Consider a Spherical Cow

A Course in Environmental Problem Solving

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Preface

This book should provide a novel, and I hope enjoyable, way of learning how to use relatively simple mathematical methods (often of the "back-of-the-envelope" variety) to understand how planet Earth and its inhabitants interact.

The idea for this text evolved from courses in environmental science I have taught at Yale University and the University of California at Berkeley over the past 15 years. These courses have ranged from the introductory undergraduate to the advanced graduate level. Regardless of the level, I have stressed quantitative problem solving in all my courses, and over the past 15 years I have invented a sizeable repertoire of homework problems. These, along with a few delightful ones contributed by my colleagues, form the basis for this book.

One thing my courses have taught *me* (or rather have recalled to mind, for I knew it all too well when I was a student) is that the ordinary combination of university math and science courses does not prepare students well for solving a frequently occurring type of "word problem." Such problems call for a quantitative answer, but their solution involves information from several disciplines scrambled together. Often the statement of these problems has that fuzzy quality characteristic of real-world situations. The problems we confront outside the classroom rarely take the streamlined form most textbooks rely upon to test how well we've read each chapter.

In this book I try to provide guidance in overcoming barriers to problem solving. To that end, the problems and solutions in the first two chapters progress through a series of insights into problem solving itself. A more customary organization by environmental topic is used in the final chapter.

At the core of the book are 44 problems, with as many worked-out solutions. Chapter I provides a set of warm-up exercises. Elementary quantitative skills, such as conversion of units and approximation methods, suffice to solve these first problems. Chapter II introduces a variety of "back-of-the-envelope" problem-solving methods. These techniques enable you to solve many problems with very little effort—often in a few lines—once you know how to begin and what tools to use. In Chapter III, Beyond the Back of the Envelope, the problem-solving methods developed in the preceding chapters are applied to more complex situations. Methods of problem solving more advanced than those in Chapter II are introduced there. In the problems of Chapter III you will recognize the real-world fuzzy quality referred to above. To solve these we will have to define our variables and system boundaries, invent models, and select tools for extracting information from the models.

Homework exercises are provided at the end of each problem. Particularly difficult exercises are indicated by an asterisk. A few of the tasks proposed are very difficult, requiring term-paper effort; these are marked with two asterisks. Numerical answers to many of the exercises are given at the back of the book.

The solutions to certain of the 44 problems, particularly the more complex ones in Chapter III, are presented at three levels. First, I provide a "hand-waving" solution (i.e., an informed guess) in which the qualitative behavior of the problem's elements is deduced. Often the sign (for example, heating versus cooling) and the order of magnitude of the response of a complex system to a disturbance can be figured out without a detailed mathematical analysis. In some cases the "hand-waving" approach gives an absurd answer. By identifying the reason for that absurdity, we can then gain insight into how the problem should really be solved. At the second level, analytical procedures and a detailed quantitative solution are presented. Because realistic environmental issues are dealt with in this book, a number of simplifying assumptions are generally made at this level to obtain a precise solution. At the third level of solution, I describe methods for deducing the approximate consequences of removing some of level two's simplifying assumptions. Obtaining the results of such deductions is often left as a homework exercise.

I believe that high-school level mathematics, properly applied, can go a long way toward elucidating complex situations. Readers with limited or no prior exposure to calculus and differential equations will be able to follow completely most of the problem solutions presented here. Only the last two problems in Chapter II and a few in Chapter III require some use of simple differential equations; even in those problems the qualitative discussion will be of value to readers with relatively little preparation in mathematics.

Solution to some of the problems presented here requires fundamental information to which not all readers have access. The Appendix to this volume, containing tabulated data about nature and technology, should help. In addition, useful source materials are cited throughout the text for readers seeking further information. A bibliography at the back of the book lists these sources. A glossary is also provided.

All the problems presented here can be solved analytically, without the use of computers. Even in the more difficult problems, the mathematical models employed are not elaborate. I believe it is preferable in environmental analysis to develop relatively simple, analytically tractable models, rather than complex ones requiring truckloads of parameters. The advantage of being able to "tinker" mentally with a simple, penetrable model, and thus explore the consequences of a variety of assumptions, outweighs in most cases the greater realism that might be attained with a complex model.

