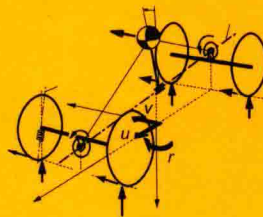
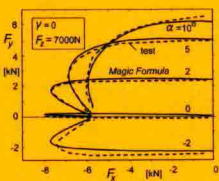


THIRD EDITION

TIRE AND VEHICLE DYNAMICS

HANS PACEJKA

SAE International®



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Tire and Vehicle Dynamics

Third edition

Hans B. Pacejka

Delft University of Technology
The Netherlands



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Butterworth-Heinemann is an imprint of Elsevier



Butterworth-Heinemann is an imprint of Elsevier
The Boulevard, Langford Lane, Oxford OX5 1GB, UK
225 Wyman Street, Waltham, MA 02451, USA

First edition 2002

Second edition 2006

Third edition 2012

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British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data

A catalog record for this book is available from the Library of Congress

ISBN: 978-0-08-097016-5

For information on all Butterworth-Heinemann publications
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Transferred to Digital Printing in 2012

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Tire and Vehicle Dynamics

The operational properties of the road vehicle are the result of the dynamic interaction of the various components of the vehicle structure, possibly including modern control elements. A major role is played by the pneumatic tire.

“The complexity of the structure and behavior of the tire are such that no complete and satisfactory theory has yet been propounded. The characteristics of the tire still presents a challenge to the natural philosopher to devise a theory which shall coordinate the vast mass of empirical data and give some guidance to the manufacturer and user. This is an inviting field for the application of mathematics to the physical world”.

In this way, Temple formulated his view on the situation almost 50 years ago (Endeavor, October 1956). Since that time, in numerous institutes and laboratories, the work of the early investigators has been continued. Considerable progress in the development of the theory of tire mechanics has been made during the past decades. This has led to better understanding of tire behavior and its role as a vehicle component. Thanks to new and more refined experimental techniques and to the introduction of the electronic computer, the goal of formulating and using more realistic mathematical models of the tire in a wide range of operational conditions has been achieved.

From the point of view of the vehicle dynamicist, the mechanical behavior of the tire needs to be investigated systematically in terms of its reaction to various inputs associated with wheel motions and road conditions. It is convenient to distinguish between symmetric and anti-symmetric (in-plane and out-of-plane) modes of operation. In the first type of mode, the tire supports the load and cushions the vehicle against road irregularities while longitudinal driving or braking forces are transmitted from the road to the wheel. In the second mode of operation, the tire generates lateral, cornering, or camber forces to provide the necessary directional control of the vehicle. In more complex situations, e.g. braking in a turn, combinations of these pure modes of operation occur. Moreover, one may distinguish between steady-state performance and transient or oscillatory behavior of the rolling tire. The contents of the book have been subdivided according to these categories. The development of theoretical models has always been substantiated through experimental evidence.

Possibly one of the more difficult aspects of tire dynamic behavior to describe mathematically is the generation of forces and moments when the tire rolls over rough roads with short obstacles while being braked and steered in

a time-varying fashion. In the book, tire modeling is discussed while gradually increasing its complexity, thereby allowing the modeling range of operation to become wider in terms of slip intensity, wavelength of wheel motion, and frequency. Formulas based on empirical observations and relatively simple approximate physical models have been used to describe tire mechanical behavior. Rolling over obstacles has been modeled by making use of effective road inputs. This approach forms a contrast to the derivation of complex models, which are based on more or less refined physical descriptions of the tire.

Throughout the book, the influence of tire mechanical properties on vehicle dynamic behavior has been discussed. For example, handling diagrams are introduced for both cars and motorcycles to clearly illustrate and explain the role of the tire non-linear steady-state side force characteristics in achieving certain understeer and oversteer handling characteristics of the vehicle. The wheel shimmy phenomenon is discussed in detail in connection with the non-steady-state description of the out-of-plane behavior of the tire and the deterioration of ABS braking performance when running over uneven roads is examined with the use of an in-plane tire dynamic model. The complete scope of the book may be judged best from the table of contents.

The material covered in the book represents a field of automotive engineering practice that is attractive to the student to deepen his or her experience in the application of basic mechanical engineering knowledge. For that purpose, a number of problems have been added. These exercises have been listed at the end of the table of contents.

