

# MANAGEMENT SCIENCE

INTRODUCTORY  
CONCEPTS  
and  
APPLICATIONS

SECOND  
EDITION

DAVID HEINZE

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## INTRODUCTORY CONCEPTS and APPLICATIONS

SECOND  
EDITION

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## PREFACE

Management science is a quantitative approach to decision making and has demonstrated itself to be a powerful decision making approach in a wide variety of managerial contexts. The primary objectives of this introductory text are

1. To familiarize the reader with the vocabulary of management science.
2. To acquaint the reader with some of the capabilities of management science in various types of managerial situations.
3. To develop in the reader an ability to construct and analyze simple management science models which will help the reader to make good decisions.

In order to achieve these objectives a broad assortment of management science models is presented. Applications are taken from finance, marketing, production, advertising, real estate development, small business operations, banking, transportation, health care, and public administration.

The development of the material in this text is oriented toward an intuitive, practical approach. Since beginning students learn most readily through the use of examples, illustrations and examples play a prominent role in the process of communicating the concepts of management science to the reader. In view of this, considerable effort has been devoted to avoiding the usual insipid examples which do little to stir the reader's interest.

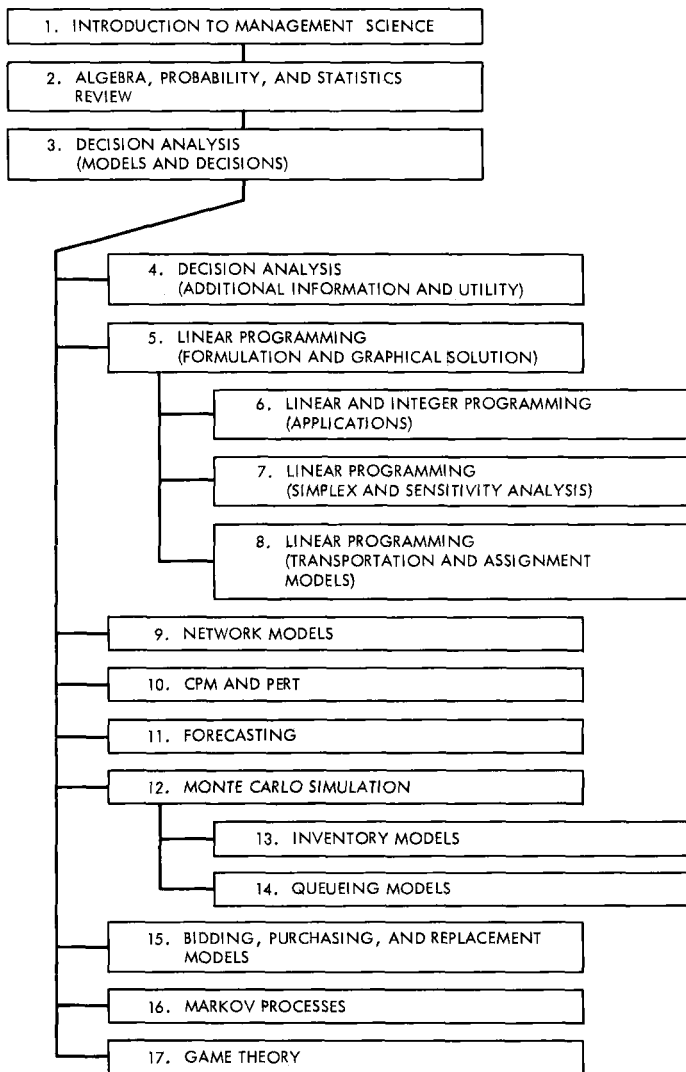
Chapter 2 gives the algebra and probability prerequisites in something of a "self-teaching" form. The reader will find that the algebra and probability used in the text does not go beyond that introduced in Chapter 2 except in a couple of places. In these cases the new mathematical technique is fully explained at its initial point of use. Even with much of the algebra and probability covered in Chapter 2, another explanation of the same type is given at the point of its first use in later chapters.

