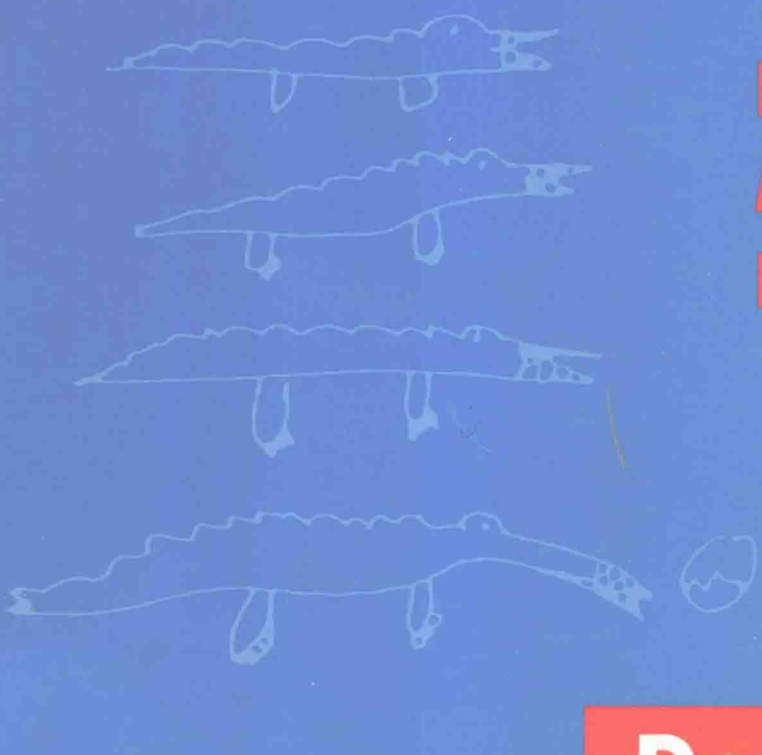
What's Your Problem?

Posing and Solving Mathematical Problems, K–2



Penny Skinner

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HEINEMANN
Portsmouth, NH

For my father, Peter Sharp, and my sister, Deborah Rudder, thanking them for their interest.

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Preface

This book describes the progress over two and a half years of one class of children in a problem-centred mathematics program.

I taught the class from midway through their Kindergarten year in 1985 until the end of 1987 at Macgregor Primary School in the A.C.T.

In outlining here how the program developed I hope to provide information that may help other teachers establish their own programs.

The program can be used in conjunction with any mathematics curriculum for its central concern is teaching style, not content. The teaching emphasis is upon the presentation of mathematical concepts and skills through problems, many of which should be generated by the children themselves.

Contents

Introduction

As problems are central to "real world" mathematics and, I believe, to effective learning, I made them the focus of the mathematics program described in this book.

What is a problem?

There is some debate in educational circles as to what constitutes a problem. Before embarking on my program I established a working definition of a problem.

I considered the many types of mathematical problems that people encounter outside schools. They range from simple "How many?", "How much?" and "How big?" problems to more complex ones involving extensive investigation. An essential aspect of each problem is that the person attempting to solve it needs or wants to do so. This became the premise for the program. The children were involved and engaged by the problems. They wanted to solve them.

My definition of a problem, stated most simply, is this:

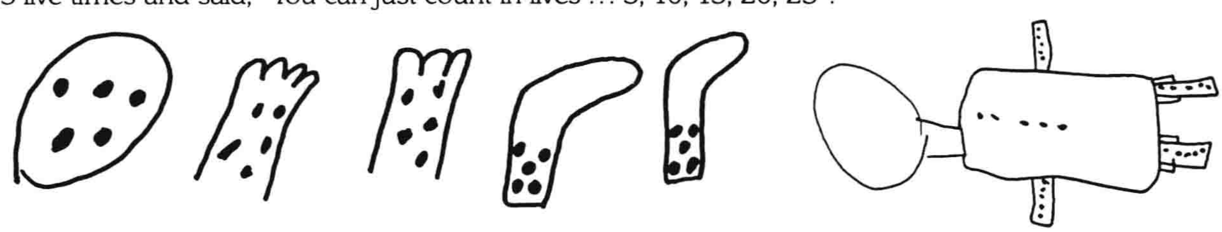
> **A problem is a question which engages someone in searching for a solution.**

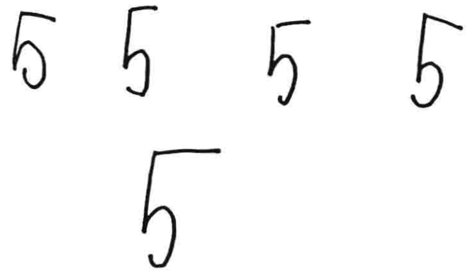
This definition implies two essential conditions of a problem:

- A problem cannot exist separately from the person who *poses* and *solves* it. Posing problems is as important as solving them. A feature of "real world" problems is that they are usually solved by the person who has posed them. Thus they fully engage the problem solver.
- The problem-solver has to *search* for a way to find a solution. A problem is not simply answered by memory recall. The children and I have a set of potential problems, each one written as a story in book form. One child aptly called them "trouble books". Problem-solvers have to go to some trouble to find answers. One of my books poses this problem:

Tim has chicken pox. He has five spots on each leg, and five spots on each arm, and five spots on his tummy. How many chicken pox spots does Tim have?

