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Joseph F. Shelley

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VECTOR MECHANICS FOR ENGINEERS Volume II: DYNAMICS

by

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To
Gabrielle
and
Stefanie, Suzanne,
Matthew, and Meredith

Your sweet love
such wealth brings,
I scorn to change
my state with kings

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TO THE STUDENT

Engineering mechanics is the study of the effects that forces produce on bodies. It has two major subdivisions: statics, in which the bodies are at rest or are moving with constant velocity; and dynamics, in which the bodies may possess any type of motion. Thus, acceleration is a necessary part of the description of dynamics problems. It is the absence of acceleration effects that distinguishes statics from dynamics.

This book is the second volume of a two-volume set. The first volume, in Chapters 1–13, treats the subject of statics. Statics is one of the beginning courses in the fields of aeronautical, civil, and mechanical engineering and is required of all engineering students. A thorough understanding of its fundamental principles is a prerequisite for further study in dynamics, strength (or mechanics) of materials, structural engineering, stress analysis, and mechanical design and analysis.

This second volume treats the subject of dynamics. The mastery of the principles of dynamics has direct, useful application in itself and is a prerequisite for further study in vibrations, dynamics of machinery, fluid mechanics, and mechanical design and analysis.

This book is a completely self-contained treatment of dynamics. All the fundamental definitions, concepts, and problem-solving techniques of dynamic force and motion analysis are introduced through questions. All groups of problems with numerical solutions are preceded by presentation of the particular definition, concept, or technique required for the solution of those problems. The material in each chapter is arranged in grouped sections of topics, and, within each section, the problems are arranged in a generally increasing order of difficulty.

The last question in each chapter is a review of the fundamental definitions, concepts, and techniques introduced and used in that chapter. The final chapter in this book is a self-study review of all the earlier questions on fundamental definitions, concepts, and techniques. This chapter contains 251 review questions, which are referenced by problem number to the original question in order to make it easy for the reader to refer to the answer. These review questions are not counted as part of the 700 Solved Problems.

Many problems are presented as easily recognizable mechanical or machine systems. The intent is to give a physical, real-world flavor to these problems with which the reader can readily identify. Additional commentary on the solutions is frequently provided at the end of problems to clarify or point out a particular characteristic or limitation of the solution. In many problems a comparison is made between the solutions when certain conditions of the original problem are varied. This gives a general engineering design flavor to these problems. The units used in this text are equally divided between U.S. Customary (USCS) units and International System (SI) units.

There is a carefully developed index, by problem number, at the end of the text. All problems involving definitions, concepts, or techniques are cross-referenced by topic. All problems in the text are listed in this index, and those which are more advanced, or have unusually lengthy solutions, are identified. The reader is encouraged to review this index and become familiar with its use, and thus be able to rapidly identify specific problems in any desired area of dynamics.

This book may be used with any textbook in dynamics. It may also be used by itself. A cross-reference of this book, by topics, with the three leading textbooks in dynamics is included in the appendices. These three texts are Beer and Johnston, *Vector Mechanics for Engineers: Statics and Dynamics*, 5th ed.; Hibbeler, *Engineering Mechanics: Statics and Dynamics*, 5th ed.; and Meriam and Kraige, *Engineering Mechanics, volume 2, Dynamics*, 2d ed.

Preparation of a work such as this is a very subjective exercise in creativity. It reflects many judgments on the part of the author with respect to organization of material and emphasis of topics. As with any other book, it receives its ultimate review by the readers. The author welcomes comments and suggestions on any matters of content, organization, or emphasis, and such information may be sent to the Schaum Division, McGraw-Hill Publishing Company, 1221 Avenue of the Americas, New York, N.Y. 10020. Every effort will be made to reply to this correspondence.

As a final note, the author wishes to thank Meredith Ann Shelley for her yeoman service in helping to perform the myriad tasks required to bring this work to its final form.