Monte Carlo simulation is used in several chapters in analyzing certain decision situations which might otherwise require a mathematical model which is too complex. Through the construction of a simulation model, the reader can readily see the relationships among many of the variables and can experiment by making changes in the controllable variables. This will give the mathematically naive reader a fuller insight into the method of management science.

With regard to the chapters that might be covered in a course, a few suggestions may be made. After the introductory material of Chapter

1 and the review of Chapter 2, it is recommended that the introductory decision analysis of Chapter 3 be studied. The decision analysis model gives the reader an understanding of the typical elements of a management science model and the primary means of reaching a decision on the basis of the model. After this, there is considerable flexibility in the sequencing of the chapters. This flexibility is shown in the following diagram.

D.C.H.



# CONTENTS

	Page
<b>Chapter 1    Introduction to Management Science</b>	<b>1</b>
1.1    Sketches    1	
1.2    Definition of Management Science    5	
1.3    Genesis and Development of Management Science    6	
1.4    Approach of Management Science    8	
1.5    Simple Models and Pedagogy    13	
1.6    Preview    14	
Problems    16	
 <b>Chapter 2    Algebra, Probability, and Statistics Review</b>	 <b>18</b>
2.1    Introduction    18	
2.2    Notation    18	
2.3    Maximums and Minimums by Graphing    21	
2.4    Simultaneous Linear Equations    23	
2.5    Matrix Algebra    25	
2.6    Simple, Joint, and Conditional Probabilities    32	
2.7    Independence    34	
2.8    Laws of Probability    36	
2.9    Random Variables and Probability Distributions    39	
2.10    Expected Value, Variance, and Standard Deviation    41	
2.11    Expected Value of a Function    44	
2.12    Cumulative Distributions    46	
2.13    Normal Distribution    49	
2.14    Confidence Intervals    58	
2.15    Conclusion    61	
 <b>Chapter 3    Decision Analysis (Models and Decisions)</b>	 <b>62</b>
3.1    Introduction    62	
3.2    Matrix Form of the Decision Model    66	
3.3    A Decision Model Example    66	
3.4    Principles of Choice    69	
3.5    Algebraic Form of the Decision Model    75	
3.6    Expectation and Breakeven    80	
3.7    Decision Trees    84	
3.8    Backward Induction    88	
Problems    94	

	Page
<b>Chapter 4    Decision Analysis (Additional Information and Utility)</b>	<b>104</b>
4.1    Introduction	104
4.2    Expected Value of Perfect Information	106
4.3    Expected Opportunity Loss and EVPI	109
4.4    Bayes' Theorem	113
4.5    Posterior Analysis	117
4.6    Utility and the Expectation Principle	119
4.7    Constructing a Utility Curve	124
4.8    Decision Trees and Utilities	130
Problems	132
<b>Chapter 5    Linear Programming (Formulation and Graphical Solution)</b>	<b>140</b>
5.1    Introduction to Linear Programming	140
5.2    A Linear Programming Model— Maximization	141
5.3    A Graphical Solution	147
5.4    The Graph of the Objective Function	157
5.5    Linear Programming and the Decision Theory Model	161
5.6    A Linear Programming Model— Minimization	163
5.7    A Graphical Solution	166
5.8    Special Considerations	171
5.9    More Applications	176
Problems	176
<b>Chapter 6    Linear and Integer Programming (Applications)</b>	<b>182</b>
6.1    Introduction	182
6.2    A Finance Application	182
6.3    A Personnel Application	187
6.4    A Production-Inventory Application	189
6.5    A Blending Application	196
6.6    Integer Programming	201
Problems	204
<b>Chapter 7    Linear Programming (Simplex and Sensitivity Analysis)</b>	<b>211</b>
7.1    Introduction	211
7.2    Algebraic Preliminaries	211
7.3    Simplex Method—An Overview	217
7.4    Simplex Method—Tableau Format and Initial Corner Solution	219

