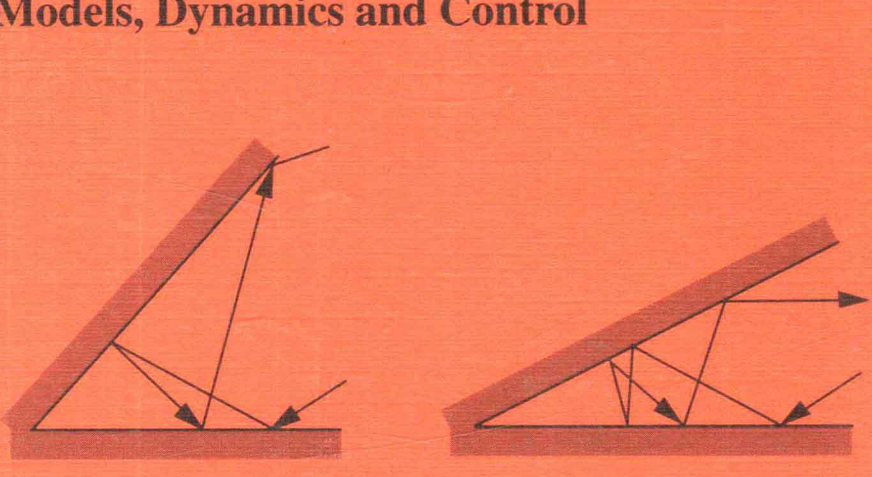
Bernard Brogliato

Nonsmooth Impact Mechanics

Models, Dynamics and Control



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Preface

This monograph is devoted to the study of a class of dynamical systems of the general form

$$\begin{aligned} \dot{x} &= g(x,u) \\ f(x,t) &\geq 0 \end{aligned} \tag{0.1}$$

where $x \in I\!R^n$ is the system's state vector, $u \in I\!R^m$ is the vector of inputs, and the function $f(\cdot,\cdot)$ represents a unilateral constraint that is imposed on the state. More precisely, we shall restrict ourselves to a subclass of such systems, namely mechanical systems subject to unilateral constraints on the position, whose dynamical equations may be written as

$$\begin{aligned} \ddot{q} &= g(q,\dot{q},u) \\ f(q,t) &\geq 0 \end{aligned} \tag{0.2}$$

where $q \in I\!R^n$ is the vector of generalized coordinates of the system and u is an input (or controller) that generally involves a state feedback loop, i.e. $u = u(q, \dot{q}, t, z)$, with $\dot{z} = Z(z, q, \dot{q}, t)$ when the controller is a dynamic state feedback. Mechanical systems composed of rigid bodies interacting fall into this subclass. A general property of systems as in (0.1) and (0.2) is that their solutions are nonsmooth (with respect to time): Nonsmoothness arises from the occurence of *impacts* (or *collisions*, or *percussions*) in the dynamical behaviour, when the trajectories attain the surface $f(x,t) = 0$. They are necessary to keep the trajectories within the subspace $\Phi = \{x : f(x,t) \geq 0\}$ of the system's statespace. It is therefore necessary, when dealing with such classes of dynamical systems, to focus on collisions dynamics.

Impact phenomenons between perfectly rigid bodies have been studied by scientists since a long time. It is worth noting that the problems related to impact dynamics have attracted the interest of physicists for at least 3 centuries. The impact physical laws were in particular studied and used initially by scientists like Newton [397] [88], Poisson [440], Huygens [225], Coriolis [113], the well-known Newton's and Poisson's restitution coefficients being still well alive as basic models for rigid bodies collisions, and later by Darboux [120], Routh [461], Appell [14] and others [583] [388] [321]. Although this fact has been a little forgotten now, rigid body (or more exactly particles) shock dynamics were extensively used in the seventeeth century to study light models [225] and also by artillerists [333] to predict the flight of cannon balls and their impacts. They are curently still used to describe motion of molecules in ideal gas [514] [105], in relationship with complex dynamical behaviour of so-called *billiards*[1]. Mathematicians (for problems related to existence of solutions to specific dynamical problems, analysis of complex dynamics of certain impacting systems like billiards[2].), researchers from mechanical engineering, robotics (to study the effect of impacts in the joints or the motion of the system after the impact, like in robot manipulators, bipede, juggling or hopping robots, multifingered hands, $\cdots$), computer sciences (graphics, virtual reality) have also been interested (and are still) by rigid body dynamics with shocks (or more exactly, as we shall see throughout this book, with *unilateral constraints*).