LIST OF SYMBOLS

$\mathbf{a}$	Acceleration
a	Magnitude of $\mathbf{a}$, acceleration in rectilinear translation
$a, b, \ldots$	Constants, lengths points
$a_1, a_2, \ldots$	Constants, lengths
a_{avg}	Average acceleration
$\mathbf{a}_a, \mathbf{a}_b$	Acceleration of points a and b
a_a, a_b	Magnitudes of $\mathbf{a}_a$ and $\mathbf{a}_b$
$\mathbf{a}_{ab}$	Relative acceleration of point a with respect to point b
a_{ab}	Magnitude of $\mathbf{a}_{ab}$
$\mathbf{a}_n$	Normal acceleration in curvilinear translation
a_n	Magnitude of $\mathbf{a}_n$
$\mathbf{a}_t$	Tangential acceleration in curvilinear translation
a_t	Magnitude of $\mathbf{a}_t$
a_x	Rectilinear acceleration in x direction
a_x, a_y	x and y components of $\mathbf{a}$
$a_0, a_1, \ldots$	Magnitudes of acceleration
a_{an}, a_{bn}	Magnitude of normal acceleration of points a and b, respectively
a_{ta}, a_{tb}	Magnitude of tangential acceleration of points a and b, respectively
$\mathbf{A}$	General vector quantity
A	Magnitude of $\mathbf{A}$, projected area of body on plane normal to direction of motion
$A, A_1, A_2, \ldots$	Plane areas
$\mathbf{a}_A, \mathbf{a}_B, \ldots$	Translational acceleration of bodies $A, B, \ldots$
$a_A, a_B, \ldots$	Magnitudes of $\mathbf{a}_A, \mathbf{a}_B, \ldots$
$\mathbf{a}_c$	Absolute acceleration of center of mass of a body
a_c	Magnitude of $\mathbf{a}_c$
c	Viscous drag force constant
c_A, c_B	Viscous drag force constant of dampers A and B, respectively
C_D	Drag coefficient
CM	Center of mass of a body
$d, d_1, d_2, \ldots$	Diameter of circular path, cylinder, gear, or sphere; axial separation distance between rotating point masses
d_i, d_0	Inside and outside diameters of cylinder or disk
d_x, d_y, d_z	Distance between center of mass of body and x, y, and z axes, respectively
D	Length
d_{0y}	Distance between center of mass of an element and y_0 axis
$d_{x1}, d_{x2}, \ldots$	Distance between center of mass of elements 1, 2, . . . and x axis
$d_{y1}, d_{y2}, \ldots$	Distance between center of mass of elements 1, 2, . . . and y axis
$d_{z1}, d_{z2}, \ldots$	Distance between center of mass of elements 1, 2, . . . and z axis
$d_{x_1}, d_{y_1}, d_{z_1}$	Distance between center of mass of a body and x_1, y_1, and z_1 axes, respectively
d_{x_2}	Distance between center of mass of a body and x_2 axis
$d_{x_{13}}$	Distance between center of mass of element 3 and x_1 axis
d_{y,y_1}	Distance between y and y_1 axes
e	Coefficient of restitution
f	Function, such as $y = f(x)$
ft	Foot (USCS unit of length)
$\mathbf{F}$	Force, friction force, resultant force acting on particle or body
F	Magnitude of $\mathbf{F}$
F_a, F_b, F_c	Forces acting on pins a, b, and c, respectively
F_c, F_d, F_e	Centrifugal force of material which would occupy holes c, d, and e, respectively
$\mathbf{F}_i, i = 1, 2, \ldots$	Forces acting on particle $1, 2, \ldots$, or body $1, 2, \ldots$
$F_i, i = 1, 2, \ldots$	Magnitude of $\mathbf{F}_i$, $i = 1, 2, \ldots$, centrifugal forces acting on mass elements $1, 2, \ldots$
F_x, F_y	x and y components, respectively, of $\mathbf{F}$

$\mathbf{F}_n$	Normal force acting on particle or body
F_n	Magnitude of $\mathbf{F}_n$
$\mathbf{F}_t$	Tangential force acting on particle or body
F_t	Magnitude of $\mathbf{F}_t$
F_{in}, F_{it}	Normal and tangential components, respectively, of force acting on mass element m_i
$F_A, F_B, \ldots$	Centrifugal forces acting on mass elements $A, B, \ldots$
F_{12}, F_{21}	Pair of action-reaction forces on mass particles m_1 and m_2
F_{12}, F_{23}	Tangential force transmitted between teeth on gears 1 and 2, and on gears 2 and 3, respectively
F_D	Drag force
F_{NC}	Nonconservative force
F_s	Spring force
$\mathbf{g}$	Acceleration of gravitational field
g	Magnitude of $\mathbf{g}$ ($g = 32.2\ \text{ft/s}^2$ or $386\ \text{in/s}^2$ in USCS units, and $9.81\ \text{m/s}^2$ in SI units)
h	Vertical displacement of a body, height of a body above the ground or above a reference point
$h_1, h_2, \ldots$	Height of cylinder $1, 2, \ldots$; height of body $1, 2, \ldots$ above the ground
$h', h'', \ldots$	Rebound height after first, second, ... impact
hp	Horsepower
$\mathbf{H}$	Angular momentum of a body
H	Magnitude of $\mathbf{H}$
$H_1, H_2, \ldots$	Magnitude of angular momentum at endpoints $1, 2, \ldots$ of a time interval
$\mathbf{H}_0$	Angular momentum about center of mass of body
H_0	Magnitude of $\mathbf{H}_0$
in	Inch (USCS unit of length)
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vectors in x, y, and z directions, respectively
i	Index of summation, $i = 1, 2, \ldots$
I_x, I_y, I_z	Mass moment of inertia of body about x, y, and z axes, respectively
I_{0x}, I_{0y}, I_{0z}	Mass moment of inertia of body about centroidal x_0, y_0, and z_0 axes, respectively
$I_{x_1}, I_{y_1}, I_{z_1}$	Mass moment of inertia of body about x_1, y_1, and z_1 axes, respectively
I_{x_2}	Mass moment of inertia of body x_2 axis
$I_1, I_2, \ldots$	Mass moment of inertia of mass elements $1, 2, \ldots$ about a specified axis
$I_{0x,1}, I_{0x,2}, \ldots$	Centroidal mass moment of inertia of mass elements $1, 2, \ldots$ about x axis
$I_{0y,1}, I_{0y,2}, \ldots$	Centroidal mass moment of inertia of mass elements $1, 2, \ldots$ about y axis
$I_{0z,1}, I_{0z,z}, \ldots$	Centroidal mass moment of inertia of mass elements $1, 2, \ldots$ about z axis
$I_{x1}, I_{x2}, \ldots$	Mass moment of inertia of mass elements $1, 2, \ldots$ about x axis
$I_{y1}, I_{yz}, \ldots$	Mass moments of inertia of mass elements $1, 2, \ldots$ about y axis
$I_{z1}, I_{z2}, \ldots$	Mass moments of inertia of mass elements $1, 2, \ldots$ about z axis
$I_0, I_{01}, I_{02}, I_{03}$	Mass moment of inertia of body about center axis or center of mass
I_{xA}, I_{yA}, I_{zA}	Moment of inertia of plane area about x, y, and z axes, respectively
I_{xM}, I_{ym}, I_{zM}	Mass moment of inertia of thin, plane body about x, y, and z axes, respectively
$I_{x1,A}, I_{x2,A}, \ldots$	Moment of inertia of plane area elements $1, 2, \ldots$ about x axis
I_{xl}, I_{yl}, I_{zl}	Mass moment of inertia of plane body formed of thin rod shapes, length moment of inertia of plane curve, about x, y, and z axes, respectively
$I_{y1,1}, I_{y1,2}, \ldots$	Mass moment of inertia of mass element $1, 2, \ldots$ about y_1 axis
I_{xy}, I_{yz}, I_{zx}	Mass product of inertia of body about x, y, and z axes
$I_{xy,A}, I_{xy,B}, I_{xy,C}$	Mass products of inertia of elements A, B, and C, respectively, about x, y, and z axes
$I_{xy,C}$	Mass product of inertia of a cylinder about x and y axes
$I_{xy,H}$	Mass product of inertia of mass, which would occupy a hole, about x and y axes
$I_{0,xy}, I_{0,yz}, I_{0,zx}$	Mass product of inertia of body about centroidal x_0, y_0, and z_0 axes
IC	Instant center
I_a	Mass moment of inertia of body about point a
I_0	Mass moment of inertia of body about center of mass
I_{assy}	Mass moment of inertia of assembly about specified axis