When the kindergarten children read this problem, they had to work out a way to solve it. Some of them asked the real Tim in the class to lie on the floor while they put red counters on his arms, legs and tummy. Tim was not too happy about this strategy, and the counters rolled off his body as he protested. The children had to devise more successful strategies. Corinne drew around Tim as he lay on a length of paper. Lisa said, "You can draw a little picture of Tim and put dots on it". Tim himself wrote the numeral 5 five times and said, "You can just count in fives . . . 5, 10, 15, 20, 25".





A wide range of types of questions can form the basis of a problem. Even a question like, "How many toy animals are here?" is a problem if it fulfills the two conditions described.

"Real world" problems

Because "real world" problems form the basis of my definition, they thereby fulfill the two conditions of what constitutes a problem. Teachers need to be alert to any real mathematical problems that arise and allow the children to work out their own ways to solve them.
"How many writing pencils do we need to order?"
"How can we cut the birthday cake into thirty pieces?"
"How shall we divide the class into teams?"
"What rules do we need for this game?"

Contrived problems

To ensure a balance in mathematical content it is sometimes necessary to provide contrived problems. In using contrived problems, there is the danger that the problem-solver has no real purpose in solving them and thus is not engaged by them. The use of contrived problems that appear in teacher resource books is often merely gimmicky. They can indeed become useful problems, but only if adopted by the problem-solver for some purpose beyond satisfying a teacher's requirements.

In this program, the contrived problems and those found in resource books were useful as starting points for the children's work. They were used to introduce ideas or as the basis for discussion. The children often extended these problems by posing related ones. Many contrived problems were presented in book form so the children could work on them in their own time.

Contrived problems as a starting point

Brady and Philip are working with the problem books at quiet reading time. The one they are reading begins with three carrots, so the boys have put out three orange Unifix cubes to represent the carrots.

These problems were useful because they motivated the children to write their own problem books and they presented a range of problem structures.

The children's problems

The most common type of problem the children posed was the story problem which involved operating on numbers.

Amber wrote the following problem.

There was a gang of robbers. The robbers broke into a bank. They stole all the money. They left 50 fingerprints on the safe. How many robbers were there?

Amber's problem was shared with the class later in the day.

The children produced problems in response to many curriculum areas. Topics dealt with in science, social studies, art and literature gave the children ideas for problems. One group wrote the following "poem" that became a problem:

The 11 little girls.
There were 11 little pillows they
found 11 little beds. They lay
down on the 11 little beds.
Along came 11 little girls and
lay in the 11 little beds. And
had 11 little dreams.
They dreamed of 11 little pets.

How many all together?

Solving problems

The children's story problems engaged the class in problem solving activity as the children were keenly interested in each other's work. They gave each other feedback on the appropriateness of the problems, and they extended each other's ideas.

At first most of the children's story problems involved simple addition, multiplication, subtraction and division. The problem solvers had to determine which operations to use and how to apply them effectively. When the children worked on the problems, they mostly used concrete materials such as Unifix cubes and MAB blocks. They constantly refined their strategies for using such materials. For instance, early strategies included grouping the Unifix cubes in tens, placing the Unifix cubes or MAB blocks (wooden blocks based on a unit measurement of one cubic centimetre, similar to Dienes blocks or Base 10 blocks) in ordered arrangements rather than haphazardly, and exchanging "tens" and "ones".

The children's problems grew in complexity and required more sophisticated solutions. Over time, they found written calculations helpful in solving problems.

Some problems required the children to do some research. Here are two examples:

There were 4 dinosaurs in days of long ago.
Heptaceratops uniceratops
Pentaceratops triceratops
How many horns altogether?

The martian ate all of Jupiter's moons
and all Africa's Deserts.

how many things did the martian eat?

Other problems led to quite extensive investigations, like this one from Craig:

27/10/87

To Mrs Skinner,
Here is a problem for the class to do on the calculaters. and this is true. playschool is 21 years old. And there's two shows a day. One show byitself is half-an-hour. I want to find out how many hours it would be all together. but! if there's 365 day's in a year (not including a leap year) playschool only on from Monday - Friday so how do you workit out?

from Craig

p.s I've allready worked it out

Realising the potential of contrived problems

The children's problems did not cover all curriculum topics so it was necessary to find a way to introduce them. Contrived problems were used as a starting point and I encouraged the children to extend them.