The interest of scientists from so different horizons for rigid bodies dynamics and unilateral constraints (which include impact dynamics) can be explained from the fact that, as we shall see, rigidity allows simplification of the dynamical contact-impact problem (an important point for engineers who have to design systems subject to percussions), but at the same time involves some deep mathematical problems (related to existence, uniqueness and approximating procedures of solutions of problems with unilateral constraints, or *nonsmooth dynamics*). Rigid body dynamics also yield strong mechanical problems, as it is well-known in mechanisms theory that rigidity may sometime create some sorts of undetermined problems (with no or several solutions). It can then become important (think of graphics and virtual reality) to possess fast enough numerical algorithms capable of providing "believable" motions (i.e. solutions not too far from what most of the experiments show).

It seems that dynamics of mechanical systems with unilateral constraints have been a little neglected in the general mechanical literature (in general, the textbooks contain -when they do- a single chapter or paragraph where impacts and/or friction dynamics are treated in a rather rapid manner. In a survey, Barmes [34] reviewed 69 textbooks containing a section on collisions. According to him, only 6 of them were theoretically sound, and only one was acceptable [442]. Certainly he was not aware of Pérès' book [429], although published before the survey was written, and of other books not written in English). One of the main reasons for this situation must be the algebraic form of shock dynamics between rigid bodies. These algebraic equations have led many people in the mechanical engineering field to consider impact dynamics as a very simple, hence not very interesting topic. One of the goals of this monograph is to prove that it is rather the contrary which is true: Dynamics of systems with unilateral constraints are far from yielding simple analysis and require sophisticated mathematical tools to be well understood. On the other hand, shocks and friction macroscopic models are still an active research area.

[1] In the literature, it seems that the word *vibro-impact systems* is used in the mechanical engineering field to name various types of systems that involve percussions. The word *billiards* refers to theoretical models of particles colliding in a closed domain, and is used mainly in mathematical physics.

[2] There has been in the past decades a great mathematical effort devoted to the study of billiards in relationship with Boltzmann's conjecture on the stochastic properties of motion of elastic balls in a closed domain, with elastic collisions, see e.g. the celebrated works of the Russian mathematician Sinai [493] [494], see also the book [292], and references therein. The study of billiards started with Coriolis [113].

The study of dynamical systems subject to impacts can be split into different classes, that more or less reflect the interest of the different groups of researchers listed above (Historically, contact-impact problems have always been studied by analytical, numerical and experimental methods [404]). Among them one may find the following ones[3]:

i) Study the *well-posedness* of impact dynamical equations, i.e. properties of solutions (existence, uniqueness, continuous dependence on initial conditions and parameters, stability ...) of differential equations (or inclusions) containing *measures*, that model contact percussions. Distributions and measures theory is at the core of this studies. This may also include the study of convergence of smooth problems towards nonsmooth ones, as well as the variational formulation of percussion dynamics.

ii) Study the global motion of systems subject to *unilateral constraints* (The preceding step being necessary but far from sufficient to solve this one). This includes in particular investigation of complex nonsmooth dynamics of multibody systems with unilateral constraints, with particular emphasis on uniqueness of the solution, problems of simultaneity and possible break of contacts.

iii) Study the impact between two rigid bodies *via* macroscopic laws that relate the motion after and before the shocks, without explicit model of contact in the compliant bodies case, in particular extend basic laws such as Newton's or Poisson's restitution laws when there is friction and slip during the impact. Study the extension of restitution rules for complex mechanical systems submitted to several unilateral constraints.

iv) Develop new sophisticated models of contact-impact laws based on Hertz, Cattanéo-Mindlin theories, finite element methods...

v) Study dynamics of systems subject to impacts (like the bouncing-ball as a benchmark example, or the linear oscillator) as *discrete-time* systems (Poincaré maps), the instants of discretization being the impact times. The core of these works is the generally complex behaviour that result from combination of very simple continuous and discrete dynamics.

vi) Investigate the complex dynamics (periodic trajectories, bifurcations, chaotic behaviour) of vibro-impact systems like the impact oscillator , that may model systems like mechanisms with clearance (In machine-tools, space structures...), or of billiards.

vii) Study numerical algorithms to integrate systems subject to unilateral constraints.

viii) Develop experimental devices and systems to substantiate the analytical models and predictions.

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[3]This list may not be exhaustive.

ix) Design control strategies to improve the behaviour of mechanical systems subject to repeated impacts with an environment (impacts are here considered as disturbances), or on the contrary use "percussive controllers" to stabilize a system.

