LECTURE NOTES IN PHYSICS

M.G.A. Paris
J. Řeháček
(Eds.)

Quantum State Estimation

Springer

Matteo Paris Jaroslav Řeháček (Eds.)

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Editors

Matteo Paris
Università di Milano
Dipartimento di Fisica
Via Celoria 16
20133 Milano
Italy

Jaroslav Řeháček
Palacky University
Department of Optics
17. listopadu 50
77200 Olomouc
Czech Republic

Matteo Paris Jaroslav Řeháček (Eds.), *Quantum State Estimation*, Lect. Notes Phys. **649** (Springer, Berlin Heidelberg 2004), DOI 10.1007/b98673

Library of Congress Control Number: 2004107804

ISSN 0075-8450
ISBN 3-540-22329-0 Springer Berlin Heidelberg New York

Springer is a part of Springer Science+Business Media

springeronline.com

Printed in Germany

Typesetting: Camera-ready by the authors/editor
Data conversion: PTP-Berlin Protago-TeX-Production GmbH
Cover design: *design & production*, Heidelberg

Printed on acid-free paper
54/3141/ts - 5 4 3 2 1 0

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Preface

This book is a comprehensive collection representing most of the theoretical and experimental developments of the last decade in the field of quantum estimation of states and operations. Though the field is fairly new, it has already been recognized as a necessary tool for researchers in quantum optics and quantum information. The subject has a fundamental interest of its own, since it concerns the experimental characterization of the quantum state, the basic object of the quantum description of physical systems. Moreover, quantum estimation techniques have been receiving attention for their crucial role in the characterization of registers at the quantum level, which itself is a basic tool in the development of quantum information technology.

The field is now mature and a stable part of many graduate curricula, but only a few review papers have been published in recent years, and no comprehensive volume with theoretical and experimental contributions has ever appeared. We anticipate readers in the areas of fundamental quantum mechanics, quantum and nonlinear optics, quantum information theory, communication engineering, imaging and pattern recognition.

As editors, we wish to thank Berge Englert for encouragement and support, and all the authors for their contributions, which will advance both the specific field and the general appreciation of it. Their efforts and the significant time they spent preparing the chapters are much appreciated. We are also grateful to Janine O'Guinn of the University of Oregon for her excellent work in copy-editing the volume. Finally, let us acknowledge support from EC project IST-2000-29681, and Czech Ministry of Education project LN00A015.

Milano and Olomouc,
April 2004

Matteo G A Paris
Jaroslav Řeháček

List of Contributors

Joseph B. Altepeter,
Dept. of Physics,
University of Illinois
at Urbana-Champaign,
Urbana IL 61801, USA
altepete@uiuc.edu

Gerald Badurek,
Atominstitut
der Österreichischen Universitäten,
Stadionallee 2,
1020 Wien, Austria

Mark Beck,
Department of Physics,
Whitman College,
Walla Walla, WA 99362, USA
beckmk@whitman.edu

János A. Bergou,
Department of Physics
and Astronomy,
Hunter College of
The City University of New York,
695 Park Avenue,
New York, NY 10021, USA
jbergou@hunter.cuny.edu

Vladimír Bužek,
Research Center
for Quantum Information,
Institute of Physics,
Slovak Academy of Sciences,
Dúbravská cesta 9,
845 11 Bratislava, Slovakia
and
Faculty of Informatics,
Masaryk University,
Botanická 68a,
602 00 Brno, Czech Republic
buzek@savba.sk

Anthony Chefles,
Department of Physical Sciences,
University of Hertfordshire,
Hatfield AL10 9AB,
Hertfordshire, UK
A.Chefles@herts.ac.uk

Giacomo Mauro D'Ariano,
QUIT Group of INFM,
Dipartimento di Fisica "A. Volta",
Università di Pavia,
via Bassi 6,
Pavia, Italy
dariano@unipv.it
www.qubit.it

Jaromír Fiurášek,
Ecole Polytechnique, CP 165,
Université Libre de Bruxelles,
1050 Bruxelles, Belgium
and
Department of Optics,
Palacky University,
17. listopadu 50,
772 00 Olomouc, Czech Republic

Christopher A. Fuchs,
Quantum Information
and Optics Research,
Bell Labs, Lucent Technologies,
600–700 Mountain Avenue,
Murray Hill, New Jersey 07974, USA

Ulrike Herzog,
Institut für Physik,
Humboldt-Universität Berlin,
Newtonstrasse 15,
12489 Berlin, Germany
ulrike.herzog
@physik.hu-berlin.de

Mark Hillery,
Department of Physics
and Astronomy,
Hunter College of
The City University of New York,
695 Park Avenue,
New York, NY 10021, USA
mhillery@hunter.cuny.edu

Zdeněk Hradil,
Department of Optics,
Palacky University,
17. listopadu 50,
772 00 Olomouc, Czech Republic
hradil@aix.upol.cz