Thus the "spherical cow" in the title of this book. The phrase comes from a joke about theoreticians I first heard as a graduate student. Milk production at a dairy farm was low so the farmer wrote to the local university, asking help from academia. A multidisciplinary team of professors was assembled, headed by a theoretical physicist, and two weeks of intensive on-site investigation took place. The scholars then returned to the university, notebooks crammed with data, where the task of writing the report was left to the team leader. Shortly thereafter the farmer received the write-up, and opened it to read on the first line: "Consider a spherical cow. . . ."

The spherical cow approach to problem solving involves the stripping away of unnecessary detail, so that only essentials remain. Of course, approaching the complex world from the spherical cow perspective can sometimes annoy others. To an expert who has labored long in the field, the cow that to you is spherical may be sacred. The trick is to know which details can be stripped away without changing the essentials. This book should help readers develop a knack for doing this.

This text should serve two functions. First, it should teach the reader how to transform realistic, qualitatively described problems into quantifiably solvable form and to arrive at an approximate solution. Second, it should teach concepts in environmental science from the novel perspective of problem solving. Readers may want to supplement this book with a general textbook in environmental science, such as the excellent *Ecoscience* by Ehrlich, Ehrlich, and Holdren (1977). Others, with more advanced backgrounds, are urged to graze among the more specialized sources cited throughout this text.

Acknowledgments

The "spherical cow" approach to problem solving is as old as science, but by reinforcing my enthusiasm for it with their own, a few teachers and colleagues contributed to the development of this book. They include Edward Purcell, Victor Weisskopf, Robert Socolow, Charles Walker, Robert Wheeler, Robert May, and John Holdren.

Early versions of the manuscript were read by Harrison Brown, Andrew Cohen, Paul Ehrlich, Steven Fetter, Peter Gleick, Ethel and Mary Ellen Harte, John Holdren, Kersten Johnson, Michael Lazarus, Michael Maniates, Robert Mann, Robert Socolow, and Kenneth Watt. I am grateful to them for their valuable comments and suggestions.

Ideas for problems and homework exercises were stimulated by conversations with Robert Adair, Andrew Gunther, Paul Ehrlich, John Holdren, Douglas McLaren, Richard Miller, James Morgan, Richard Schneider, Stephen Schneider, Robert Socolow, Frank Starr, Charles Walker, James C. G. Walker, and Thomas Powell. Others whose writings provided inspiration for problems are acknowledged in the text.

Over the years numerous students in my courses have generously shared with me their frustration and anger at the devils herein with which I plagued them. This feedback has been of great value in helping me get the "bugs" out of the problems.

For the unearthing of data I am grateful to Mari Wilson, Pi Chao Chen, John Holdren, and Anthony Nero. John Cairns supplied the apt epigraph from Aristotle. A conversation with Donald Goldsmith clinched the title. William Kaufmann provided the quotation in Chapter II from Herbert Simon and essential advice on the structure of the book. With Maggie Duncan at the helm, the manuscript weathered the storms of production in fine style. Ike Burke did a superb, insightful editing job.

I wrote most of the text during two summers at the Rocky Mountain Biological Laboratory, where my colleagues and the incomparable mountains provided a most enjoyable work place. Financial support from the Hewlett Foundation during that time is gratefully acknowledged.

Finally, I am deeply thankful to my wife, Mary Ellen, for her help in all stages of the project. She drew the original illustrations, did all the word processing of the text, located information, and thoroughly edited the entire manuscript both for grammar and content. For all of this, and for her love especially, I am lucky.

Contents

Chapter I
Warm-up Exercises

Only basic problem-solving skills are needed in this chapter. The first problem requires you to guess the approximate values of some numbers that you probably haven't thought about very much. (Guessing makes some people uncomfortable, but give it a try.) Problems I.2 and I.3 can be solved using only basic ideas about areas, volumes, and density; the second homework exercise in Problem I.3 will get you thinking about probabilities. The next two problems are about depletion and growth, determining how long resources will last at present consumption rates and how rapidly population density increases at the present growth rate. If your logarithms are rusty, now is the time to polish them up. The last two problems in this chapter show how to derive measures of the magnitude of the human presence on Earth today. Solving them requires skill in selecting relevant data and in converting from one kind of unit to another. All but the first problem in this chapter will send you scurrying to the tables of information in the Appendix. Familiarize yourself with what the Appendix offers. You'll refer to it often in solving problems later on.