	7.5	Simplex Method—Implementation	224
	7.6	Special Considerations	243
	7.7	Shadow Prices	246
	7.8	Simplex for Minimization Models—Preliminaries	248
	7.9	Simplex for Minimization Models—Completion	252
	7.10	Artificial Variables in Maximization Models	257
	7.11	Equality Constraints	257
	7.12	Inconsistent Constraints	258
	7.13	Sensitivity Analysis—Introduction	260
	7.14	Sensitivity Analysis—Changes in $B_i$ and $C_j$	261
	7.15	Duality	268
		Problems	269
<b>Chapter 8</b>		<b>Linear Programming (Transportation and Assignment Models)</b>	<b>274</b>
	8.1	Introduction	274
	8.2	The Transportation Model	274
	8.3	Northwest Corner Rule	282
	8.4	Stepping-Stone Technique	284
	8.5	Degeneracy	298
	8.6	Unbalanced Models and Impossible Routes	303
	8.7	More Applications	307
	8.8	The Assignment Model	310
	8.9	The Assignment Method	313
	8.10	Special Considerations	319
		Problems	321
<b>Chapter 9</b>		<b>Network Models</b>	<b>327</b>
	9.1	Introduction	327
	9.2	Minimal Spanning Tree Model	328
	9.3	Maximal Flow Model	335
	9.4	Traveling Salesman Problem	336
		Problems	338
<b>Chapter 10</b>		<b>CPM and PERT</b>	<b>342</b>
	10.1	Introduction	342
	10.2	Symbols and Definitions	342
	10.3	Constructing the Network	345
	10.4	Activity Identifications	349
	10.5	Finding the Critical Path	350
	10.6	Slacks	356



10.7	A Time-Scaled Network	357
10.8	Crashing	358
10.9	PERT	362
	Problems	366
<b>Chapter 11</b>	<b>Forecasting</b>	<b>372</b>
11.1	Introduction	372
11.2	Subjective Forecasts	374
11.3	Subjective Forecasts by Individuals	375
11.4	Subjectively Assessed Normal Distribution	380
11.5	Subjective Forecasts by Groups	386
11.6	Time Series and an Overview	387
11.7	A Short-Term Probabilistic Forecast	389
11.8	Exponential Smoothing	392
11.9	Regression Analysis	403
	Problems	412
<b>Chapter 12</b>	<b>Monte Carlo Simulation</b>	<b>415</b>
12.1	Introduction	415
12.2	Random Number Table	416
12.3	Two Elementary Examples of Simulation	416
12.4	Package Delivery Simulation	421
12.5	Ambulance Simulation	423
12.6	Risk Analysis Simulation	429
12.7	The Computer and Simulation	434
12.8	Statistics and Simulation	434
	Problems	436
<b>Chapter 13</b>	<b>Inventory Models</b>	<b>439</b>
13.1	Introduction	439
13.2	A Simple Deterministic Inventory Model	439
13.3	Sensitivity Analysis	449
13.4	Quantity Discounts	451
13.5	Reorder Level	453
13.6	An Inventory-Production Model	457
13.7	A Discrete Probabilistic Inventory Model With No Reordering	461
13.8	A Continuous Probabilistic Inventory Model With No Reordering	469
13.9	Sensitivity Analysis	472
13.10	A Probabilistic Inventory Model With Reordering	475
13.11	Another Inventory Model	485
	Problems	485

	Page
<b>Chapter 14      Queueing Models</b>	<b>491</b>
14.1      Introduction	491
14.2      Queueing Model Components	493
14.3      A Single-Server Model	497
14.4      Economic Implications	508
14.5      A Two-Server Queueing Model	511
14.6      The Simulation of a Queueing System	518
14.7      Another Queueing Simulation	525
Problems	532
 <b>Chapter 15      Bidding, Purchasing, and Replacement Models</b>	 <b>537</b>
15.1      Bidding Strategy	537
15.2      Historical Costs and Estimates	537
15.3      Bids and Profits	540
15.4      Winning Against One Competitor	541
15.5      Optimal Bidding Against One Competitor	544
15.6      Complications	546
15.7      Optimal Bidding Against Two or More Competitors	548
15.8      Introduction to Transitory Purchasing Opportunities	550
15.9      A Transitory Purchasing Model	551
15.10      Replacement Policy	557
15.11      A Probabilistic Renewal Model	558
15.12      A Deterministic Replacement Model	562
Problems	567
 <b>Chapter 16      Markov Processes</b>	 <b>572</b>
16.1      Introduction	572
16.2      Markov States and Transitions	573
16.3      Probability Trees and Transition Probabilities	575
16.4      Transition Matrices	577
16.5      Other Characteristics	577
16.6      Probability Trees for Multi-Step Transitions	578
16.7      Matrices for Multi-Step Transitions	581
16.8      Steady State and a Brand Switching Example	583
16.9      Geometric Calculation of Steady State	592
16.10      Absorbing States	595
16.11      Summary and Other Applications	598
Problems	600