When using a contrived problem I often pretended I was making it up and thus modelled problem posing. By my giving reasons for posing each problem the children felt encouraged to contribute ideas.

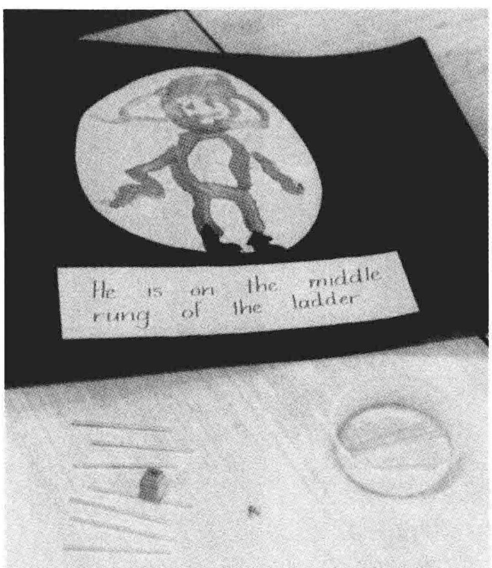
The topic of odd and even numbers illustrates how the children adopted and extended contrived problems. The children had already investigated what odd and even numbers were before commencing this work.

A group of children wrote a story which began: "A building is on fire! Here comes the fire engine! Here comes the fireman! The fireman is climbing a ladder."

I recalled a problem I had read elsewhere and used it to extend the children's story. Before writing, I explained to the class that I was going to make up some clues about the size of the ladder. The clues were:

He is on the middle rung of the ladder. He goes up three more steps to get to the top of the ladder. How many rungs are on the ladder?

We made our story into an illustrated book and the children devised ways to work out the answer to the problem. Alison used straws for the rungs of the ladder.

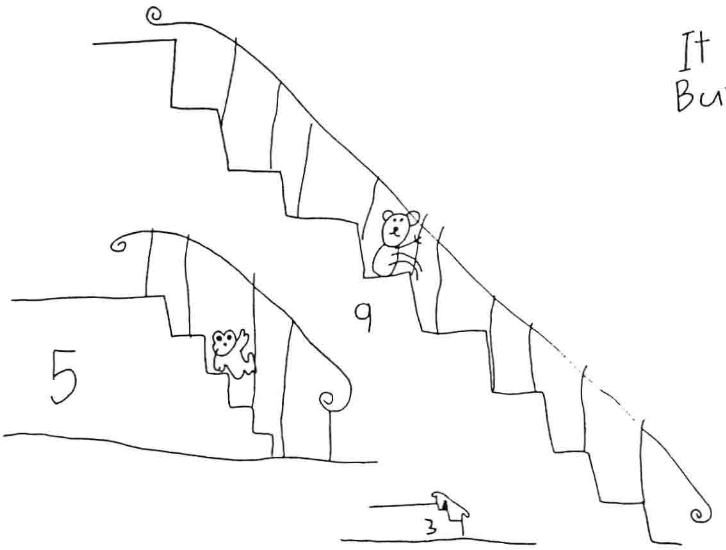


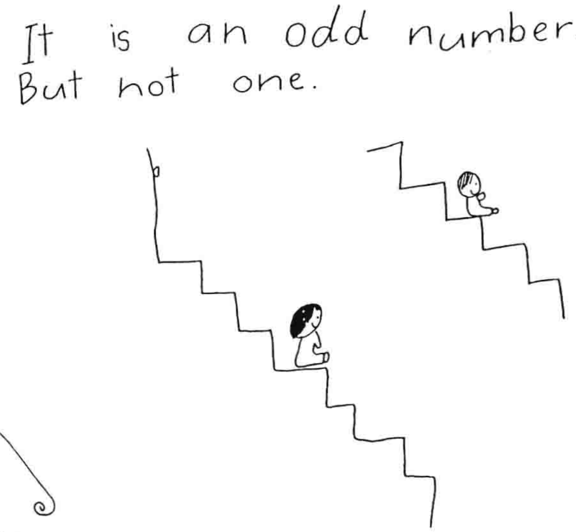
The children extended the problem by changing the number of rungs on the ladder and then making up some clues of their own but always starting with the firemen on the middle rung. Several children noted the ladder always had an odd number of rungs and we discussed why this was so.

Some time later I read to the class A.A. Milne's poem "Halfway down", in which a child sits on the middle stair of a staircase. One child asked how many stairs there were in the staircase and the class then investigated how many there might be.

Some children made models of staircases. Teresa's is pictured overleaf.

Some children drew staircases, like these ones by Shannon and Cameron.