Much of the work described in this book has been carried out at the Vehicle Research Laboratory of the Delft University of Technology, Delft, the Netherlands. This laboratory was established in the late 1950s through the efforts of professor Van Eldik Thieme. With its unique testing facilities realistic tire steady-state (over the road), transient, and obstacle traversing (on flat plank) and dynamic (on rotating drum) characteristics could be assessed. I wish to express my appreciation to the staff of this laboratory and to the Ph.D. students who have given their valuable efforts to develop further knowledge in tire mechanics and its application in vehicle dynamics. The collaboration with TNO Automotive (Delft) in the field of tire research opened the way to produce professional software and render services to the automotive and tire industry, especially for the *Delft-Tire* product range that includes the *Magic Formula* and *SWIFT* models described in Chapters 4, 9, and 10. I am indebted to the Vehicle Dynamics group for their much appreciated help in the preparation of the book.

Professors Peter Lugner (Vienna University of Technology) and Robin Sharp (Cranfield University) have carefully reviewed major parts of the book (Chapters 1–6 and Chapter 11, respectively). Igo Besselink and Sven Jansen of TNO Automotive reviewed the Chapters 5–10. I am most grateful for their

valuable suggestions to correct and improve the text. Finally, I thank the editorial and production staff of Butterworth-Heinemann for their assistance and cooperation.

Hans B. Pacejka
Rotterdam,
May, 2002

NOTE ON THE SECOND EDITION

In this new edition, many small and larger corrections and improvements have been introduced. Recent developments on tire modeling have been added. These concern mainly camber dynamics (Chapter 7) and running over three-dimensional uneven road surfaces (Chapter 10). Section 10.2 has been added to outline the structure of three advanced dynamic tire models that are important for detailed computer simulation studies of vehicle dynamic performance. In the new Chapter 12, an overview has been given of tire testing facilities that are designed to measure tire steady-state characteristics both in the laboratory and over the road, and to investigate the dynamic performance of the tire subjected to wheel vibrations and road unevennesses.

Hans B. Pacejka
Rotterdam,
September, 2005

NOTE ON THE THIRD EDITION

In this new edition, again many improvements have been introduced. Some chapters have been reorganized, notably Chapter 10, and a new Chapter 13 has been introduced outlining three advanced dynamic tire models. We express our thanks to the two guest authors Michael Gipser and Christian Oertel (the original model developers) for their contributions in this chapter. Igo Besselink has contributed to the preparation of the new edition. He wrote his part of Chapter 13 and the extended Appendices. He prepares and maintains information online, notably on dynamic vehicle measurements, see website: <http://www.elsevierdirect.com/companion.jsp?ISBN=9780080970165>.

We much appreciate and are grateful for the help provided by TNO Automotive, Helmond, the Netherlands, especially for making available motorcycle tire measurement data obtained with the new test facility that can handle large camber angles. An image of the device appears on the front cover of the book. This TNO facility is shown mounted on the old Delft Tire Test Trailer but is now installed in the new TNO Tire Test Semi-Trailer, cf. Chapter 12. We also thank TNO for allowing us to use their vehicle measurement data. We are grateful to Antoine Schmeitz, who checked and made important remarks, notably on Chapter 4 and on the revised Chapter 10.

We thank Manfred Plöchl of the Vienna Technical University for carefully checking Chapters 1 and 11. Bill Milliken (now 101 years old! See his wonderful engineering autobiography (2006)) responded very competently on a question of mine regarding the origin of the *Similarity Method*. We are also grateful to readers who sent – often small – remarks that have served to improve the book.

In the book, vehicle dynamic problems have been addressed and a number of tire models have been discussed. Applications of these tire models serve to illustrate their use and the influence of relevant aspects in vehicle dynamic behavior. Two tire models are not in the first place meant to be used seriously in applications. They have been discussed for providing insight and for studying the main typical aspects of tire force and moment generation in the steady state (the *Brush Model*, Chapter 3) and in the transient state (the *String Model*, Chapter 5). In Chapter 1, the basic form of the *Magic Formula* has been introduced. This empirical model is well suited to be applied in vehicle dynamics studies. For high standard applications requiring great accuracy, the full model discussed in Chapter 4 may be employed. For relatively low frequency and larger wavelength phenomena, the transient tire model featuring the relaxation length, developed in Chapter 7 and applied in Chapter 8, is often sufficiently accurate. In case one is interested in higher frequency and shorter wavelength responses of the tire with the first natural frequencies included, the advanced dynamic tire model developed in Chapter 9 is recommended. This so-called *SWIFT* model works in conjunction with the *Magic Formula*. Rolling over uneven roads including short obstacles can be handled by using the special geometric filtering technique treated in Chapter 10.