	Page
<b>Chapter 17    Game Theory</b>	<b>604</b>
17.1    Introduction	604
17.2    Types of Games	605
17.3    Payoffs, Strategies, and the Game Matrix	608
17.4    Solution by Dominance	613
17.5    Pure Strategies and Saddle Points	615
17.6    Mixed Strategies	620
17.7    Graphical Solutions	627
17.8    Other Games	633
17.9    Other Applications	634
Problems	638
 <b>Appendix Tables</b>	 <b>641</b>
 <b>Answers to Odd-Numbered Problems</b>	 <b>647</b>
 <b>Index</b>	 <b>661</b>

# **1. INTRODUCTION TO MANAGEMENT SCIENCE**

## **1.1 SKETCHES**

For the sake of introducing management science, several brief sketches of decision situations will be given which are representative of those where management science can make a significant contribution to decision making.

### **Kearfott Case**

A large aerospace company has been awarded the contract to build a new mobile ICBM, which, when launched from any spot in the northern hemisphere, must “read” the stars as it ascends, thereby determining its precise location, and then guide itself to the predetermined target. The company is not in a position to develop and produce the guidance system for the missile because of the missile’s sophisticated nature and because of time constraints. It has, consequently, decided to accept bids from subcontractors for the work to be done. One such subcontractor is Kearfott, Inc.

Ernesto Carlos is Kearfott’s executive responsible for the preparation and submission of a bid for the stellar guidance system contract. What are some of the difficulties facing Carlos in this situation? For one, there is the matter of determining the cost of developing and producing the guidance system. While the R & D, engineering, and production departments can all estimate their respective costs for such a project, Carlos must determine the reliability of such estimates. Considerable attention must be devoted to accurately projecting costs. If one of these departments significantly underestimates and if Kearfott gets the contract, it is obvious that serious consequences for the profitability of the contract will follow.

Even if Carlos knew with certainty the costs to perform this contract, there would also be the matter of determining the actual bid. If the estimated costs were \$10 million, should Kearfott bid \$10.1 million or \$12 million for the guidance system contract? A low bid would offer a good chance of obtaining the contract; but if such a bid were submitted and subsequently

won, it would offer Kearfott only a small profit. A bid of \$12 million, on the other hand, would offer a substantial profit, but would be less likely to win against the bids of the other companies. Thus, Carlos is beset with various difficulties in attempting to prepare a bid for the guidance system.

It will be seen in Chapter 15 that management science offers a concrete, logical method of grappling with a bidding situation such as Kearfott faces.

### **Port of San Francisco Authority Case**

The George Washington Bridge extends from San Francisco to Alcatraz Island, the financial center of Western civilization. Since the George Washington Bridge is a toll bridge, visitors to Alcatraz must pay a toll of 50¢ upon crossing it and then another toll of 50¢ upon returning. The other way to get to the island from San Francisco is through the Holland Tunnel which was dug by the Mole Man of Alcatraz. The tunnel, like the bridge, bears a one-way toll of 50¢. With economic woes besetting the city of San Francisco, the city put pressure on the Port of San Francisco Authority to cut the costs of operating the bridge and tunnel while keeping the toll revenues stable. The story of how this was done is one of the great monuments to human ingenuity.

