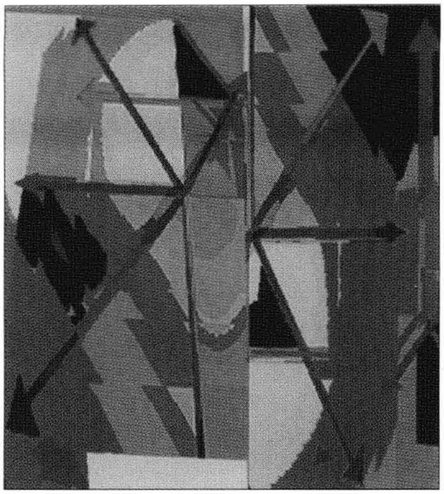
ELEMENTARY LINEAR ALGEBRA

7 EDITION

HOWARD ANTON



ELEMENTARY LINEAR ALGEBRA

SEVENTH EDITION

HOWARD ANTON
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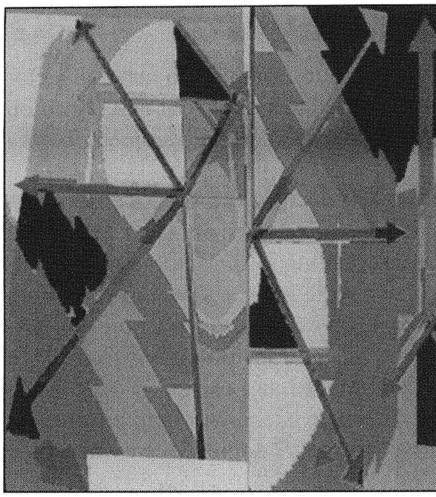
To my wife Pat
and
my children
Brian, David, and Lauren



PREFACE

As with previous editions, this new edition gives an elementary treatment of linear algebra that is suitable for students in their freshman or sophomore year. My aim is to present the fundamentals of linear algebra in the clearest possible way—pedagogy is the main consideration. Calculus is not a prerequisite, but there are exercises and examples for students with calculus backgrounds; those exercises and examples are clearly marked as such and can be omitted with no loss of continuity.

SUMMARY OF CHANGES IN THIS EDITION

Although this edition has much in common with its predecessors, it is a substantial revision. I have tried to maintain the clarity and style of the earlier editions, yet reflect the changing needs of a new generation of students. To this end I have implemented a number of the recommendations of the *Linear Algebra Curriculum Study Group*. I have also made some organizational changes that should make it easier for instructors to cover the fundamentals of *all* major topics, even with severe time constraints. A chapter-by-chapter description of the changes is given later in this preface, but here is a summary of the more noteworthy changes:

- **Greater Emphasis on Relationships Between Concepts:** One of the important goals of a course in linear algebra is to establish the intricate thread of relationships between systems of equations, matrices, determinants, vectors, linear transformations, and eigenvalues. In this edition that thread of relationships is developed through the following crescendo of theorems that link each new idea with ideas that preceded it: 1.5.3, 1.6.4, 2.3.6, 4.3.4, 5.6.9, 6.2.7, 6.4.5, 7.1.5. These theorems not only bring a coherence to the linear algebra landscape but also serve as a constant source of review.

- **Smoother Transition to Abstraction:** The transition from R^n to general vector spaces is traumatic for most students, so I have tried to smooth out that transition by discussing R^n in detail with more emphasis on the underlying geometry before proceeding to general vector spaces.
- **Early Exposure to Linear Transformations and Eigenvalues:** To ensure that the material on linear transformations and eigenvalues does not get lost at the end of the course, some of the basic concepts relating to those topics are developed earlier in the text and then reviewed when the topic is developed in more depth in the later part of the text. For example, characteristic equations are discussed briefly in the section on determinants. Linear transformations from R^n to R^m are discussed immediately after R^n is introduced and reviewed later in the context of general linear transformations. These revisions will help ensure that students are exposed to the fundamentals of all major topics, even when time is tight.
- **Greater Emphasis on Visualization:** In keeping with the current interest in visualization and the growing applications of linear algebra to graphics, I have placed greater emphasis on the geometric aspects of rotations, projections, and reflections in R^2 and R^3.
- **New Material on Least Squares and *QR*-Decomposition:** New material on least squares and the *QR*-decomposition has been added in response to the growing interest in those topics.
- **More Proofs:** A number of proofs that were previously omitted have been added. All proofs in the text are written in a style tailored for beginners, and special care has been exercised to ensure that the accessibility and friendliness of the text has not been adversely affected by the additional proofs. Those who want a tighter course mathematically will find the new edition better suited for that purpose, and those who want a more conceptual course will have a greater choice in the proofs to include or exclude.