Hans B. Pacejka
Rotterdam,
August, 2011

Contents

Exercises	xi
Preface	xiii

1. Tire Characteristics and Vehicle Handling and Stability

1.1. Introduction	2
1.2. Tire and Axle Characteristics	3
1.2.1. Introduction to Tire Characteristics	3
1.2.2. Effective Axle Cornering Characteristics	7
1.3. Vehicle Handling and Stability	16
1.3.1. Differential Equations for Plane Vehicle Motions	17
1.3.2. Linear Analysis of the Two-Degree-of-Freedom Model	22
1.3.3. Nonlinear Steady-State Cornering Solutions	35
1.3.4. The Vehicle at Braking or Driving	49
1.3.5. The Moment Method	51
1.3.6. The Car-Trailer Combination	53
1.3.7. Vehicle Dynamics at More Complex Tire Slip Conditions	57

2. Basic Tire Modeling Considerations

2.1. Introduction	59
2.2. Definition of Tire Input Quantities	61
2.3. Assessment of Tire Input Motion Components	68
2.4. Fundamental Differential Equations for a Rolling and Slipping Body	72
2.5. Tire Models (Introductory Discussion)	81

3. Theory of Steady-State Slip Force and Moment Generation

3.1. Introduction	87
3.2. Tire Brush Model	90
3.2.1. Pure Side Slip	92
3.2.2. Pure Longitudinal Slip	97
3.2.3. Interaction between Lateral and Longitudinal Slip (Combined Slip)	100
3.2.4. Camber and Turning (Spin)	112

3.3.	The Tread Simulation Model	128
3.4.	Application: Vehicle Stability at Braking up to Wheel Lock	140
4.	Semi-Empirical Tire Models	
4.1.	Introduction	150
4.2.	The Similarity Method	150
4.2.1.	Pure Slip Conditions	152
4.2.2.	Combined Slip Conditions	158
4.2.3.	Combined Slip Conditions with F_x as Input Variable	163
4.3.	The <i>Magic Formula</i> Tire Model	165
4.3.1.	Model Description	165
4.3.2.	Full Set of Equations	176
4.3.3.	Extension of the Model for Turn Slip	183
4.3.4.	Ply-Steer and Conicity	191
4.3.5.	The Overturning Couple	196
4.3.6.	Comparison with Experimental Data for a Car, a Truck, and a Motorcycle Tire	202
5.	Non-Steady-State Out-of-Plane String-Based Tire Models	
5.1.	Introduction	212
5.2.	Review of Earlier Research	212
5.3.	The Stretched String Model	215
5.3.1.	Model Development	216
5.3.2.	Step and Steady-State Response of the String Model	225
5.3.3.	Frequency Response Functions of the String Model	232
5.4.	Approximations and Other Models	240
5.4.1.	Approximate Models	241
5.4.2.	Other Models	256
5.4.3.	Enhanced String Model with Tread Elements	258
5.5.	Tire Inertia Effects	268
5.5.1.	First Approximation of Dynamic Influence (Gyroscopic Couple)	269
5.5.2.	Second Approximation of Dynamic Influence (First Harmonic)	271
5.6.	Side Force Response to Time-Varying Load	277
5.6.1.	String Model with Tread Elements Subjected to Load Variations	277
5.6.2.	Adapted Bare String Model	281
5.6.3.	The Force and Moment Response	284
6.	Theory of the Wheel Shimmy Phenomenon	
6.1.	Introduction	287
6.2.	The Simple Trailing Wheel System with Yaw Degree of Freedom	288