Daniel F.V. James,
Theoretical Division T-4,
Los Alamos National Laboratory,
Los Alamos, New Mexico 87545,
USA
dfvj@lanl.gov

Miroslav Ježek,
Department of Optics,
Palacky University,
17. listopadu 50,
772 00 Olomouc, Czech Republic

Paul G. Kwiat,
Dept. of Physics,
University of Illinois
at Urbana-Champaign,
Urbana IL 61801, USA
kwiat@uiuc.edu

Paoloplacido Lo Presti,
QUIT Group of INFM,
Dipartimento di Fisica "A. Volta",
Università di Pavia,
via Bassi 6,
Pavia, Italy
lopresti@unipv.it
www.qubit.it

Alexander Lvovsky,
Universität Konstanz,
Fakultät für Physik,
Fach 696,
78457 Konstanz, Germany
www.uni-konstanz.de/
quantum-optics/quantech/

Francesco De Martini,
Dipartimento di Fisica
and
Istituto Nazionale
per la Fisica della Materia,
Università "La Sapienza",
Roma, 00185 - Italy

Gabriel Molina-Teriza
Institut für Experimentalphysik,
Universität Wien,
Boltzmanngasse 5,
1090 Wien, Austria

Matteo G.A. Paris,
Unità INFM
and
Dipartimento di Fisica,
Università di Milano,
Milano, Italia

Helmut Rauch,
Atominstitut
der Österreichischen Universitäten,
Stadionallee 2,
1020 Wien, Austria
rauch@ati.ac.at

Michael G. Raymer,
Department of Physics,
University of Oregon,
Eugene, OR 97403, USA
raymer@uoregon.edu

Jaroslav Řeháček,
Department of Optics,
Palacky University,
17. listopadu 50,
772 00 Olomouc, Czech Republic

Marco Ricci,
Dipartimento di Fisica
and
Istituto Nazionale
per la Fisica della Materia,
Università "La Sapienza",
Roma, 00185 - Italy

Massimiliano F. Sacchi,
Unità INFM
and
Dipartimento di Fisica "A. Volta",
Università di Pavia,
via Bassi 6,
Pavia, Italia

Rüdiger Schack,
Department of Mathematics,
Royal Holloway,
University of London,
Egham, Surrey TW20 0EX, UK

Fabio Sciarrino,
Dipartimento di Fisica
and
Istituto Nazionale
per la Fisica della Materia,
Università "La Sapienza",
Roma, 00185 - Italy

Alipasha Vaziri,
Institut für Experimentalphysik,
Universität Wien,
Boltzmanngasse 5,
1090 Wien, Austria

Michael Zawisky,
Atominstitut
der Österreichischen Universitäten,
Stadionallee 2,
1020 Wien, Austria

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Contents

1 Introduction

Matteo G.A. Paris[1] and Jaroslav Řeháček[2]

[1] Dipartimento di Fisica dell'Università di Milano, Italy
[2] Department of Optics, Palacky University, Olomouc, Czech Republic

The state of a physical system is the mathematical description of our knowledge of it, and provides information on its future and past. A state estimation technique is a method that provides the complete description of a system, *i.e* achieves the maximum possible knowledge of the state, thus allowing one to make the best, at least the best probabilistic, predictions on the results of any measurement that may be performed on the system.

In classical physics the state of a system is a set of numbers, and it is always possible, at least in principle, to devise a procedure consisting of multiple measurements that fully recovers the state of the system. In Quantum Mechanics this is no longer possible, and this impossibility is inherently related to fundamental features of the theory, namely its linearity and the Heisenberg uncertainty principle. On one hand linearity implies the no-cloning theorem [1], which forbids us to create perfect copies of an arbitrary system in order to make multiple measurements on the same state. On the other hand, the uncertainty principle [2] says that one cannot perform an arbitrary sequence of measurements on a single system without disturbing it in some way, *i.e.* inducing a back-action which modifies the state itself. Therefore, it is not possible, even in principle, to determine the quantum state of a single system without having some prior knowledge on it [3]. This is consistent with the very definition of a quantum mechanical state, which in turn prescribes how to gain information about the state: many identical preparations taken from the same statistical ensemble are needed and different measurements should be performed on each of the copies.

Despite its fundamental interest the problem of inferring the state of a quantum system from measurements is *not* as old as quantum mechanics, and the first systematic approach was the work of U. Fano in the late fifties [4]. In the last decade a constantly increasing interest has been devoted to the subject. On one side, new developments in experimental techniques, especially in the fields of photodetection and nonlinear optical technology, resulted in a set of novel and beautiful experiments about quantum mechanics. On the other, increasing attention has been directed to quantum information technology, which is mostly motivated by the promising techniques of error correction and purification, which make possible fault tolerant quantum computing and long distance teleportation and cryptography. In particular, the development of suitable purification protocols, and the possibility of a quantum charac-

M.G.A. Paris, J. Řeháček, Introduction, Lect. Notes Phys. **649**, 1–4 (2004)
http://www.springerlink.com/

terization of communication channels, rely heavily on quantum estimation techniques.