The primary objective in doing this work was about control of mechanical systems with unilateral constraints, and more expressly extension of classical schemes on hybrid force/position control of manipulators submitted to holonomic frictionless constraints [324] [606]. As noted in [72], *...it is an oversimplification to imagine that the goal of a robot is simply to traverse a prescribed path in position and orientation space.*, and impacts play a major role in many robotic manipulation tasks. Also, *the contact problem is unsolved for rigid manipulator, rigid sensor, rigid environment problems.* [422], and a necessary step towards a solution is a clear understanding of percussion dynamics. The subject may well possess other important applications as we noted above, like bipedal locomotion, which has become an important topic of investigations within the mechanical engineering and robots control communities, and whose dynamics are generally based on rigid bodies impacts assumptions. Also the control of all mechanisms with clearances taking into account impact dynamics is a topic deserving future works. At some places of the book applications of impact dynamics are mentioned. Consequently, this work aims at gathering and if possible comparing different works that concern rigid percussion dynamics, and also at giving an overview as clear and as complete as possible on the very nature of rigid body impact dynamics. It is clear that mathematicians will not learn much mathematics here, and that mechanicians will not learn much mechanics! But I hope that those who desire to get acquainted with this topic and get a general point of view on it, will find what they are looking for, whatever their background may be.

Some topics (points **iv**,**vi**, **viii**) are not or only partially covered in this monograph. Point **vii** is rapidly commented. The interested reader can consult concerning **iv** the papers [243] [244] [614] where more than 500 references can be found on the subject. A survey concerning point **vi** on systems with clearances can be found in [184], although many studies on dynamics of such systems (points **v**,**vi**, **vii**) have been done since that time. Here only a small part of point **vi** will be treated, concerning existence and stability of periodic trajectories. I admit that some place could have been devoted these problems, and that they certainly are of interest to the control of mechanical systems. One therefore realizes how numerous and various the problems involving impacts are, and that they can hardly be treated all at the same time!

The work rather focusses on points **i**,**ii**,**iii**, **v** and **ix**, and I try to introduce the different works that have been devoted to this field in a logical way. Also since the goal is to provide solid theoretical bases for control theory of impacting mechanical systems, I insist on the mathematical sides of the subject (which is a necessary step to study stability of such systems and extend previous works on control of constrained manipulators [324] [606]). A state-of-the-art of percussion models that may be useful in this context is also presented.

To the best of my knowledge it is the first time that all the presented works are gathered and discussed[4]. This monograph is therefore to be considered as what should be the first version of a more detailed and complete work, making all the various sides of nonsmooth dynamics accessible in one volume. I am therefore open to all the remarks and comments one might have on my work. Although this might not be too important, let us mention that around 90 references have been found in the journals ASME J. of Applied Mechanics (34 papers), PMM USSR translated in J. of Applied Mathematics and Mechanics (30 papers) and the journal of Sound and Vibrations (24 papers), which seem to provide the most important source of references on rigid body impact dynamics (around 15 percent of the total number of references, but a much higher percentage of journal papers). Also 22 papers from the American Journal of Physics (dedicated to teachers) prove that impact dynamics are present in teaching. From my own recent experience I feel that there are some "gaps to fill" between researchers from very different horizons that work on the same topic. Control theory and practice of mechanical systems with unilateral constraints may be a domain requiring knowledge from all the other parts, hence the necessity to have a general point of view on this topic. Finally the interested reader is invited to have a look at some other recently published monographs with related contents: The book of R.M. Brach [54] is oriented towards rigid body impacts and Newton like restitution rules, sustained by many experimental results, and belongs to the Mechanical Engineering field. In [292], V.V. Kozlov and D.V. Treshchev provide a detailed analysis of billiards dynamics. In [366], M. Monteiro-Marques deals with existential results for the sweeping process evolution problem. Both monographs [292] and [366] belong to the Applied Mathematics field, although the first one is oriented towards dynamical systems analysis (see also parts of [262] and [181] dedicated to that field), whereas the second one belongs to the functional analysis field. Let us also mention the book by D.D. Bainov and P.S. Simeonov [27] which deals with stability of dynamical systems submitted to impulses, although the systems analyzed in [27] do not fit with systems subject to unilateral constraints (mainly the authors analyze systems subject to impulses at fixed exogeneous instants). Among older books on mechanics, the ones by Pèrés [429] and by Moreau [376] contain an important chapter dedicated to rigid body impacts. Many other books contain a part devoted to this topic, and it is neither possible nor worth citing them here. There are also several monographs on impact dynamics and vibro-impact systems in the Russian literature, whose references can be found in most of the bibliographical notes of the cited translated Russian papers.

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[4]I have not included in the bibliography those references not written in English or French (except for very few in Italian and German); This is due to the fact that I am confident only with these two languages, and that it is already hard enough to understand science when it is explained in your native language! In particular, many references in Russian and German are not included, but can be found in the cited ones.

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