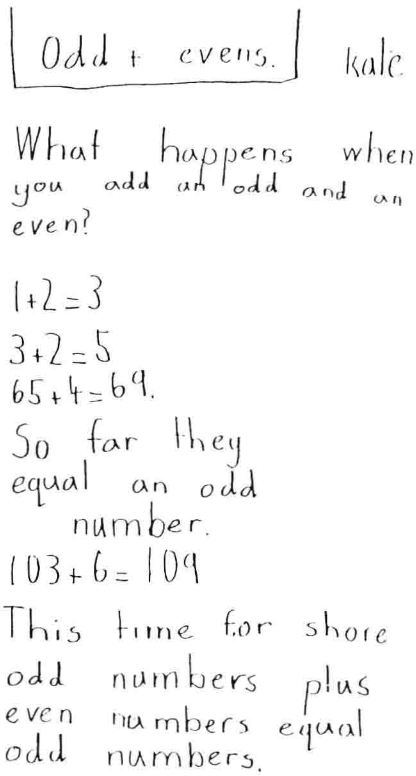


In a follow-up discussion the children concluded that there could be any odd number of stairs except one, because you cannot have a middle stair if there is an even number of stairs, or if there is only one. The children looked at the illustration in the A.A. Milne book which shows a child sitting on a stair, the stairs above that one, but not the stairs below. The picture reminded some of them about the fireman's ladder problem and we discussed the relationship between the two situations.

I asked the children if they could pose any other problems about odd numbers. This led them to pose problems focusing on the addition of odd and even numbers:

- What happens if you add two odd numbers?
- What happens if you add three odd numbers?
- What happens if you add any odd number of odd numbers?
- What happens if you add two even numbers?
- What happens if you add one even number and one odd number?

These questions formed the basis of investigations like this one:


Odd + evens. Kate

What happens when you add an odd and an even?

1+2=3
3+2=5
65+4=69.
So far they equal an odd number.
103+6=109
This time for shore odd numbers plus even numbers equal odd numbers.

The class then posed the following problem related to odd and even numbers:

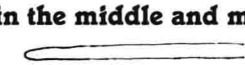
Fold a sheet of paper down the middle. With a hole puncher, punch holes through both layers of paper. Open up the paper. Do you think you have an even number of holes or an odd number of holes? Check.

Some children found that by punching a hole at the very edge of the fold they could make an odd number of holes. They added to our problem:

How can you make an odd number of holes? Because it is possible.

David wrote a problem based on experiments he had done with lengths of yarn:

Fold a piece of yarn in the middle and make two layers like this:



Now cut through BOTH layers as many times as you like. Now count your pieces. Will there be an odd number or an even number?

Games

Many of the children's investigations inspired them to make games. Our work on odd and even numbers led Renee to make a game called Noah's Ark. She made a wooden boat and used planks of wood for a river. She put the boat on the river and installed a ramp leading up to the boat. She marked the ramp into sections. Next she got some Unifix cubes and called them animals. Her idea was for two players each to choose an animal and proceed up the ramp onto the boat. The first player to land an animal on the boat was the winner. To determine how to proceed up the ramp, Renee decided to use a twenty-sided die. The players took turns to throw the die. If they threw an even number they could move one step up the ramp. If they threw an odd number they would end up in the river. These rules frustrated the players as they always ended up in the river. Renee therefore amended the rules so that when an odd number was thrown, the animal simply stayed where it was.

With the failure of the early rule whereby the animal fell into the river when an odd number was thrown, Renee and her friends diverged into investigating aspects of probability.

Later we changed the game so that two regular dice were used. If the combination of the numbers on the two faces was odd, then the animal would remain still. If the combination was even, then the animal would move up the ramp. This reinforced what the children found out about adding two odd numbers.

The children helped me make another game based on their work with odd and even numbers. It is called "The Spot Box game" and had its beginnings in a book we read, *Put me in the Zoo* by Robert Lopshire, (Random House, 1966). There is a "spot box" in the story and the spots jump around from one object to another. Our spot box game has two "animals", which are simply two squares of felt; forty cardboard circles, which are the "spots"; and an ordinary die. Two players each lay out an animal and pick up an odd number of spots (the same number each). They then take it in turns to throw the die and put their spots onto their animal according to the number thrown; a condition is that an animal can only have an odd number of spots at any time. The winner is the first player to get all his or her spots on their animal.

Mathematics concepts and skills

As the children solved problems and worked on investigations, they developed a wide range of mathematical concepts and skills. Within a purposeful context, they competently measured and calculated.

They devised ways to add, subtract, multiply and divide. They were proficient at mental calculations and they came to find written algorithms an effective alternative to the use of concrete materials. Written number work was not a separate entity in the program. Whenever it arose it was in the context of problem solving and investigation.

Thus the children gained the skills traditionally taught in school mathematics programs, and much more as well.

ORGANISATION

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