DETAILS OF THE CHANGES IN THE SEVENTH EDITION

The wide acceptance of the first six editions has been most gratifying, and I am appreciative of the many constructive suggestions received from users and reviewers. Portions of the text have been revised for greater clarity, and substantial changes in content and organization have been made in response to both reviewer and user suggestions as well as recommendations of the *Linear Algebra Curriculum Study Group*.

There are many ways in which one can order the material in a linear algebra course; the ordering of the chapters that I have selected reflects my adherence to the axiom that one should proceed from the familiar to the unfamiliar and from the concrete to the abstract.

Here is a chapter-by-chapter summary of the major changes in the seventh edition.

- **Chapter 1** There is a new section on matrices with special forms: diagonal, triangular, and symmetric. By reorganizing the material slightly, the number of sections in this chapter has not increased.
- **Chapter 2** New introductory material on eigenvalues, eigenvectors, and characteristic equations has been added to this determinant chapter. This material is reviewed and then discussed in more detail in Chapter 7. The proof of the formula $\det(AB) = \det(A)\det(B)$ has been added.
- **Chapter 3** There is new material on the vector equations of lines and planes and the geometric interpretation of 2×2 and 3×3 determinants.
- **Chapter 4** This is a new chapter devoted exclusively to R^n. Basic concepts are developed, and there is an introduction to linear transformations from R^n to R^m with emphasis on the geometry of projections, rotations, and reflections. Unlike the previous edition, this material now comes *before* the development of general vector spaces. The material in this chapter is reexamined later in the context of general vector spaces.
- **Chapter 5** This chapter corresponds to Chapter 4 in the previous edition. Many of the proofs that were omitted in the previous edition have been added. There is also new material on the Wronskian for students who have studied calculus, and there is new material on the four fundamental spaces of a matrix.
- **Chapter 6** This chapter corresponds to Chapter 5 in the previous edition. There is new material on orthogonal complements, QR-decomposition, and least squares.
- **Chapter 7** This chapter corresponds to Chapter 6 in the previous edition. Material developed earlier on eigenvalues and eigenvectors is reviewed. There is new material on geometric and algebraic multiplicity and an improved explanation of the requirements for diagonalizability.
- **Chapter 8** This chapter corresponds to Chapter 7 in the previous edition. The material has been rewritten substantially to reflect the fact that linear transformations from R^n to R^m were introduced in Chapter 4.
- **Chapter 9** This chapter corresponds to Chapter 8 and Sections 9.1 and 9.2 in the previous edition. The section on the geometry of linear operators on R^2 has been rewritten to build on concepts developed in Section 4.2.
- **Chapter 10** This chapter corresponds to Chapter 10 in the previous edition. The changes are minor.

ABOUT THE EXERCISES

Each exercise set begins with routine drill problems, progresses to problems with more substance, and concludes with theoretical problems. At the end of most chapters there is a set of Supplementary Exercises that are more challenging and force the student to draw ideas from an entire chapter rather than a specific section.

SUPPLEMENTARY MATERIALS

FOR THE STUDENT

- ***Student Solutions Manual*** *Charles A. Grobe, Jr.* (*Bowdoin College*) *and Elizabeth M. Grobe* provides detailed solutions to most theoretical exercises and many computational exercises in the text. Included is a solution to at least one nonroutine exercise of every type. (ISBN: 0-471-30622-3)

FOR THE INSTRUCTOR

- ***Test Bank*** *Randy Schwartz* (*Schoolcraft College*) includes approximately 50 free-form questions and 5 essay questions for each chapter and one sample cumulative final exam. Also given are worked out solutions to each question in the test bank. (ISBN: 0-471-30848-X)

- ***Computerized Test Bank*** provides instructors with the entire test bank on disk and the capability to create customized tests by scrambling questions or choosing specific questions and sequences. This software is available for IBM PC's and compatibles and Macintosh computers. (IBM ISBN: 0-471-00824-9; Macintosh ISBN: 0-471-00826-5)

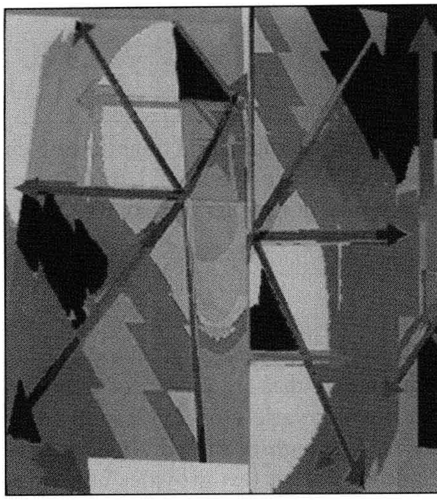
FOR A MICROCOMPUTER-ASSISTED COURSE

- ***Linear Algebra with Derive*** *Benny Evans and Jerry Johnson* (*Oklahoma State University*) is a supplementary manual designed to help students use the Derive program as a tool to solve problems in the course, and to provide unusual problems that encourage exploration and discovery. (ISBN: 0-471-59194-7)