6.3. Systems with Yaw and Lateral Degrees of Freedom	295
6.3.1. Yaw and Lateral Degrees of Freedom with Rigid Wheel/Tire (Third Order)	296
6.3.2. The Fifth-Order System	297
6.4. Shimmy and Energy Flow	311
6.4.1. Unstable Modes and the Energy Circle	311
6.4.2. Transformation of Forward Motion Energy into Shimmy Energy	317
6.5. Nonlinear Shimmy Oscillations	320
 7. Single-Contact-Point Transient Tire Models	
7.1. Introduction	329
7.2. Model Development	330
7.2.1. Linear Model	330
7.2.2. Semi-Non-Linear Model	335
7.2.3. Fully Nonlinear Model	336
7.2.4. Nonlagging Part	345
7.2.5. The Gyroscopic Couple	348
7.3. Enhanced Nonlinear Transient Tire Model	349
 8. Applications of Transient Tire Models	
8.1. Vehicle Response to Steer Angle Variations	356
8.2. Cornering on Undulated Roads	356
8.3. Longitudinal Force Response to Tire Nonuniformity, Axle Motions, and Road Unevenness	366
8.3.1. Effective Rolling Radius Variations at Free Rolling	367
8.3.2. Computation of the Horizontal Longitudinal Force Response	371
8.3.3. Frequency Response to Vertical Axle Motions	374
8.3.4. Frequency Response to Radial Run-out	376
8.4. Forced Steering Vibrations	379
8.4.1. Dynamics of the Unloaded System Excited by Wheel Unbalance	380
8.4.2. Dynamics of the Loaded System with Tire Properties Included	382
8.5. ABS Braking on Undulated Road	385
8.5.1. In-Plane Model of Suspension and Wheel/Tire Assembly	386
8.5.2. Antilock Braking Algorithm and Simulation	390
8.6. Starting from Standstill	394
 9. Short Wavelength Intermediate Frequency Tire Model	
9.1. Introduction	404

9.2. The Contact Patch Slip Model	406
9.2.1. Brush Model Non-Steady-State Behavior	406
9.2.2. The Model Adapted to the Use of the <i>Magic Formula</i>	426
9.2.3. Parking Maneuvers	436
9.3. Tire Dynamics	444
9.3.1. Dynamic Equations	444
9.3.2. Constitutive Relations	453
9.4. Dynamic Tire Model Performance	462
9.4.1. Dedicated Dynamic Test Facilities	463
9.4.2. Dynamic Tire Simulation and Experimental Results	466
 10. Dynamic Tire Response to Short Road Unevennesses	
10.1. Model Development	475
10.1.1. Tire Envelopment Properties	476
10.1.2. The Effective Road Plane Using Basic Functions	478
10.1.3. The Effective Road Plane Using the 'Cam' Road Feeler Concept	485
10.1.4. The Effective Rolling Radius When Rolling Over a Cleat	487
10.1.5. The Location of the Effective Road Plane	493
10.2. SWIFT on Road Unevennesses (Simulation and Experiment)	497
10.2.1. Two-Dimensional Unevennesses	497
10.2.2. Three-Dimensional Unevennesses	504
 11. Motorcycle Dynamics	
11.1. Introduction	506
11.2. Model Description	508
11.2.1. Geometry and Inertia	509
11.2.2. The Steer, Camber, and Slip Angles	511
11.2.3. Air Drag, Driving or Braking, and Fore-and-Aft Load Transfer	514
11.2.4. Tire Force and Moment Response	515
11.3. Linear Equations of Motion	520
11.3.1. The Kinetic Energy	521
11.3.2. The Potential Energy and the Dissipation Function	523
11.3.3. The Virtual Work	524
11.3.4. Complete Set of Linear Differential Equations	525
11.4. Stability Analysis and Step Responses	529
11.4.1. Free Uncontrolled Motion	529
11.4.2. Step Responses of Controlled Motion	536
11.5. Analysis of Steady-State Cornering	539
11.5.1. Linear Steady-State Theory	540
11.5.2. Non-Linear Analysis of Steady-State Cornering	555
11.5.3. Modes of Vibration at Large Lateral Accelerations	563
11.6. The Magic Formula Tire Model	565