Wilbur Wright and his brother Orville were retained by the Port Authority to expend their genius for the accomplishment of the cost-cutting task. It was Wilbur who said on that occasion, "What goes up must come down." The quick-of-mind Orville took hold of this profound insight and adapted it to the situation at hand, "What goes out (to Alcatraz Island) must come back." Because of this astute comprehension of the situation, the Port Authority dismissed half of its toll-takers and retained only those who collected the tolls from the travellers going to the island via the bridge or the tunnel. The new toll, of course, was \$1. Toll revenues remained the same, and the travellers were pleased in having to pay a toll once instead of twice. In this way, the payroll for toll-takers was cut in half and the Port Authority was successful in cutting costs.

The city administrators were impressed with the cost reduction and with the ease with which it was accomplished. Nevertheless, they felt that more could be done. The Port Authority had to therefore consider the possibility of reducing the number of toll-takers who serviced the travellers crossing to the island. (Remember that the toll-takers servicing the travellers returning to San Francisco had already been eliminated.) Because the residents of San Francisco were tiring of great social experiments, the Port

Authority was afraid to experimentally alter the number of toll-takers in order to see the effects on the waiting time of the travellers crossing the bridge or the tunnel to the island. It was forced to undertake an extensive “theoretical” study through which consulting management scientists were able to predict how much extra waiting would result if the number of toll-takers were reduced by varying amounts. The Port Authority and the consultants were then able to decide how many toll-takers could safely be dismissed without causing excessive line-ups by the cars crossing to the island.

In Chapter 14, management science techniques which deal with decisions concerning service and waiting lines will be explored. These sorts of models were employed by the consulting management scientists working for the Port Authority.

### **Colemans Case**

Colemans is a large department store which has replaced its old standard cash registers with computerized registers. While still performing the traditional functions, the computerized register also serves as a component in the inventory system. When a customer purchases a harvest gold electric popcorn popper, for example, the clerk types into the register the inventory identification number for this specific product. The computer to which the cash register is tied then automatically accounts for this sale of one harvest gold popper. When the number of harvest gold poppers in inventory falls below a prescribed number, the computer which is keeping track of inventory prints out this fact or might even print an order to replenish the supply of harvest gold poppers.

To operate this elaborate system, Colemans must specify two numbers for the harvest gold poppers. First, a reorder level must be determined. The reorder level is the level of inventory, such as three harvest gold poppers, which triggers the computer to print a request, or order, for the replenishing of the harvest gold poppers. Should this reorder level be a low number, like three, or a high number such as six? Quite obviously, if Colemans reorders a shipment of harvest gold poppers when inventory falls to a relatively high number like six, it is not probable that it will run out of poppers while waiting for the new shipment to arrive. Letting the inventory level fall to three before reordering, however, runs the risk of selling out the poppers before the new shipment arrives. It would seem, then, that a customer-oriented store like Colemans ought to always use a

high reorder level so that it will never have to turn away customers seeking harvest gold poppers.

On the other hand, high reorder levels have some unpleasant ramifications. With the higher reorder level Colemans will end up carrying extra harvest gold poppers in inventory. Then, of course, if a higher reorder level is used for harvest gold poppers, it should for the same reason also be used for avocado poppers, white poppers, aluminum poppers, and for poppy red poppers. In fact, in order to be consistent the higher reorder level should prevail for all Colemans products (for example, for newborn disposable diapers, toddler disposable diapers, . . . , 3' chain link fencing, 4' chain link fencing, . . . , 4 H.P. rototillers, 5 H.P. rototillers, . . . , gold curtain rods, white curtain rods, and so on). Quite obviously, Colemans, in uniformly using higher reorder levels, will be forced to carry a huge extra inventory. This means that money which could have been used elsewhere will be tied up in inventory and that more storage space will be required. It is clear that there are disadvantages to high reorder levels as well as disadvantages to low levels.

Assuming that Colemans could optimally set reorder levels for each product sold, the second problem concerns the size of the order that Colemans should place for the sake of replenishing inventory. Should Colemans order 10 or 30 harvest gold poppers? The matter of determining reorder levels and order sizes is a problem which is best handled through a management science approach. Management science has made a substantial contribution in the area of inventory control. Chapter 13 will deal with some inventory models which are helpful in the management of inventory.