$I_{0A}, I_{0B}, \ldots$	Centroidal mass moments of inertia of bodies $A, B, \ldots$
$I_{01}, I_{02}, \ldots$	Centroidal mass moments of inertia of bodies $1, 2, \ldots$
$\mathbf{I}'$	Impulse of a force
I'	Magnitude of $\mathbf{I}'$
I'_x, I'_y	x and y components of $\mathbf{I}'$
J	Polar mass moment of inertia of a thin, plane body
J	Joule
k	Spring constant
kg	Kilogram (SI unit of mass)
kn	Knot (speed of one nautical mile per hour)
k_x, k_y, k_z	Radii of gyration of body about x, y, and z axes, respectively
k_{0x}, k_{0y}, k_{0z}	Radii of gyration of body about centroidal x_0, y_0, and z_0 axes, respectively
k_0	Radius of gyration of a body about center of mass
l	Length, length of a plane curve
lb	Pound (USCS unit of force)
m	Mass of particle or body
m	Meter (SI unit of length)
$m_A, m_B, \ldots$	Mass of bodies $A, B, \ldots$
$m_1, m_2, \ldots$	Mass of elements $1, 2, \ldots$
M_a	Moment about point a
M_a, M_b	Dynamic moment about points a and b, respectively
m_a, m_b, m_c, m_d	Mass of dynamic balance correction weights
mi	mile (1 mi = 5280 ft)
M_0	Moment about center of mass of body
M_{ai}	Moment required to produce F_{it}
M_x, M_y, M_z	Moment about x, y, and z axes, respectively
M_{ce}, M_{df}	Moment due to distributed centrifugal forces
M_{AB}, M_{BC}, M_{CD}	Torque transmitted through lengths AB, BC, and CD, respectively, of shaft
M_{NC}	Nonconservative moment
M_C	Moment due to centrifugal forces acting on mass element C
$\mathbf{N}$	Normal force
N	Magnitude of $\mathbf{N}$
N	Newton (SI unit of force)
$N_1, N_2, \ldots$	Number of teeth on gears $1, 2, \ldots$
$\mathbf{N}_a, \mathbf{N}_b, \ldots$	Normal forces at points $a, b, \ldots$
$N_a, N_b, \ldots$	Magnitude of $\mathbf{N}_a, \mathbf{N}_b, \ldots$
$\mathbf{N}_A, \mathbf{N}_B, \ldots$	Normal forces acting on bodies $A, B, \ldots$
$N_A, N_B, \ldots$	Magnitudes of $\mathbf{N}_A, \mathbf{N}_B, \ldots$
$\mathbf{N}_{AB}$	Normal force transmitted between bodies A and B
N_{AB}	Magnitude of $\mathbf{N}_{AB}$
N_L, N_R	Left-side and right-side normal reaction forces, respectively
$\mathbf{N}_F, \mathbf{N}_R$	Normal reaction forces on front and rear automobile wheels, respectively
n, t	Normal and tangential axes
N_F, N_R	Magnitudes of $\mathbf{N}_F$ and $\mathbf{N}_R$, respectively
N'	Tangential impulse
O_a, O_b	Center of radii of curvatures ρ_a and ρ_b, respectively
$\mathbf{P}$	Force
P	Magnitude of $\mathbf{P}$, power, resultant centrifugal force
P_a	Actual power requirement
P_t	Theoretical power requirement
P_1, P_2	Resultant centrifugal force on mass elements 1 and 2, respectively
P_{AB}, P_{BC}, P_{CD}	Power transmitted through lengths AB, BC, and CD, respectively, of shaft
P_y, P_z	y and z components of resultant centrifugal force
q	Factor equal to ρt, where t is thickness of a thin, plane body; distance between center of percussion and axis of rotation
$\mathbf{r}$	Displacement, or position, vector
r	Magnitude of $\mathbf{r}$, radius of circular path of motion, radius of cylinder or sphere, radial coordinate of a mass element

