FOURTH EDITION

CALCULUS
with Analytic Geometry

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Preface

The addition of a junior author provided the impetus for making a major revision of *Calculus with Analytic Geometry*. Previous users will discover that the text has been completely reworked. Yet we have been careful to preserve the basic integrity of a book that has met the needs of several hundred thousand students in its first three editions. In particular, this new edition continues to be a mainstream textbook for a three semester (four or five quarter) calculus course. Like its predecessors, it stresses the seven fundamental concepts of calculus (function, limit, continuity, derivative, antiderivative, definite integral, and infinite series) and applies them in a myriad of practical situations. In style, it is simple and direct with clear explanations, an abundance of illustrative examples, and carefully graded problem sets.

While one would need to read a few sections to sense the true flavor of the book, we can at least suggest some of its special features.

- A **new specially designed format** invites students to read the book. We have aimed for a clean uncluttered appearance in which important results stand out clearly. Theorems are labeled with letters and, more importantly, the major ones are named (Monotonicity Theorem, Mean Value Theorem, and so on) and referred to that way when needed later in the text. Sections are of approximately equal length (one lecture) and are divided into subsections, thus breaking up the large doses of prose that seem to frighten some students. A larger page size has allowed us to greatly increase the number of illustrative figures (about 1100 in all).

• To give the book a more human quality, we have opened each chapter with the **picture and brief biography of a mathematician** associated with that chapter. Calculus was discovered and molded by real people with real faces. Our experience indicates that students appreciate attempts to introduce bits of history into a calculus course. Note the capsule history of calculus that appears on the front end papers.

• While the first chapter offers a rapid survey of precalculus, it is intended mainly as a reference. Many instructors will choose to begin at Chapter 2, where three of the fundamental concepts of calculus (function, limit, and continuity) make their appearance. Note that separate sections are devoted to **an intuitive and a rigorous definition of limit.** The latter can be deemphasized if desired.

• We introduce the **trigonometric fuctions** in Chapter 2 and find their derivatives in Chapter 3, thus making these important functions available for examples and problems throughout the book.

• **Maxima-minima theory** is presented in a logical and unified way (Chapters 4 and 15). The term *critical point* is used to include stationary points (where the derivative is zero), singular points (where the derivative fails to exist), and endpoints. Thus, an extremum, if it exists, must occur at a critical point. Also global extrema are automatically local extrema, which we think conforms with good usage of the English language.

• The concept of **linearity** is highlighted throughout. The fact that all the major operators of calculus (lim, D, $\int$, $\sum$, ∇) are linear is demonstrated and used repeatedly. A final and significant application of this concept occurs in the last chapter, which treats linear differential equations.

• A simple technique—**slice**, **approximate**, **integrate**—is used consistently in showing how the definite integral arises in applications (see, for example, pages 263, 269, 289, and 685).

• Recognizing that most students have electronic calculators, we have greatly increased the emphasis on **numerical techniques**. We use the symbol [C] to designate those problems that are simplified by the use of calculators. More significant is a whole chapter devoted to numerical calculus (Chapter 10). While there is an advantage in treating this material as a unit, instructors can spread it throughout the course if desired.

• The **gradient** has been defined in a thoroughly modern way (Chapter 15) and plays a central role in an expanded treatment of vector calculus (Chapter 17), which includes the theorems of Green, Gauss, and Stokes.

• We think many instructors will like our decision to define the **multiple integral** of a function f first for a rectangular box. The general case is obtained by the simple device of extending f as the zero function outside of a given region (see Chapter 16).

• Finally, we mention that the **problem sets** have been enlarged in two ways—more elementary problems and more applied problems. Each chapter concludes with a section titled Chapter Review Problems. This includes a **true-false quiz** centering on the theory of calculus and **miscellaneous problems** of the kind a typical instructor might put on a chapter examination.

ACKNOWLEDGMENTS AND RECOMMENDATIONS

We express sincere gratitude to a host of reviewers who have improved our efforts in innumerable ways. Special thanks go to four colleagues who read and offered detailed comments on the whole manuscript:

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Lou Guillou, St. Marys College
Joseph Konhauser, Macalester College
William Wardlaw, United States Naval Academy

In addition to reviewing, Fleming provided answers to all the problems, Guillou independently worked all the problems and prepared both the instructor's and student's manuals, and Konhauser joined us in reading and correcting the galley proofs.

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Our book has profited greatly from the hard work and expertise of the staff at Prentice-Hall. To Eleanor Henshaw Hiatt, Zita de Schauensee, Robert Sickles, and Jayne Conte go special accolades.

We learned our calculus in many ways but want to recommend five books that have helped us and will supplement what we do here. In particular, the three advanced calculus books give the proofs that we have omitted.