12. Tire Steady-State and Dynamic Test Facilities	567
13. Outlines of Three Advanced Dynamic Tire Models	
Introduction	577
13.1. The <i>RMOD-K</i> Tire Model (Christian Oertel)	578
13.1.1. The Nonlinear FEM Model	578
13.1.2. The Flexible Belt Model	579
13.1.3. Comparison of Various <i>RMOD-K</i> Models	581
13.2. The <i>FTire</i> Tire Model (Michael Gipser)	582
13.2.1. Introduction	582
13.2.2. Structure Model	583
13.2.3. Tread Model	584
13.2.4. Model Data and Parametrization	586
13.3. The <i>MF-Swift</i> Tire Model (Igo Besselink)	586
13.3.1. Introduction	586
13.3.2. Model Overview	587
13.3.3. MF-Tire/MF-Swift	588
13.3.4. Parameter Identification	589
13.3.5. Test and Model Comparison	589
References	593
List of Symbols	603
Appendix 1. Sign Conventions for Force and Moment and Wheel Slip	609
Appendix 2. Online Information	611
Appendix 3. MF-Tire/MF-Swift Parameters and Estimation Methods	613
Index	627

Exercises

Exercise 1.1. Construction of effective axle characteristics at load transfer	14
Exercise 1.2. Four-wheel steer, at the condition that the vehicle slip angle vanishes	35
Exercise 1.3. Construction of the complete handling diagram from pairs of axle characteristics	41
Exercise 1.4. Stability of a trailer	57
Exercise 2.1. Slip and rolling speed of a wheel steered about a vertical axis	71
Exercise 2.2. Slip and rolling speed of a wheel steered about an inclined axis (motorcycle)	71
Exercise 2.3. Partial differential equations with longitudinal slip included	81
Exercise 3.1. Characteristics of the brush model	112
Exercise 4.1. Assessment of off-nominal tire side force characteristics and combined slip characteristics with F_x as input quantity	207
Exercise 4.2. Assessment of force and moment characteristics at pure and combined slip using the <i>Magic Formula</i> and the <i>Similarity Method</i> with κ as input	208
Exercise 5.1. String model at steady turn slip	240
Exercise 6.1. Influence of the tyre inertia on the stability boundary	295
Exercise 6.2. Zero energy circle applied to the simple trailing wheel system	317
Exercise 7.1. Wheel subjected to camber, lateral and vertical axle oscillations	353
Exercise 8.1. Response to tyre stiffness variations	378
Exercise 8.2. Self-excited wheel hop	379

We may also refer to the online information, cf. App. 2, containing MATLAB applications.

Tire Characteristics and Vehicle Handling and Stability

Chapter Outline

1.1. Introduction	2	Stability of the Motion	28
1.2. Tire and Axle Characteristics	3	Free Linear Motions	28
1.2.1. Introduction to Tire Characteristics	3	Forced Linear Vibrations	31
1.2.2. Effective Axle Cornering Characteristics	7	1.3.3. Nonlinear Steady-State Cornering Solutions	35
Effective Axle Cornering Stiffness	9	Stability of the Motion at Large Lateral Accelerations	38
Effective Nonlinear Axle Characteristics	12	Assessment of the Influence of Pneumatic Trail on Handling Curve	43
1.3. Vehicle Handling and Stability	16	Large Deviations with Respect to the Steady-State Motion	44
1.3.1. Differential Equations for Plane Vehicle Motions	17	1.3.4. The Vehicle at Braking or Driving	49
1.3.2. Linear Analysis of the Two-Degree-of-Freedom Model	22	1.3.5. The Moment Method	51
Linear Steady-State Cornering Solutions	24	1.3.6. The Car-Trailer Combination	53
Influence of the Pneumatic Trail	27	1.3.7. Vehicle Dynamics at More Complex Tire Slip Conditions	57