r_1, r_2	Radii of circle or cylinder
$\mathbf{R}$	Reaction force, dynamic bearing force acting on shaft
R	Gyroscopic force, magnitude of $\mathbf{R}$, radius of circular arc
R_x, R_y	x and y components of $\mathbf{R}$
r_x, r_y, r_z	Coordinates of differential mass element
r_c	Distance between z and z_0 axes and between center of mass and axis of rotation
$\mathbf{r}_i$	Position vector from center of mass to mass particle m_i
r_i	Magnitude of $\mathbf{r}_i$, radius of path of mass particle m_i
$\ddot{\mathbf{r}}_i$	Relative acceleration of mass particle m_i with respect to center of mass of body
r_{ix}, r_{iy}	x and y components, respectively, of $\mathbf{r}_i$
r_0	Position of line of action of resultant centrifugal force
$\mathbf{R}_a, \mathbf{R}_b$	Dynamic bearing forces, normal bearing forces, reaction forces
R_a, R_b	Magnitude of $\mathbf{R}_a$ and $\mathbf{R}_b$, respectively
$R_{ay}, R_{az}, R_{by}, R_{bz}$	y and z components of dynamic bearing force at a and b, respectively
r_A, r_B, r_C	Radii of cylinders A, B, and C, respectively
$\mathbf{R}_C, \mathbf{R}_D$	Reaction forces on bodies C and D, respectively
R_C, R_D	Magnitude of $\mathbf{R}_C$ and $\mathbf{R}_D$, respectively
$R_{cx}, R_{cy}, R_{Dx}, R_{Dy}$	x and y components of $\mathbf{R}_C$ and $\mathbf{R}_D$, respectively
$\mathbf{s}$	Displacement
s	Magnitude of $\mathbf{s}$, rectilinear displacement, coordinate of length along curvilinear path
s	second (unit of time)
$\dot{s}, \ddot{s}$	First and second time derivatives of s, respectively
s_0	Initial displacement
$s_1, s_2, \ldots$	Displacement at times $t_1, t_2, \ldots$; displacement at points $1, 2, \ldots$
$\mathbf{s}_a, \mathbf{s}_b$	Displacement of points $a, b, \ldots$
$s_a, s_b, \ldots$	Magnitude of $\mathbf{s}_a, \mathbf{s}_b, \ldots$
$\mathbf{s}_A, \mathbf{s}_B, \ldots$	Displacement of bodies $A, B, \ldots$
$s_A, s_B, \ldots$	Magnitude of $\mathbf{s}_A, \mathbf{s}_B, \ldots$
$\mathbf{s}_{ab}, \mathbf{s}_{ba}$	Relative displacement of point a with respect to point b, and point b with respect to point a, respectively
s_{ab}, s_{ba}	Magnitudes of $\mathbf{s}_{ab}$ and $\mathbf{s}_{ba}$, respectively
t	Time, thickness of plane body
t_m	Time for particle acted on by drag force to reach maximum displacement
$t_1, t_2, \ldots$	Time, time intervals
$\mathbf{T}$	Cable tensile force
$\mathbf{T}_A, \mathbf{T}_B, \ldots$	Cable tensile force acting on bodies $A, B, \ldots$
$T_A, T_B, \ldots$	Magnitude of $\mathbf{T}_A, \mathbf{T}_B, \ldots$
$T_1, T_2, \ldots$	Cable tensile forces
$T_{AC}, T_{BD}, T_{BE}, T_{CD}, T_{DE}$	Cable tensile force between bodies A and C, B and D, B and E, C and D, and D and E, respectively
T	Kinetic energy of particle or body; magnitude of T
$T_1, T_2, \ldots$	Kinetic energy of particle or body at points $1, 2, \ldots$
$T_{A1}, T_{A2}, \ldots, T_{B1}, T_{B2}, \ldots$	Kinetic energy of bodies $A, B, \ldots$ at points $1, 2, \ldots$
$\mathbf{v}$	Velocity
v	Magnitude of $\mathbf{v}$, rectilinear velocity
v_x, v_y	x and y components of $\mathbf{v}$
$\dot{v}$	First time derivative of v
v_0	Initial velocity
v_{0x}, v_{0y}	x and y components of v_0
$v_{0a}, v_{0b}, \ldots$	Initial velocities of points $a, b, \ldots$
$v_{0A}, v_{0B}, \ldots$	Initial velocities of bodies $A, B, \ldots$
$v_1, v_2, \ldots$	Velocity at times $t_1, t_2, \ldots$; velocity at points $1, 2, \ldots$
$\mathbf{v}_a, \mathbf{v}_b, \ldots$	Velocity of points $a, b, \ldots$
$v_a, v_b, \ldots$	Magnitude of $\mathbf{v}_a, \mathbf{v}_b, \ldots$
$\mathbf{v}_A, \mathbf{v}_B, \ldots$	Velocity of bodies $A, B, \ldots$
$v_A, v_B, \ldots$	Magnitude of $\mathbf{v}_A, \mathbf{v}_B, \ldots$
v_{Ax}, v_{Ay}	x and y components of v_A