- ***Linear Algebra Applications Software*** (*IntelliPro*) consists of 10 application modules featuring graphic representations, animations, and simulation problems requiring a high degree of student interaction. Available for IBM PC's and compatibles running *Windows*. This software may be purchased independently of the text (ISBN: 0-471-00827-3) or shrinkwrapped with the text at a substantial discount price (ISBN: 0-471-00828-1).

- ***LINEAR-KIT*** is a software package that can perform most of the basic linear algebra computations using either fractions or decimals and will save the student many hours of homework time that can be used for other pedagogical purposes. It is available free to departments adopting this text or can be purchased by individuals. This software is available for IBM PC's and compatibles. (ISBN: 0-471-61998-1)

If any of these supplements is not in your bookstore, ask the bookstore manager to order a copy for you.



A GUIDE FOR THE INSTRUCTOR

POSSIBLE SCHEDULES FOR A STANDARD COURSE

I have reviewed a large number of course outlines for linear algebra courses. The variation between institutions is wide, but courses tend to fall into two categories—those with about 25–30 lectures (excluding tests and reviews) and those with about 35–40 lectures (excluding tests and reviews). Based on my examination of the course outlines, I have provided two templates for constructing your course outline. These will have to be adjusted to reflect your local interests and requirements, but they may be helpful as a starting point. In the long template, it is assumed that all sections in the chapter are covered, and in the short template it is assumed that the instructor selects material to fit the available time.

Two changes in the organization of the text make it easier to construct shorter courses: the brief introduction to eigenvalues and eigenvectors that occurs in Sections 2.3 and 4.3 and the earlier placement of linear transformations from R^n to R^m in Chapter 4. These changes ensure that the student will have some familiarity with these basic concepts, even if the time available for Chapters 7 and 8 is limited. Note also that Chapter 3 can be omitted without loss of continuity for students who are already familiar with the material.

	Long Template	Short Template
Chapter 1	7 lectures	6 lectures
Chapter 2	4 lectures	3 lectures
Chapter 4	3 lectures	3 lectures
Chapter 5	8 lectures	7 lectures
Chapter 6	6 lectures	3 lectures
Chapter 7	4 lectures	3 lectures
Chapter 8	6 lectures	2 lectures
Total	38 lectures	27 lectures

VARIATIONS OF THE STANDARD COURSE

Many variations of the standard course are possible. For example, one might create an alternative long template by following the time allocations in the short template and devoting the remaining 11 lectures to some of the topics in Chapters 9 and 10.

APPLICATIONS-ORIENTED COURSE

Chapter 9 contains selected applications of linear algebra that are mostly of a mathematical nature. Instructors who are interested in a wider variety of applications may want to consider the alternative version of this text, ***Elementary Linear Algebra, Applications Version,*** by Howard Anton and Chris Rorres. That text provides numerous applications to business, biology, engineering, economics, the social sciences, and the physical sciences.

Linear Algebra Applications Software by IntelliPro consists of 10 application modules featuring graphic representations, animations, and simulation problems requiring a high degree of student interaction. For IBM PC's and compatibles running *Windows*, this software may be purchased independently of the text (ISBN: 0-471-00827-3) or shrinkwrapped with the text at a substantial discount price (ISBN: 0-471-00828-1).

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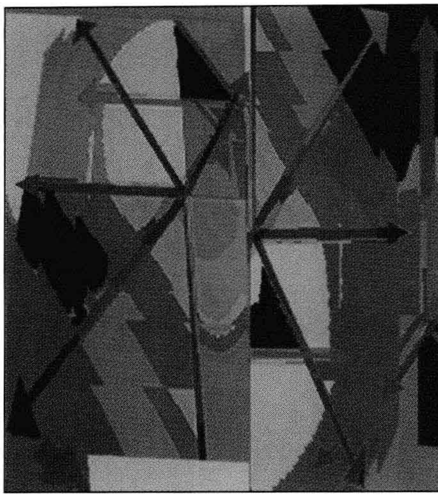

