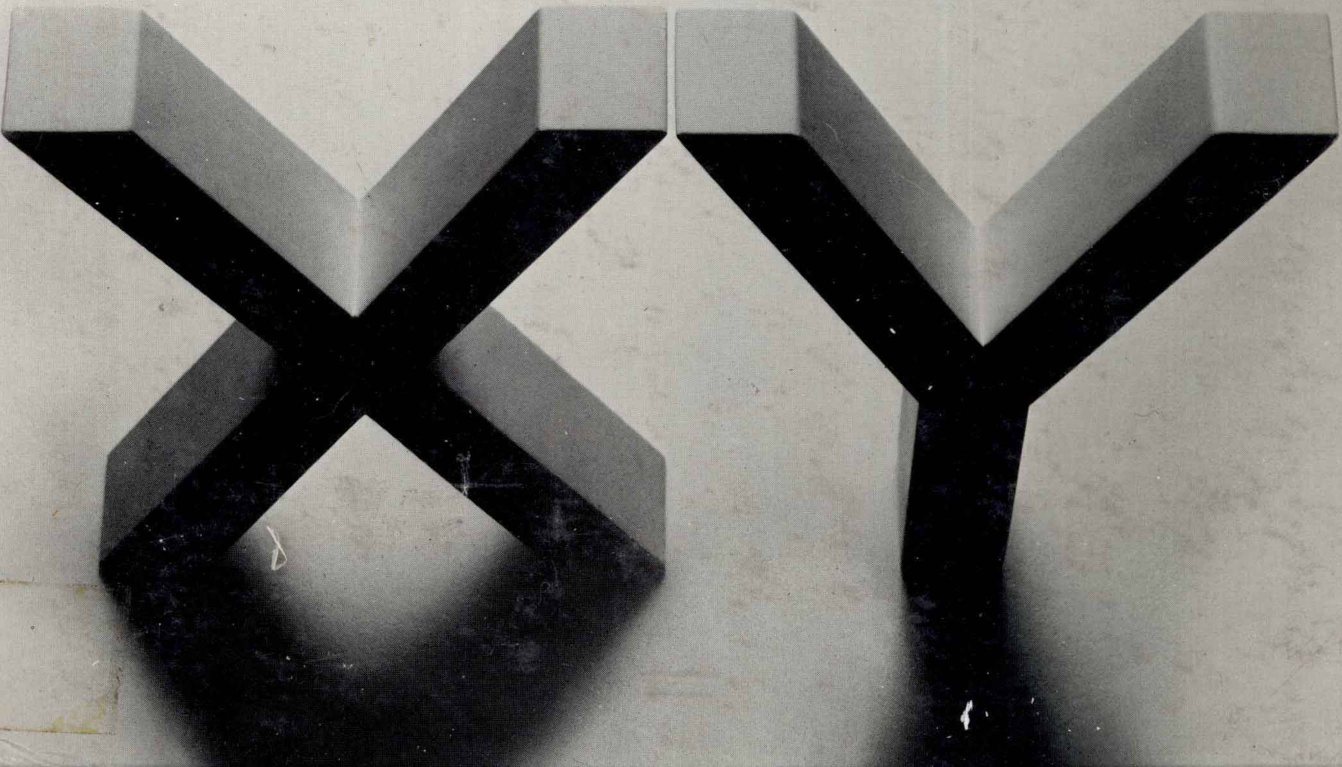
CHARLES P. McKEAGUE

ELEMENTARY ALGEBRA



Elementary Algebra

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CUESTA COLLEGE

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To The Instructor

This book grew out of my experiences as a teacher of mathematics at Cuesta Community College in San Luis Obispo. I enjoy teaching beginning algebra and feel that the students in this course are a unique and challenging group. Many are returning students—veterans who have spent time in the service, or women who are back in school after having raised a family. They are generally very capable people, but many of them lack a solid background in mathematics, while others may have some difficulty with motivation. I feel I have been quite successful in finding ways to teach and motivate these students, and have written this text with them in mind.

I realize that not all of the students in this course are science majors. Few, if any, are used to reading material in mathematics, and so this book is written in everyday language. The vocabulary necessary to communicate effectively in algebra is clearly defined and explained. The properties and definitions are highlighted so reference back to them is simple. When memorization is necessary I say so.

I have also kept you, the instructor, in mind while writing this text. The book is organized into sections, each of which can easily be presented in a 45 or 50 minute lecture or class period. The problem sets are organized so that the students get a good start on new material before they encounter the more difficult problems.

Organization of the Text: The book begins with a preface to the student explaining what study habits are necessary to ensure success in a beginning algebra course.

The rest of the book is divided into chapters. Each chapter is organized as follows:

1. A preface to the student explains in a very general way what he or she can expect to find in the chapter. The preface includes a list of previous material that is used to develop the concepts in the chapter.
2. The body of the chapter, which is divided into sections. Each section contains three main parts:
   a. *Explanations:* The explanations are made as simple and as intuitive as possible. The ideas, properties, and definitions from Chapter One are used continuously throughout the book. The idea is to make as few rules and definitions as possible and then refer back to them when new situations are encountered. This has been a very successful teaching technique for me and I feel confident about applying it to this text.
   b. *Examples:* The examples are chosen to clarify the explanations and preview the problem sets. They cover all situations encountered in the problem sets and can easily be referred to when trouble arises.
   c. *Problem Sets:* I have incorporated three main ideas into each problem set.
      i. *Drill:* There are enough problems in each set to insure student proficiency in the material.
      ii. *Progressive Difficulty:* The problems increase in difficulty as the problem set progresses.
      iii. *Odd-Even Similarities:* Each pair of consecutive problems is similar. The answers to the odd problems are listed in the back of the book. This gives the students a chance to check their work and then try a similar problem.
3. The chapter summary and review, which lists the new properties and definitions found in the chapter. (Often it also contains a list of common mistakes, clearly marked as such, so that the student can learn to recognize and avoid them.)
4. The chapter test, designed to give the student an idea of how well he or she has mastered the material in the chapter. The problems are representative of all problems in the chapter. All answers for these chapter tests are included in the back of the book to let the student evaluate his or her grasp of the material, and decide if and where further effort is needed.