v_t	Tangential velocity in curvilinear translation
$\mathbf{v}_{ab}, \mathbf{v}_{ba}$	Relative velocity of point a with respect to point b, and point b with respect to point a, respectively
v_{ab}, v_{ba}	Magnitudes of $\mathbf{v}_{ab}$ and $\mathbf{v}_{ba}$, respectively
$v_{ba,x}, v_{ba,y}$	x and y components of v_{ba}
$\mathbf{v}_{AB}, \mathbf{v}_{BA}$	Relative velocity of body A with respect to body B, and body B with respect to body A, respectively
v_{AB}, v_{BA}	Magnitudes of $\mathbf{v}_{AB}$ and $\mathbf{v}_{BA}$, respectively
v_{avg}	Average velocity
V	Potential energy of particle or body, volume
$V_1, V_2, \ldots$	Potential energy at endpoints $1, 2, \ldots$; volume of elements $1, 2, \ldots$
$\mathbf{v}_A, \mathbf{v}_B$	Velocity of bodies A and B before impact
v_A, v_B	Magnitude of $\mathbf{v}_A$ and $\mathbf{v}_B$, respectively
$\mathbf{v}'_A, \mathbf{v}'_B$	Velocity of bodies A and B after impact
v'_A, v'_B	Magnitude of $\mathbf{v}'_A$ and $\mathbf{v}'_B$
$v_{Ax}, v_{Ay}, v_{Bx}, v_{By}$	x and y components of v_A and v_B
$v'_{Ax}, v'_{Ay}, v'_{Bx}, v'_{By}$	x and y components of v'_A and v'_B
$v_{Ax_1}, v_{Ay_1}, v_{Bx_1}, v_{By_1}$	x_1 and y_1 components of v_A, v_B
$v'_{Ax_1}, v'_{Ay_1}, v'_{Bx_1}, v'_{By_1}$	x_1 and y_1 components of v'_A, v'_B
v'_x, v'_y	x and y components of velocity of center of mass of body after impact
v_T	Terminal velocity
v^*_T	Constant defined by $\sqrt{\xi/\zeta}$
$\mathbf{W}$	Weight force
W	Magnitude of $\mathbf{W}$, work done
W	Watt
$\mathbf{W}_A, \mathbf{W}_B, \ldots$	Weight force of bodies $A, B, \ldots$
$W_A, W_B, \ldots$	Magnitude of $\mathbf{W}_A, \mathbf{W}_B, \ldots$
$W_1, W_2, \ldots$	Weight of elements $1, 2, \ldots$
W_{12}	Work done between points 1 and 2
W_{NC}	Nonconservative work done
x	Length
x, y, z	Cartesian coordinate axes
$\ddot{x}, \ddot{y}, \ddot{z}$	x, y, and z components of $\mathbf{a}$
x_m	Maximum displacement of particle acted on by drag force
x_0, y_0, z_0	Centroidal axes
x_0, y_0	Coordinates of differential mass element in x_0, y_0 coordinates
x_0	Position of line of action of resultant centrifugal force
x_{ab}, y_{ab}	x and y components of $\mathbf{s}_{ab}$
x_A, y_A, x_B, y_B	Coordinates of point on body in positions A and B, position coordinates of mass elements A and B, respectively
x_c, y_c	Coordinates of centroid of plane area
x_c, y_c, z_c	Coordinates of centroid of body, or volume
x_i, y_i, z_i	Centroidal coordinates of n mass, weight, or volume elements $1, 2, \ldots, i, \ldots, n$
x_1, y_1	Coordinate axes
x'_c, y'_c	Centroidal coordinates of plane body formed of thin rod shapes or of plane curve
$x_{\text{CP}}, y_{\text{CP}}$	Coordinates of center of percussion

GREEK SYMBOLS

$\boldsymbol{\alpha}$	Angular acceleration
α	Angle, magnitude of $\boldsymbol{\alpha}$
$\alpha_A, \alpha_B, \ldots$	Angular acceleration of bodies $A, B, \ldots$
α_{avg}	Average angular acceleration
β	Angle, direction of acceleration in plane curvilinear translation
β_A, β_D	Direction of dynamic balance holes in elements A and D, respectively
β_a, β_b	Direction of dynamic unbalance forces R_a and R_b, respectively
β_f	Angle
δ, δ_0	Spring deflection, initial spring deflection

$\Delta\theta$	Included angle, change in angular displacement
Δ	Change in a quantity
Δs	Arc length
ΔT	Change in kinetic energy T
ΔV	Change in potential energy V
$\Delta v_n, \Delta v_t$	Normal and tangential components, respectively, of change in velocity
γ	Specific weight
$\boldsymbol{\theta}$	Angular displacement
θ	Angle, magnitude of $\boldsymbol{\theta}$
θ_A, θ_B	Angular positions of line on body in positions A and B, respectively
$\theta_A, \theta_B, \ldots$	Angular displacement of bodies $A, B, \ldots$
θ_A, θ_D	Direction of centrifugal force acting on mass elements A and D, respectively
$\theta_1, \theta_2, \theta_3$	Endpoints of angular motion, angles, angular displacement of gears 1, 2, and 3, respectively
θ_0	Initial angular displacement
$\dot{\theta}, \ddot{\theta}$	Variable angular velocity and acceleration, respectively
ρ, ρ_a, ρ_b	Density, radius of curvature, radius of curvature of points a and b, respectively
ρ_0	Mass density per unit length of homogenous rod material of constant cross section
$\boldsymbol{\omega}$	Angular velocity
ω	Magnitude of $\boldsymbol{\omega}$
ω'	Angular velocity after impact
ω_{avg}	Average angular velocity
ω_{bc}	Angular velocity of line bc
ω_0	Initial angular velocity
ω_A, ω_B	Angular velocity of bodies A and B, respectively, before impact
ω_A', ω_B'	Angular velocity of bodies A and B, respectively, after impact
$\omega_1, \omega_2, \omega_3$	Angular velocity of gears 1, 2, and 3, respectively
$\boldsymbol{\omega}_1$	Angular velocity of axis of shaft, or of direction of vector
ω_1	Magnitude of $\boldsymbol{\omega}_1$
ω_C, ω_D	Angular velocity of bodies C and D, respectively
ζ	Angle, constant defined by $C_D\rho A/2m$
μ	Absolute viscosity, coefficient of friction
π	3.14159. . .
η	Energy efficiency, constant defined by c/m
ξ	Constant defined by p/m
ΔW	Energy dissipated by viscous forces