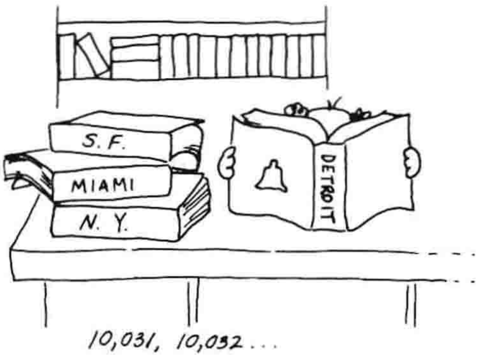
1. Counting Cobblers

How many cobblers are there in the United States?[1]

....

One excuse for including this problem in a book about the environment is that getting your shoes repaired consumes less resources than buying a new pair. It is here mainly, however, to illustrate the ease with which a few plausible guesses can be combined to answer a question that at first glance seems resistant to guesswork. Can you estimate the order of magnitude[2] of the answer?

To do so, you could find out if there are cobbler licensing boards and, if so, write to them for their statistics. Or you could walk to the library and check the yellow pages of telephone directories for representative U. S. cities. However, why not be lazy and let your mind do the walking? Start by assuming that cobblers are generally busy most of the work week. As a rough estimate, they spend about 10 minutes on a heel job and perhaps 30 minutes on full heels and soles. More complicated repairs are rare, so ignore them. If time out for cleaning shop and dealing with customers is included, an average of 30 minutes per job is a reasonable guess. (Remember, the answer is an order of magnitude.)



1. A variation on this problem is found in the text *Used Math* by Swartz (1973), which is an excellent introductory review of applied mathematics.
2. When a number characterizing something is known imprecisely, either because the measurements are poor or because the "something" varies alot, an order-of-magnitude estimate is often given. Usually, orders of magnitude are expressed as powers of 10; a number that is in the range of 0.3 to 3 is said to be on the order of magnitude of 1; a number between 3 and 30 is on the order of magnitude of 10, and so forth.

By this reasoning, a cobbler can finish perhaps 15 jobs in a workday, or about 4000 a year. All you need to know now is how many repair jobs are done each year in the United States. I get a pair of shoes or boots repaired about every four years. Assuming I am typical, the 2.3×10^8 people in the United States (1983) have about $2.3 \times 10^8/4$ or 5.75×10^7 repair jobs carried out each year. Since one cobbler can repair 4000 shoes in a year, we need $5.75 \times 10^7/4000$ or 14,375 cobblers to do all the repair work in the United States.

You should be careful not to write your answer as 14,375, however. That number has five significant figures: the 1, the 4, the 3, the 7, and the 5, and a pretense to such accuracy is unjustified. An order-of-magnitude answer was wanted, so 10^4 will suffice. A more precise answer—one valid to five significant figures—would require input data precise to five significant figures, and we used no such data. Nonsignificant figures have a habit of accumulating in the course of a calculation, like mud on a boot, and you must wipe them off at the end. It is still good policy to keep one or two nonsignificant figures during a calculation, however, so that the rounding off at the end will yield a better estimate.

Try your hand at the exercises below. Provide order-of-magnitude answers.

EXERCISE 1: Suppose you have never watched a cobbler work, so you have no idea how long each job takes; but you have paid the cobbler's bill. How will you estimate the number of cobblers in the United States now?

EXERCISE 2: How many dentists are there in New York, a city of roughly 10^7 people? How many fresh tarts (along with cobblers and other fruit pastries) are there in the "Big Apple"?

EXERCISE 3: How many pairs of shoes can be made from a cow? (Hint: consider a spherical cow—and a spherical shoe, to boot.)

EXERCISE 4: About what fraction of a cubic centimeter of rubber is worn off an automobile tire with each revolution of the wheel?

2. Measuring Molecules

> Benjamin Franklin dropped oil on a lake's surface and noticed that a given amount of oil could not be induced to spread out beyond a certain area.[3] If the number of drops of oil was doubled, then so was the maximum area to which it would spread. His measurements revealed that 0.1 cm^3 of oil spread to a maximum area of 40 m^2. How thick is such an oil layer?

. . . .