This book aims to review all of the relevant quantum estimation techniques, and to assess the state of art in this novel field which has provoked renewed interest in fundamentals quantum mechanics. A number of leading experts have cooperated to describe the main features of the field. The rest of this introduction gives a brief description of their chapters. The volume is divided in two parts. The first is devoted to quantum estimation in the strict sense, both of quantum states and quantum operations, whereas the second (much shorter) part addresses the problem of state discrimination.

Part I of the book starts with Chapt. 2 by G.M. D'Ariano *et al.*, which reviews quantum tomography, *i.e.* the determination of the expectation value of any operator (including nondiagonal projectors needed to construct a matrix representation of the density operator) for a generic quantum system from the measurement of a suitable set of observables (a quorum) on repeated preparations of the system. Topics include characterization of quora, determination of *pattern functions*, effect of instrumental noise, and examples of tomographic procedures for harmonic and spin systems.

Quantum estimation is in principle a deterministic problem, given that a quorum of observables is measured on the system of interest. However, often only partial information of the system can be achieved. Therefore, a question arises about what one can say about a quantum system given an arbitrary set of observations on repeated preparations of the system. In Chap. 3, Z. Hradil *et al.* give a statistical answer to this question using the maximum-likelihood principle. The formalism is applied to quantum-state estimation and discrimination as well as the estimation of quantum measurements and processes.

The polarization state of a photon is a natural experimental realization of a two-level quantum system – a qubit. For many experiments in quantum theory and quantum information it is very important to develop reliable sources of arbitrary polarization-entangled quantum states. Quantum estimation is important for the development of new quantum sources, since the quantum reconstruction techniques are natural means of calibration and tuning of experimental apparatuses. A detailed account of the production, characterization, and utilization of entangled states of light qubits is given by J.B. Altepeter *et al.* in Chap. 4.

Even in the realistic case of small ensembles, when the expectation values are not accessible, one can still infer the quantum state by means of the Bayesian principle of inference that provides a unique rule for updating the prior information about the quantum system after a measurement has been made. Although the principle itself is well justified, the notion of prior information is a highly subjective element of the theory. Therefore, in the Bayesian approach, the subjective interpretation of quantum states and operations is stressed. The formulation of the quantum Bayesian inference is by Ch. Fuchs and R. Schack in Chap. 5, and they will then apply it to the reconstruction of quantum states and quantum operations.

Yet another principle of inference based on partial knowledge – Jaynes' principle of maximum entropy – comes from the information theory. Unlike the maximum likelihood estimation of Chap. 3 that always selects the most likely configuration, the principle of maximum entropy leads to the least biased estimate consistent with the given information. Its typical applications are momentum problems: the determination of the quantum state from the expectation values of a few, tomographically incomplete observations. An overview of the applications of Jaynes' principle to quantum reconstruction is reported by V. Bužek in Chap. 6.

The development of quantum estimation techniques started with the proposal by Vogel and Risken [5] and with the first experiments (which already showed reconstructions of coherent and squeezed states of a radiation field mode) performed in Michael Raymer's group at the University of Oregon. [6]. Chapter 7, by M. Raymer and M. Beck is a detailed review of the theoretical and experimental work on quantum state measurement based on homodyne detection, and discuss the determination of the quantum state of one or more modes of the radiation field.

Tomographic methods were initially employed only for measuring radiation states. However, they can profitably be used also to characterize devices through imprinting of quantum operations on quantum states. In Chap. 8 G.M. D'Ariano and P.L. Presti give a self-contained presentation of the theoretical bases of the method, together with examples of experimental setups based on homodyne tomography. As a contrast, Chap. 9 by F. De Martini *et al.* is devoted to reviewing the experimental realization of many unitary and non unitary operations on light qubit and their effective characterization by Pauli tomography of the polarization state.

The utility of the maximum-likelihood principle in experimental quantum estimation is demonstrated by G. Badurek *et al.* in Chap. 10, which closes the first part of the book. The ideas presented in Chap. 3 are systematically applied to experiments with quantum systems of increasing complexity starting with the quantum phase or simple two-dimensional systems and eventually coming to an infinite-dimensional mode of light.

The second part of the book consists of two chapters devoted to decisions among quantum hypotheses. Here we have a quantum system prepared in a state chosen from a discrete set, rather than from the whole set of possible states, and we want to discriminate among the set starting from the results of certain measurements performed on the system. To the extent that the quantum states to be discriminated are nonorthogonal, the problem is highly non-trivial, and of practical importance. Indeed, the increasing need for faster communication implies the steady decrease of the energy used for the transmission of a bit of information through the communication channel. When the carriers of information became truly microscopic systems the classical information they carry is encoded into their quantum state. A fundamental theorem of quantum theory tells us that it is not possible perfectly to distinguish between two non-orthogonal quantum states. This places a fundamental limit on the error rate of the communication because the or-