OTHER SYMBOLS

1, 2, . . .	Endpoints of an interval of motion, elements
%D	Percent difference

CONTENTS

CHAPTER 14
Kinematics of Particles

14.1 RECTILINEAR MOTION, DISPLACEMENT, VELOCITY, AND ACCELERATION

14.1 (*a*) What is the definition of the term *kinematics*?
(*b*) What is the difference between a *particle* and a *body*?

(*a*) Kinematics is the science which studies the motions of particles, or points, and bodies, *without concern for the forces which produce these motions*. A particle is a physical body which has mass and whose physical dimensions are assumed to be vanishingly small. The consequence of this assumption is that all rotation effects, of the body which is represented as a particle, about any axis through the body may be neglected. The particle may thus be thought of as a *point* in space.

(*b*) If a physical element is assumed to not be a particle, it is referred to as a *body*. All the bodies considered in this text are assumed to be rigid. In a rigid body, the distance between any two points is always the same, no matter what type of force system acts on the body. Thus a *rigid body* may be viewed as a body with unchanging dimensions. It is left as an exercise for the reader to show why the assumption of rigidity has no meaning when applied to a particle.

14.2 What are the only two possible motions of a particle?

Two general types of particle motion may be identified. The first type is referred to as *rectilinear translation*, and this type of motion is straight-line motion. In Fig. 14.2*a* a particle, shown by the dark circle, is constrained to move along the straight line *ab*. The motion of the particle in this case is described as rectilinear translation.

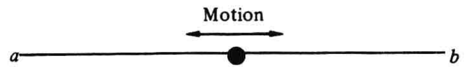

Fig. 14.2*a*

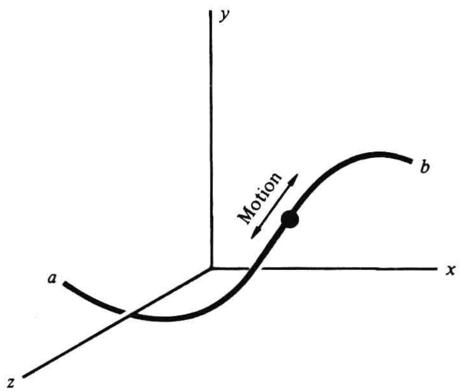

Fig. 14.2*b*

The second general type of particle motion is called *curvilinear translation*. In Fig. 14.2b, the particle is constrained to move along the curved path *ab*. For this case, the motion of the particle is described as curvilinear translation. It is emphasized that, in *both* of the above definitions, the term *translation* implies that no rotation, or angular motion, effects are required for the complete kinematic description of the motion of the particle. The reason for this, from the definition of a particle, is that all dimensions of the particle are effectively zero.

14.3 (*a*) What is the definition of the term *displacement*?
(*b*) Show that displacement is a vector quantity.

▮ (*a*) Figure 14.3 shows a particle which is constrained to move along the straight line *ab*. The *displacement*, or position, of the particle is the location of the particle with respect to a fixed reference point. For the situation shown in Fig. 14.3, the fixed reference point is 0. The coordinate *s* describes the displacement of the particle with respect to this point. The positive sense of the coordinate *s* is the sense of increasing *s*, or to the right in the figure. If a value of *s* is positive, the particle is located to the right of the origin. If this coordinate has a negative value, the particle is located to the left of the origin.

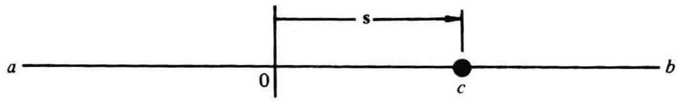

Fig. 14.3

(*b*) From the description in part (*a*), it follows that the displacement is a vector quantity. The *magnitude* of the displacement is the value of the *s* coordinate. The *direction* of the displacement is along the line *ab*. Finally, the *sense* of the displacement is determined by the sign of the value of *s*. There is a single, known direction in rectilinear translation problems.

14.4 (*a*) What is the definition of the term *velocity*?

(*b*) What is the relationship between the positive senses of velocity and displacement?

(*c*) What is meant by the *speed* of a particle?

▮ (*a*) The velocity v of a particle is defined to be the first time derivative of the displacement. The velocity may be thought of as the time rate of change of displacement.

In Fig. 14.4, the particle moves from c to c' during the time interval Δt. The average velocity v_{avg} is defined to be

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t}$$

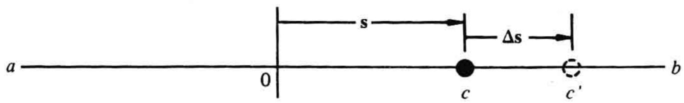

Fig. 14.4

The time interval Δt is now allowed to decrease without limit. The formal definition of the velocity v of the particle is then

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Throughout this text, a dot over a quantity will indicate the first time derivative of this quantity, and two dots will represent a second time derivative. Thus,

$$v = \frac{ds}{dt} = \dot{s}$$

The units of velocity are length divided by time. Typical U.S. Customary System (USCS) units for velocity are feet per second or inches per second. In SI units, the velocity is expressed in meters per second.