Let's denote the thickness of the layer by the symbol d. If d is expressed in units of meters, then the volume of that layer is $40d$ m^3. Since oil does not change volume much under changes in pressure or temperature, it is reasonable to assume that the volume of the oil sample does not change significantly simply by being spread out on a surface. Therefore, we can equate the initial volume, 0.1 cm^3, to the final volume, $40d$ m^3, and thus determine d. First, though, we must express both volumes in the same units. If we select cubic meters as our unit of volume, then we have to express 0.1 cm^3 in m^3. Since 1 m = 100 cm, it follows by cubing both sides that 1 $m^3 = (100)^3$ cm^3 or 1 $m^3 = 10^6$ cm^3. Hence, 1 $cm^3 = 10^{-6}$ m^3 and 0.1 $cm^3 = 10^{-7}$ m^3. Now that the units are consistent, we can equate $40d$ m^3 with 10^{-7} m^3 to get $d = 10^{-7}/40 = 25 \times 10^{-10}$, in units of meters.

A length of 10^{-10} m is called an angstrom and is denoted by the symbol Å. Thus, d equals 25 Å. The angstrom is a convenient unit because the lighter atoms such as hydrogen, carbon, and oxygen are on the order of magnitude of 1 Å in diameter. The distance between atoms in the molecules of common liquids and solids is also on the order of magnitude of 1 Å. The oil layer, then, is on the order of magnitude of ten atoms thick. For the kind of oil Franklin used, this is equivalent to being approximately one molecule thick. That is why such thin oil layers are called "monomolecular layers," and it is also why the oil layer would not spread thinner.

The pragmatic Franklin was interested in these experiments because he wished to explore the use of oil to calm rough waters and thereby prevent wave damage to ships. In Franklin's time, no one knew about molecules, but his creative experimental approach enabled him to make, in effect, the first estimate of a molecule's size!

EXERCISE 1: Franklin actually showed that 1 teaspoon of oil would spread to cover about 0.5 acre. Using the information in the

3. See Goodman (1956) for more on Franklin's many scientific achievements.

Appendix (I.3) that 10^4 m^2 = 2.47 acres, determine how many cubic centimeters there are in a teaspoon.

EXERCISE 2: Estimate the average spacing between H_2O molecules in liquid water by making use of two pieces of information: (*a*) liquid water has a density of 1 g/cm^3, and (*b*) every 18 g of water contain Avogadro's number (6.02×10^{23}) of H_2O molecules.

3. The Size of an Ancient Asteroid

It has been proposed that dinosaurs and many other organisms became extinct 65 million years ago because Earth was struck by a large asteroid (Alvarez et al. 1980). The idea is that dust from the impact was lofted into the upper atmosphere all around the globe, where it lingered for at least several months and blocked the sunlight reaching Earth's surface. On the dark and cold Earth that temporarily resulted (Pollack et al. 1983), many forms of life then became extinct. Available evidence (Alvarez et al. 1980) suggests that about 20% of the asteroid's mass ended up as dust spread uniformly over Earth after eventually settling out of the upper atmosphere. This dust amounted to about 0.02 g/cm^2 of Earth's surface. The asteroid very likely had a density of about 2 g/cm^3. How large was the asteroid?

. . . .

To solve this problem we proceed in two steps. First we estimate the mass of the dust, and then we determine how big the asteroid must have been to contain that mass. The dust surrounds the Earth. According to the Appendix, Earth has an area of 5.1×10^{14} m^2, or 5.1×10^{18} cm^2. Since every square centimeter contained 0.02 g of dust from the asteroid, the dust layer contained a mass of $(0.02\ g/cm^2)(5.1 \times 10^{18}\ cm^2) = 1.02 \times 10^{17}$ g.

This much dust is 20% of the mass, M, of the asteroid, so the asteroid had a mass of

$$M = \frac{1.02 \times 10^{17}}{0.20}\ g = 5.1 \times 10^{17}\ g. \qquad (1)$$

Now, consider a spherical asteroid with a radius, R. Its volume, V, is given by

$$V = \frac{4}{3}\pi R^3. \qquad (2)$$

The mass of material in the asteroid is equal to the density, ρ, times the volume, or

$$M = \rho V = \rho\frac{4}{3}\pi R^3. \qquad (3)$$