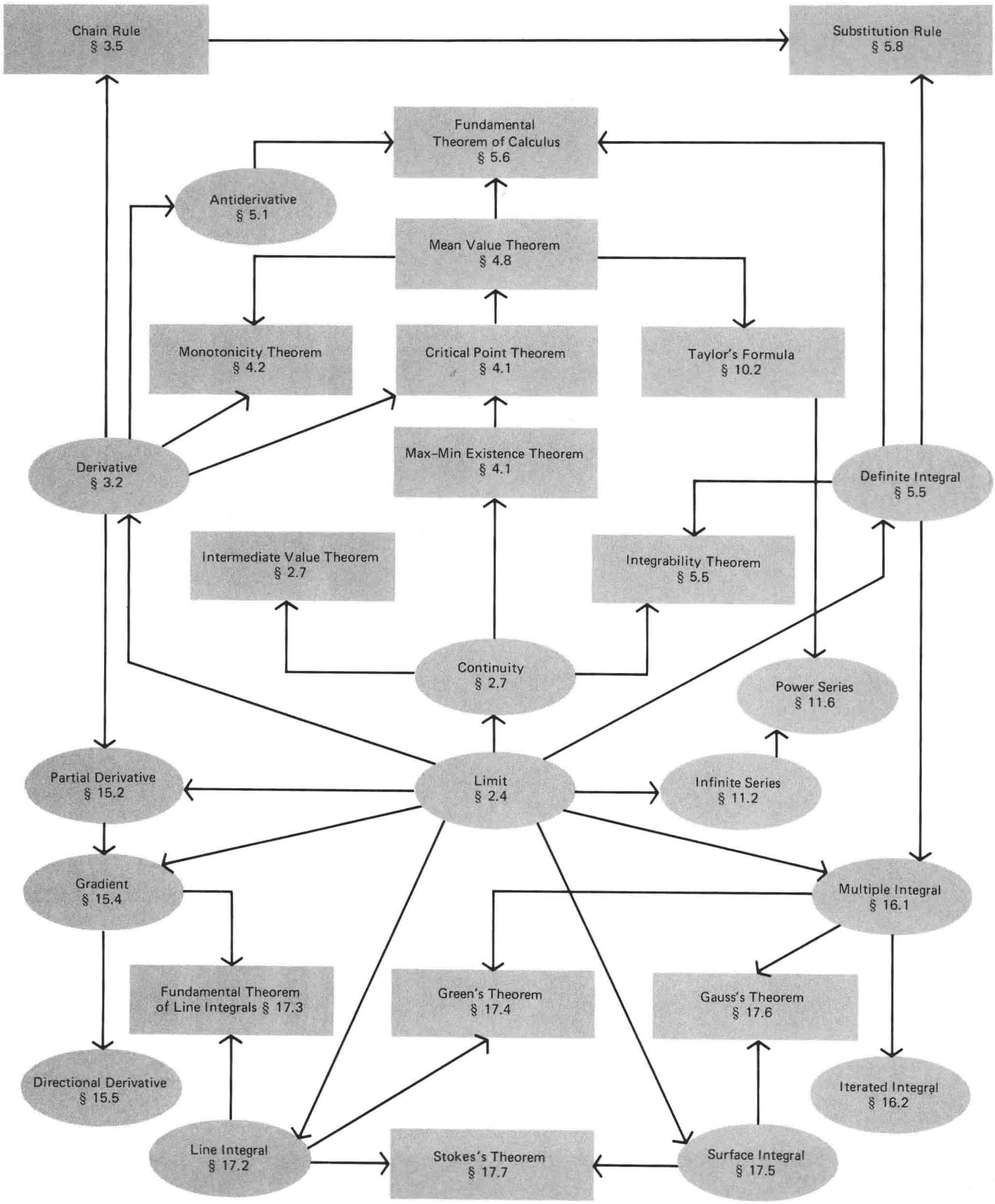
Creighton Buck, *Advanced Calculus* (3rd ed.). New York: McGraw-Hill, 1978
Morris Kline, *Mathematical Thought from Ancient to Modern Times*. New York: Oxford University Press, 1972

J. M. H. Olmsted, *Advanced Calculus*. New York: Prentice-Hall, 1961
Walter Rudin, *Principles of Mathematical Analysis* (3rd ed.). New York: McGraw-Hill, 1976
Angus E. Taylor and W. Robert Mann, *Advanced Calculus* (3rd ed.). New York: John Wiley, 1983

Our preface concludes with a dependence chart. It is not the usual dependence chart relating the various chapters to their prerequisites. Rather, the chart announces the major ideas of calculus and shows how they fit together. Issuing from the central notion of *limit* are the principal concepts (in ovals) and theorems (in boxes). We think a student will profit from studying this simple one-page skeleton not so much at the beginning but later, when the growing mass of material may seem overwhelming.

Edwin J. Purcell

Dale Varberg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1 Preliminaries

[Coordinate geometry], far more than any of his metaphysical speculations, immortalized the name of Descartes, and constitutes the greatest single step ever made in the progress of the exact sciences.
John Stuart Mill

René Descartes
1596–1650

René Descartes is best known as the first great modern philosopher. He was also a founder of modern biology, a physicist, and a mathematician.

Descartes was born in Touraine, France, the son of a moderately wealthy lawyer who sent him to a Jesuit school at the age of eight. Because of delicate health, Descartes was permitted to spend his mornings studying in bed, a practice he found so useful that he continued it throughout the rest of his life. At age 20, he obtained a law degree and thereafter lived the life of a gentleman, serving for a few years in the army and living at times in Paris, at others in the Netherlands. Invited to instruct Queen Christina, he went to Sweden, where he died of pneumonia in 1650.

Descartes searched for a general method of thinking that would give coherence to knowledge and lead to truth in the sciences. The search led him to mathematics, which he concluded was the means of establishing truth in all fields. His most influential mathematical work was *La Géométrie*, published in 1637. In it, he attempted a unification of the ancient and venerable *geometry* with the still infant *algebra*. Together with another Frenchman, Pierre Fermat (1601–1665), he is credited with the union that we today call analytic geometry, or coordinate geometry. The full development of calculus could not have occurred without it.