The direction of the velocity is the limiting direction of Δs, as $\Delta s \to 0$. Thus, the velocity is a vector quantity.

(*b*) The term Δs, by definition, using Fig. 14.4, is

$$\Delta s = s_{c'} - s_c$$

Since $s_{c'} > s_c$, Δs *is positive* in the same sense as s. Δt is always positive, since time may only increase. From consideration of the above equation, it follows that *the velocity is positive in the same sense as the displacement coordinate*. Thus, the choice of a positive sense for the displacement automatically establishes the positive sense of the velocity.

If the velocity of the particle in Fig. 14.4 is positive, then the particle moves to the right. If this velocity is negative, the particle moves to the left. These conclusions are independent of the *positions* of the particle along line *ab*.

(*c*) The term speed is a *scalar* quantity used to describe the *magnitude* of the velocity.

14.5 (*a*) What is the definition of the term *acceleration*?

(*b*) What is the relationship among the positive senses of acceleration, velocity, and displacement?

(*c*) What is meant by the term *deceleration*?

(*d*) Give the form for the average acceleration of a particle.

▌ (*a*) Figure 14.5 shows the two velocities of the particle at the locations c and c'. The acceleration a of the particle is defined to be the first time derivative of the velocity. The acceleration may thus be thought of as describing the time rate of change of velocity. The formal definition of the acceleration of the particle is

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

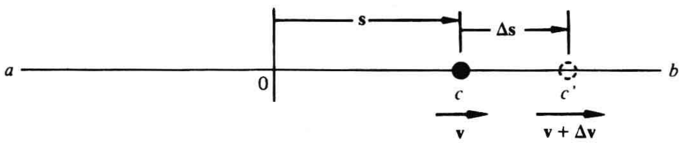

Fig. 14.5

The above results may be expanded further as

$$a = \frac{dv}{dt} = \frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2} = \ddot{s}$$

where two dots over a variable represent the second time derivative of the quantity. The fundamental units of acceleration, from the above equations, are velocity divided by time, or length divided by time squared. In the U.S. Customary System, typical acceleration units are feet per second squared or inches per second squared. The SI units for acceleration are meters per second squared.

(*b*) From comparison of the forms for a and v, it may be seen that the positive senses of a and v are the same. As was shown in Prob. 14.4, the positive senses of v and s are the same. Thus, *the positive sense of the acceleration is always the same as the positive sense of the coordinate s*. It follows from the discussion that the choice of a positive sense for the displacement coordinate automatically establishes the *same* positive senses for the velocity and the acceleration.

(*c*) The term deceleration is commonly used to describe an acceleration of negative sense.

(*d*) The average value a_{avg} of the acceleration in the region of motion between c and c' in Fig. 14.5 is

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

14.6 A particle moves with rectilinear translation. From a prior calculation, the displacement of the particle is known to be

$$s = (100 - 4t^2)$$

where t is in seconds and s is in meters.

(*a*) Find the general forms of the velocity and acceleration of the particle.

(*b*) Find the values of the displacement, velocity, and acceleration when $t = 0$. Show these results on the axis of motion.

(*c*) Do the same as in part (*b*) for the time when the particle passes through the origin.

(*d*) Find the value of the average velocity for the interval between $t = 0$ and the time found in part (*c*).

(*e*) Where is the particle when $t = 10$ s? What are the values of the velocity and acceleration at this time? Sketch these results on the axis of motion.

▮ (*a*) The general forms of the velocity and acceleration are

$$v = \dot{s} = \frac{ds}{dt} = \frac{d}{dt}(100 - 4t^2) = -8t \text{ m/s} \qquad a = \dot{v} = \ddot{s} = \frac{d}{dt}(\dot{s}) = \frac{d}{dt}(-8t) = -8 \text{ m/s}^2$$

(*b*) When $t = 0$,

$$s = 100 - 4(0) = 100 \text{ m} \qquad v = \dot{s} = -8(0) = 0 \qquad a = \dot{v} = \ddot{s} = -8 \text{ m/s}^2$$

The results are shown in Fig. 14.6*a*.

(*c*) When the particle passes through the origin, $s = 0$. Using the equation for s, v, and a,

$$0 = 100 - 4t^2 \qquad t = 5 \text{ s} \qquad v = -8t = -8(5) = -40 \text{ m/s} \qquad a = -8 \text{ m/s}^2$$

These results are shown in Fig. 14.6*b*.

(*d*) The average velocity between $t = 0$ and $t = 5$ s is

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{s|_{t=5} - s|_{t=0}}{5 - 0} = \frac{0 - 100}{5} = -20 \text{ m/s}$$

At the beginning of this interval the velocity is zero. At the end of the interval, when $t = 5$ s, the velocity has a magnitude of 40 m/s. It may be seen that the average value of the velocity is representative of *neither* of these two endpoints. This example illustrates that *caution must be exercised in using average values to describe a problem.* The average value of velocity will more closely approach the true value of the velocity as the time interval Δt decreases and approaches zero.

