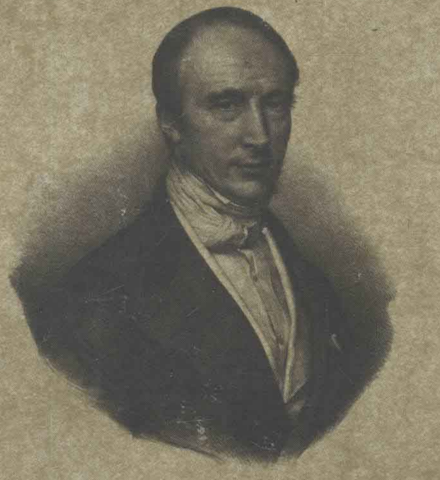
about mathematics



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about

to gretchen
without whose assistance and
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never have been finished.

preface

This book began as a set of notes for a course offered to liberal arts students at Syracuse University. The majority of these students were freshmen and most of them had decided that their principal interests were not in the sciences. The purpose of the course was to provide these students with a better understanding of the nature of mathematics.

The person whose contact with mathematics has been limited to the high-school courses in geometry and algebra has usually received an erroneous impression about what mathematics is and what mathematicians do. The geometry that he learned, for the most part, was known and compiled by the time of Euclid in 300 B.C. Although the notation used is more recent, the algebraic techniques that he has seen are almost as old. He has little or no knowledge of the mathematics developed in the last 300 years. That such a person should find it difficult to understand how anyone could enjoy or appreciate mathematics should not be surprising; most mathematicians find very little to appreciate in high-school mathematics.

Those students who take "serious" courses in mathematics usually find themselves in a similar position. The courses that they take offer them thorough discussions of very limited topics in mathematics. Seldom, if ever, are they provided with any significant amount of background or a framework for the mathematics they have learned.

It is hoped that this book will be of interest and value to all those who wish to know more *about* mathematics. The mathematical prerequisites for its use are found in the traditional high-school algebra and geometry courses. The reader must also have the perseverance, however motivated, to carefully consider a number of new, often nontrivial, concepts.

The first part of the book is devoted to a historical introduction that considers the mathematics known in the ancient civilizations and the contributions of the Greeks. Each of the other four parts of the book contains a more or less chronologically ordered discussion of the development of ideas and techniques that are important in modern mathematics. The emphasis throughout is on concepts rather than on computational agility or logical rigor. Each part of the book begins with topics that should be recognizable as mathematics to the student and traces the growth and shift of emphasis within a major branch of mathematics. The goal is to explain, by showing the actual historical stages of the metamorphosis, what mathematics is and why it has become the way it is.

There are exercises and discussion questions at the end of each chapter. The exercises are routine and generally phrased so that the answer is provided. They are designed to allow an easy check on the more computational aspects of the material. The discussion questions, on the other hand, seldom have brief or even clear-cut answers. They are meant to be thought about and discussed. The discussions can proceed at a variety of levels depending on the mathematical sophistication of the participants. They may also be used as the starting points for "independent research" by students.

The table of contents should provide those with some mathematical background with sufficient information about the selection of topics. These topics are, in general, simply surveyed and the basic concepts introduced. If each were treated vigorously or in detail, there would be material sufficient for several courses. By proceeding at a fairly rapid pace and resisting the temptation to elaborate further, the material can be covered in a one-semester course by students of average ability and background.

A two-semester course for those less well prepared is possible by proceeding at a slower pace and making judicious use of the problems. Students who have had several previous mathematics courses can complete the entire book in one semester with ample time for discussion of all the problems and the numerous related questions that will arise naturally. (The instructor will undoubtedly find the experience stimulating.)

I wish to acknowledge the assistance and cooperation given me by the staff of the George Arents Research Library and Mrs. Nancy Rude of the Mathematics Library at Syracuse University. I wish also to thank my colleagues William Groening and J. Kevin Doyle for the suggestions they made on the basis of their use of the manuscript of the book.

R. S. H.

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part one

introduction

one

mathematics in ancient civilizations

Modern mathematics is the result of centuries of development and refinement. Each succeeding generation of mathematicians has built upon the work of those that preceded. As a result it is necessary to know something of the history of mathematics when trying to understand the nature of the mathematics of today.

No one knows, of course, when man first began to do mathematics. As early as 30,000 B.C. our ancestors carved symbols on animal bones to keep a count of the progress of the phases of the moon. A carved bone with markings that may be a primitive multiplication table has been found near Lake Edward in central Africa and probably dates back to 6000 B.C.

Later civilizations have left massive evidence of the ability to make extensive and accurate measurements. The Great Pyramid at Giza, for example, was built about 2600 B.C. and the errors made in laying out the sides and angles of its square base are extremely small. The famous stone ruins at Stonehenge in England are apparently the remains of a structure built about the same time. The ruins consist of giant stones that have been accurately positioned in large circles. It has been shown recently that the number and location of the stones is such that they can be, and may have been originally, used to predict the movements of the sun and moon in the sky and thereby predict the changes in the seasons and the occurrences of eclipses.[1]

In general, little tangible evidence of mathematical activity has survived from civilizations earlier than those that flourished in Egypt and

[1] For a discussion of one of the recent theories, see Gerald S. Hawkins, *Stonehenge Decoded* (New York: Dell Publishing Co., 1965).

Mesopotamia beginning about 2000 B.C. The sophistication of the mathematics done at that time indicates, however, that man had been doing simple arithmetic and practical geometry for generations. It is only because of the climate in these two regions and because durable materials were chosen on which to inscribe records that we have a considerable amount of information about the mathematical abilities and accomplishments of the Egyptians and the Babylonians.

Egyptian Mathematics

The most extensive record of Egyptian mathematics known today is the papyrus found by A. H. Rhind in 1858. This papyrus is a copy made about 1650 B.C. of an older work that has not survived. The prefatory remarks in the text state that the content has to do with "accurate reckoning" and that "mysterious secrets" are to be revealed.[2] The text of the papyrus is written in the hieratic, or sacred, script rather than the demotic script used for everyday matters. It is reasonable to infer from this that the facts contained in the papyrus were known only to a select group of people.

It was possible to translate the Rhind papyrus almost immediately because of the knowledge gained from the Rosetta Stone which was found by officers of Napoleon's army in 1799. This stone contains an inscription carved in triplicate in Greek, hieroglyphic, and demotic. (Hieroglyphic is the form of hieratic used when carving on stone.) The Rhind papyrus was found to be a textbook for government officials and consists of a large number of practical problems. The solution was provided for each problem. The reader of the papyrus was expected to be able to add and subtract positive whole numbers. The details of multiplying two numbers were shown each time it was done.

Multiplication was accomplished by repeatedly doubling one of the numbers and then adding the appropriate "doubles" to form the product. For example, the products $6 \cdot 13$ and $11 \cdot 13$ would have been found as shown below.

1	13		\1	13	
\2	26		\2	26	
\4	52	Total 78	4	52	
			\8	104	Total 143

[2] A. B. Chace, L. S. Bull, H. P. Manning, and R. C. Archibald, *The Rhind Papyrus* (Oberlin, Ohio: M.A.A., 1927–1929), two volumes.