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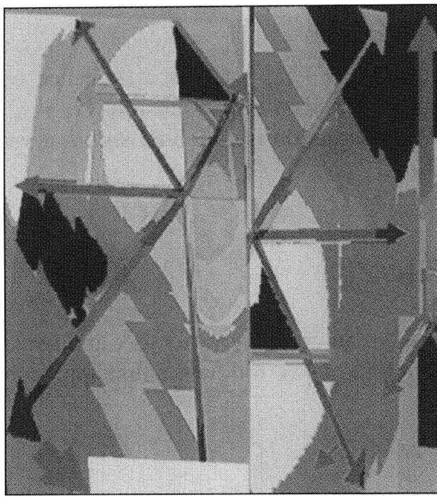

CHAPTER 1

SYSTEMS OF LINEAR EQUATIONS AND MATRICES

1.1 INTRODUCTION TO SYSTEMS OF LINEAR EQUATIONS

The study of systems of linear equations and their solutions is one of the major topics in linear algebra. In this section we shall introduce some basic terminology and discuss a method for solving such systems.

LINEAR EQUATIONS

A line in the xy-plane can be represented algebraically by an equation of the form

$$a_1x + a_2y = b$$

An equation of this kind is called a linear equation in the variables x and y. More generally, we define a ***linear equation*** in the n variables $x_1, x_2, \ldots, x_n$ to be one that can be expressed in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, a_2, \ldots, a_n$, and b are real constants. The variables in a linear equation are sometimes called the ***unknowns***.

Example 1 The following are linear equations:

$$x + 3y = 7 \qquad x_1 - 2x_2 - 3x_3 + x_4 = 7$$
$$y = \tfrac{1}{2}x + 3z + 1 \qquad x_1 + x_2 + \cdots + x_n = 1$$

Observe that a linear equation does not involve any products or roots of variables. All variables occur only to the first power and do not appear as arguments for trigonometric, logarithmic, or exponential functions. The following are *not* linear equations:

$$x + 3y^2 = 7 \qquad 3x + 2y - z + xz = 4$$
$$y - \sin x = 0 \qquad \sqrt{x_1} + 2x_2 + x_3 = 1$$

A ***solution*** of a linear equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ is a sequence of n numbers $s_1, s_2, \ldots, s_n$ such that the equation is satisfied when we substitute $x_1 = s_1$, $x_2 = s_2, \ldots, x_n = s_n$. The set of all solutions of the equation is called its ***solution set*** or sometimes the ***general solution*** of the equation.

Example 2 Find the solution set of

(a) $4x - 2y = 1$ (b) $x_1 - 4x_2 + 7x_3 = 5$

Solution (a). To find solutions of (a), we can assign an arbitrary value to x and solve for y, or choose an arbitrary value for y and solve for x. If we follow the first approach and assign x an arbitrary value t, we obtain

$$x = t, \qquad y = 2t - \tfrac{1}{2}$$

These formulas describe the solution set in terms of the arbitrary parameter t. Particular numerical solutions can be obtained by substituting specific values for t. For example, $t = 3$ yields the solution $x = 3$, $y = \frac{11}{2}$; and $t = -\frac{1}{2}$ yields the solution $x = -\frac{1}{2}$, $y = -\frac{3}{2}$.

If we follow the second approach and assign y the arbitrary value t, we obtain

$$x = \tfrac{1}{2}t + \tfrac{1}{4}, \qquad y = t$$

Although these formulas are different from those obtained above, they yield the same solution set as t varies over all possible real numbers. For example, the previous formulas gave the solution $x = 3$, $y = \frac{11}{2}$ when $t = 3$, while the formulas immediately above yield that solution when $t = \frac{11}{2}$.

Solution (b). To find the solution set of (b) we can assign arbitrary values to any two variables and solve for the third variable. In particular, if we assign arbitrary values s and t to x_2 and x_3, respectively, and solve for x_1, we obtain

$$x_1 = 5 + 4s - 7t, \qquad x_2 = s, \qquad x_3 = t$$

LINEAR SYSTEMS

A finite set of linear equations in the variables $x_1, x_2, \ldots, x_n$ is called a ***system of linear equations*** or a ***linear system***. A sequence of numbers $s_1, s_2, \ldots, s_n$ is called a ***solution*** of the system if $x_1 = s_1$, $x_2 = s_2, \ldots, x_n = s_n$ is a solution of every equation in the system. For example, the system

$$\begin{aligned} 4x_1 - x_2 + 3x_3 &= -1 \\ 3x_1 + x_2 + 9x_3 &= -4 \end{aligned}$$

has the solution $x_1 = 1$, $x_2 = 2$, $x_3 = -1$ since these values satisfy both equations. However, $x_1 = 1$, $x_2 = 8$, $x_3 = 1$ is not a solution since these values satisfy only the first of the two equations in the system.

Not all systems of linear equations have solutions. For example, if we multiply the second equation of the system

$$\begin{aligned} x + y &= 4 \\ 2x + 2y &= 6 \end{aligned}$$