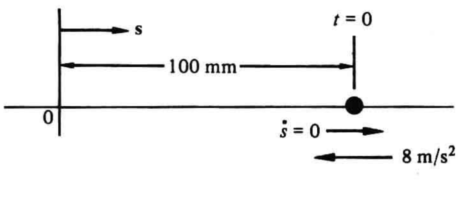

Fig. 14.6*a*

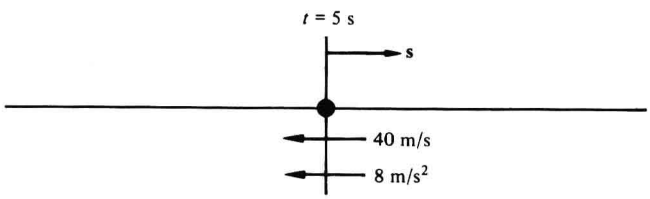

Fig. 14.6*b*

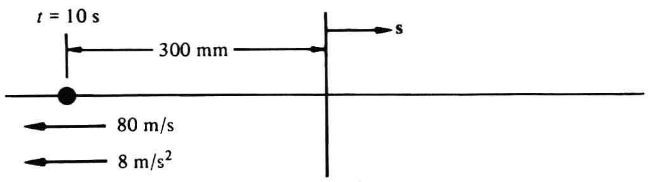

Fig. 14.6*c*

(*e*) When $t = 10$ s,

$$s = 100 - 4t^2 = 100 - 4(10)^2 = -300 \text{ m} \qquad v = -8(10) = -80 \text{ m/s} \qquad a = -8 \text{ m/s}^2$$

These values are shown in Fig. 14.6*c*.

14.7 A particle moves in rectilinear translation with the displacement-time function $s = 1 - e^{-0.5t}$, where s is in meters and t is in seconds.

(*a*) Find the general forms of the velocity and acceleration of the particle.

(*b*) Find the displacement, velocity, and acceleration when $t = 0$ and when $t = 4$ s. Show these values on the axis of motion.

(*c*) Find the average values of the velocity and acceleration in the time interval between $t = 0$ and $t = 4$ s.

▮ (*a*) The velocity and acceleration have the forms

$$s = 1 - e^{-0.5t} \text{ m} \qquad v = \dot{s} = -(-0.5)e^{-0.5t} = 0.5e^{-0.5t} \text{ m/s}$$
$$a = \dot{v} = \ddot{s} = -0.5(0.5)e^{-0.5t} = -0.25e^{-0.5t} \text{ m/s}^2$$

(*b*) At $t=0$,

$$s=0 \qquad v=0.5e^0=0.5\text{ m/s} \qquad a=-0.25e^0=-0.25\text{ m/s}^2$$

At $t=4$ s,

$$s=1-e^{-0.5(4)}=0.865\text{ m} \qquad v=0.5e^{-0.5(4)}=0.0677\text{ m/s} \qquad a=-0.25e^{-0.5(4)}=-0.0338\text{ m/s}^2$$

The above results are shown in Fig. 14.7.

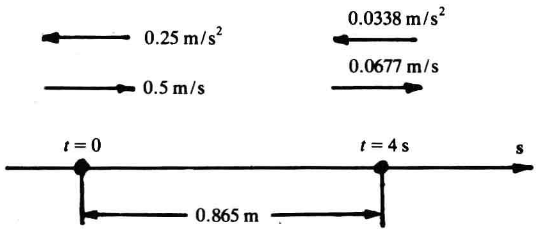

Fig. 14.7

(*c*) In the interval $0 \le t \le 4$ s,

$$v_{avg}=\frac{\Delta s}{\Delta t}=\frac{0.865-0}{4}=0.216\text{ m/s} \qquad a_{avg}=\frac{\Delta v}{\Delta t}=\frac{0.0677-0.5}{4}=-0.108\text{ m/s}^2$$

14.8 Solve Prob. 14.7, if $s=25\sin \pi t/2$, where s is in inches and t is in seconds.

▌ (*a*) The velocity and acceleration have the forms

$$s=25\sin\frac{\pi t}{2}\text{ in} \qquad v=\dot{s}=25\left(\frac{\pi}{2}\right)\cos\frac{\pi t}{2}=39.3\cos\frac{\pi t}{2}\text{ in/s}$$

$$a=\dot{v}=\ddot{s}=-25\left(\frac{\pi}{2}\right)\left(\frac{\pi}{2}\right)\sin\frac{\pi t}{2}=-61.7\sin\frac{\pi t}{2}\text{ in/s}^2$$

(*b*) At $t=0$,

$$s=25\sin 0=0 \qquad v=39.3\cos 0=39.3\text{ in/s} \qquad a=-61.7\sin 0=0$$

At $t=4$ s,

$$s=25\sin\frac{\pi(4)}{2}=0 \qquad v=39.3\cos\frac{\pi(4)}{2}=39.3\text{ in/s} \qquad a=-61.7\sin\frac{\pi(4)}{2}=0$$

The above results are shown in Fig. 14.8.

It may be seen that the motion is periodic, and $t=0$ s, 4 s are the endpoints of one cycle.

(*c*) In the interval $t=0$ to $t=4$ s,

$$v_{avg}=\frac{\Delta s}{\Delta t}=\frac{0-0}{4}=0 \qquad a_{avg}=\frac{\Delta v}{\Delta t}=\frac{39.3-39.3}{4}=0$$

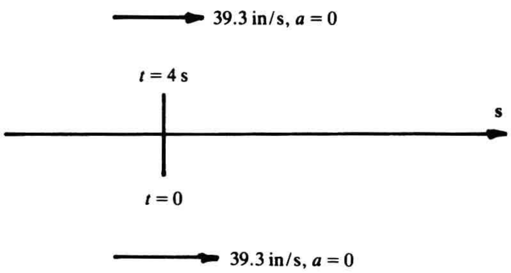

Fig. 14.8

14.9 The displacement of a particle which moves with rectilinear translation is known to have the functional form $s=at^2+bt+8$, where s is in feet and t is in seconds, and a and b are constants to be chosen.