### **Beetle, Beezel, & Gookin Case**

Beetle, Beezel & Gookin is an advertising agency out of Philadelphia which puts together packages of television commercials. BB&G buys television time from stations all over the United States. One day the makers of Acnil, an unknown but promising new acne skin creme, approached BB&G with the intent of hiring BB&G to put together a national television promotion for Acnil. The makers of Acnil were specific in communicating to BB&G how many young people, old people, middle-income people, women, men, and so forth should be exposed to Acnil commercials. Since BB&G had information as to how many persons in each of these groups were watching various television programs, it was next up to BB&G to put together a minimum cost television campaign which met all of Acnil's requirements. In other words, BB&G needed to purchase, for as little as possible,

a wide enough assortment of television spots to expose the desired number of viewers in the various categories specified by the makers of Acnil. Acnil ads, as a result of BB&G's efforts, appeared on a broad variety of television programs over several months.

How did BB&G put together this minimum cost television campaign? A management science technique known as linear programming was used to select the set of television programs which would meet the manufacturer's requirements while keeping costs to a minimum. Linear programming has many sorts of applications in various disciplines. LP is presented in Chapters 5 through 8.

## 1.2 DEFINITION OF MANAGEMENT SCIENCE

In attempting to define or describe something, there are two possible approaches that may be taken. A definition can be given or some examples can be given. If you were called upon to describe a stone to someone unfamiliar with the same, you could turn to the dictionary which states that a stone is "concreted earthy or mineral matter." Such a definition might prove to be less than informative. A better method of communication would be to display to the person an assortment of stones of various sizes, colors, and shapes.

In attempting to define or describe management science, there are also these two alternative approaches. This chapter will try to define management science according to the dictionary approach, although excessive time will not be spent in that pursuit. The most effective and interesting way to communicate the character of management science is through the use of examples; accordingly, the succeeding chapters of this text will serve as the display of management science in its various "sizes, colors, and shapes." (The four cases outlined in the first section give a preliminary glimpse into the variety of situations with which management science deals.)

One of many definitions of *management science* is that it is *a scientific approach to making decisions in a managerial context*. A narrower definition would state that it is a scientific approach which utilizes mathematical and statistical methods in making decisions in managerial situations.

The term "management science" suggests that this discipline has something to do with management and something to do with science. Consider first the management aspect, then the science aspect.

Decision making is the very heart of management; and having been convinced of the centrality of decision making to management, one would



be correct in concluding that management science is concerned with decision making. It is further recognized that the management process is indigenous to all kinds of organizations. Management is not confined to profit-making corporations, but is common to every organization whether that organization be a hospital, a university, a governmental agency, or a business. The organization lives, moves, and adapts only through the implementation of decisions. And not only is the management process operating in all types of organizations, but also it is present at the various levels in the hierarchies of these organizations. Hence it may be expected that "management" science will have a wide spectrum of application.

The term "science" connotes a couple of ideas which are relevant to management science. In the first place, we think of science as being an explicit, systematic, structured undertaking. So it is that management "science" is an explicit, systematic, and well-structured approach to decision making. In the second place, the term brings to mind the concepts of hypothesis, observation, and experimentation. In management science alternative courses of action (hypotheses) are evaluated. These evaluations are accomplished through the method of experimenting with a model. Just as the aerodynamic engineer uses a model or replica of a plane in the wind tunnel for experimentation purposes, the management scientist uses a model of the decision situation in order to evaluate the choices that can be made. In this sense also management science becomes a scientific approach to making decisions.

In addition to the above definitions, some of the predominant characteristics of management science should be noted. First, management science is particularly concerned with decisions which have economic ramifications or consequences, creating thereby an abundant mention of revenues, costs, and profits. The fact that economic consequences are relatively easy to quantify leads to the second characteristic of management science. As with most sciences, the liberal use of mathematics is an efficient way to describe managerial decision situations. This is not to say, however, that management science is a branch of mathematics. As Bertrand Russell has argued in *Principia Mathematica*, mathematics is an extension of logic. Thus, in using mathematics, the management scientist is applying a powerful form of logic in order to assist in making decisions.

### 1.3 GENESIS AND DEVELOPMENT OF MANAGEMENT SCIENCE

How long has management science been in use? The answer to this question depends upon how strictly management science is defined. Most