1.1. INTRODUCTION

This chapter is meant to serve as an introduction to vehicle dynamics with emphasis on the influence of tire properties. Steady-state cornering behavior of simple automobile models and the transient motion after small and large steering inputs and other disturbances will be discussed. The effects of various shape factors of tire characteristics (cf. Figure 1.1) on vehicle handling properties will be analyzed. The slope of the side force F_y vs slip angle α near the origin (the cornering or side slip stiffness) is the determining parameter for the basic linear handling and stability behavior of automobiles. The possible offset of the tire characteristics with respect to their origins may be responsible for the occurrence of the so-called tire-pull phenomenon. The further nonlinear shape of the side (or cornering) force characteristic governs the handling and stability properties of the vehicle at higher lateral accelerations. The load dependency of the curves, notably the nonlinear relationship of cornering stiffness with tire normal load, has a considerable effect on the handling characteristic of the car. For the (quasi)-steady-state handling analysis, simple single track (two-wheel) vehicle models will be used. Front and rear axle effective side force characteristics are introduced to represent effects that result from suspension and steering system design factors such as steering compliance, roll steer, and lateral load transfer. Also, the effect of possibly applied (moderate) braking and driving forces may be incorporated in the effective characteristics. Large braking forces may result in wheel lock and possibly large deviations from the undisturbed path. The motion resulting from wheel lock will be dealt with in an application of the theory of a simple physical tire model in Chapter 3 (the brush model). The application of the handling and stability theory to the dynamics of heavy trucks will also be briefly dealt with in this chapter. Special attention will be given to the phenomenon of oscillatory instability that may show up with the car-trailer combination.

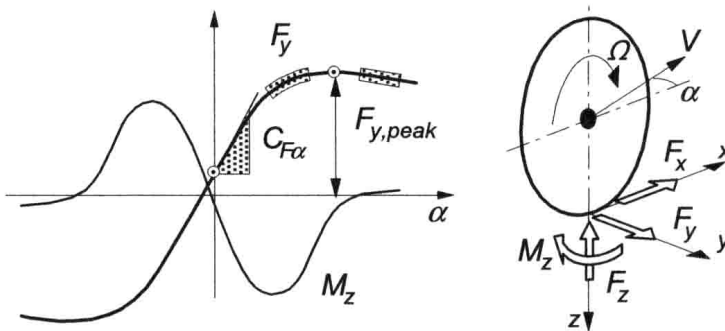


FIGURE 1.1 Characteristic shape factors (indicated by points and shaded areas) of tire or axle characteristics that may influence vehicle handling and stability properties. Slip angle and force and moment positive directions, cf. App. 1.

When the wavelength of an oscillatory motion of the vehicle that may arise from road unevenness, brake torque fluctuations, wheel unbalance, or instability (shimmy) is smaller than say 5 m, a non-steady-state or transient description of tire response is needed to properly analyze the phenomenon. In Chapters 5–8, these matters will be addressed. Applications demonstrate the use of transient and oscillatory tire models and provide insight into the vehicle dynamics involved. Chapter 11 is especially devoted to the analysis of motorcycle cornering behavior and stability.

1.2. TIRE AND AXLE CHARACTERISTICS

Tire characteristics are of crucial importance for the dynamic behavior of the road vehicle. In this section, an introduction is given to the basic aspects of the force- and moment-generating properties of the pneumatic tire. Both the pure and combined slip characteristics of the tire are discussed and typical features presented. Finally, the so-called effective axle characteristics are derived from the individual tire characteristics and the relevant properties of the suspension and steering system.

1.2.1. Introduction to Tire Characteristics

The upright wheel rolling freely, that is without applying a driving torque, over a flat level road surface along a straight line at zero side slip, may be defined as the starting situation with all components of slip equal to zero. A relatively small pulling force is needed to overcome the tire-rolling resistance, and a side force and (self)-aligning torque may occur as a result of the not completely symmetric structure of the tire. When the wheel moves in a way that the condition of zero slip is no longer fulfilled, wheel slip occurs that is accompanied by a buildup of additional tire deformation and possibly partial sliding in the contact patch. As a result, (additional) horizontal forces and the aligning torque are generated. The mechanism responsible for this is treated in detail in the subsequent chapters. For now, we will suffice with some important experimental observations and define the various slip quantities that serve as inputs into the tire system and the moment and forces that are the output quantities (positive directions according to Figure 1.1). Several alternative definitions are in use as well. In Appendix 1, various sign conventions of slip, camber, and output forces and moments together with relevant characteristics have been presented.

For the freely rolling wheel, the forward speed V_x (longitudinal component of the total velocity vector V of the wheel center) and the angular speed of revolution Ω_o can be taken from measurements. By dividing these two quantities, the so-called effective rolling radius r_e is obtained:

$$r_e = \frac{V_x}{\Omega_o} \quad (1.1)$$