An Instructor's Manual, available upon request, provides answers to the even numbered problems and all answers to the chapter tests. Also included are additional chapter tests, with answers.

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Preface

An' here I sit so patiently
    waiting to find out what price
You have to pay to get out of
    going through all these things twice.

—Bob Dylan, *Stuck Inside of Mobile with the Memphis Blues Again,*

Although I think Bob Dylan had something else in mind when he wrote these lyrics, he does ask the same question many first year algebra students ask. "Why can't I understand this stuff the first time?" The answer is, "You're not expected to." Learning algebra is not the same as reading a novel. Once through this book is not going to do it for you. You must proceed one section at a time, working problems and reading the material as many times as you need in order to master it. You can master it.

Here are answers to some more questions that are often asked by students in beginning algebra classes.

How much math do I need to know before taking algebra?

You should be able to do the four basic operations (add, subtract, multiply, and divide) with whole numbers, fractions, and decimals. Most important is your ability to work with whole numbers. If you can't add or multiply you should take a basic math class first. If you are a bit weak at working with fractions because you haven't worked with them in awhile don't be too concerned; it will come back to you. I have had students who eventually did very well in algebra, even though they were initially unsure of themselves when working with fractions.

What is the best way to study?

The best way to study is to study consistently. You must work problems every day. The more time you spend on your homework in the beginning, the easier the sections in the rest of the book will seem. In the first two chapters, the basic properties that are a basis for the rest of the book are developed. Do well on these and you will have set a successful pace for the rest of the book.

If I understand everything that goes on in class, can I take it easy on my homework?

Not necessarily. There is a big difference between understanding a problem someone else is working and working the same problem yourself. You are watching someone else think through a problem. There is no substitute for thinking it through yourself. The concepts and properties are not going to come to you through osmosis. You will ultimately have to convince yourself they work.

I'm worried about not understanding it. I've tried algebra before and just didn't get it.

There will probably be few times when you can understand completely everything that goes on in class. This is standard with most math classes. It doesn't mean you can't understand it. As you read through the book and try the problems on your own, you will understand more and more of the material. If you don't try the problems on your own you are almost guaranteed confusion. I haven't found a student yet who couldn't be successful at algebra. This isn't to say all my students are successful. They all could be. You can be too.

If you have made up your mind to be successful, here is a list of things you can do to attain that success.

1. Attend class.
2. Read the book and work problems every day.
3. Spend as much time as necessary to master the material. (You have to do whatever it takes to get it.)
4. Do it on your own. Don't be misled into thinking someone else's work is your own.
5. Don't expect to understand it the first time. It can only cost you your confidence.
6. Relax. It's not as difficult as you think.

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1 The Basics

Chapter 1 contains some of the most important material in the book. It also contains some of the easiest material to understand. Be sure that you master it. Your success in the following chapters depends upon how well you understand Chapter 1. Here is a list, in order of importance, of the ideas you must know after having completed Chapter 1.

1. You *must* know how to add, subtract, multiply, and divide positive and negative numbers. There is no substitute for consistently getting the correct answers.
2. You *must* understand and recognize when the commutative, associative, and distributive properties are being used. These properties are used continuously throughout the book to create other properties and definitions. They are the fundamental properties on which our algebraic system is built.
3. You should know the major classifications of numbers. That is, you should know the difference between whole numbers, integers, rational numbers, and real numbers.

If the material in Chapter 1 seems familiar to you, you may have a tendency to skip over some of it lightly. Don't do it. Look at the above list as you proceed through the chapter and make sure you understand these topics as they come up in the chapter. You will be off to a good start and increase your chance for success with the rest of the material in the book.

1.1 Notation and Symbols

Since much of what we do in algebra involves comparison of quantities, we will begin by listing some symbols used to compare mathematical quantities. The comparison symbols fall into two major groups, equality symbols and inequality symbols.

We will let the letters a and b stand for (represent) any two mathematical quantities.

Comparison Symbols

Equality:	$a = b$	a is equal to b (a and b represent the same number)
	$a \neq b$	a is not equal to b
Inequality:	$a < b$	a is less than b
	$a \not< b$	a is not less than b
	$a > b$	a is greater than b
	$a \not> b$	a is not greater than b

The symbols for inequality $<$ and $>$ always point to the smaller of the two quantities being compared. $3 < x$ means 3 is smaller than x. $5 > y$ means y is smaller than 5.

Along with symbols for comparison we have symbols for operations, which we've all seen before. We should be familiar with most of them.

Operation Symbols

Addition:	$a + b$	(The *sum* of a and b)
Subtraction:	$a - b$	(The *difference* of a and b)
Multiplication:	$a \cdot b$, $(a)(b)$, $a(b)$, $(a)b$, ab	(The *product* of a and b)
Division:	$a \div b$, a/b, $\frac{a}{b}$, $b\overline{)a}$	(The quotient of a and b)

When we encounter the word *sum* the implied operation is addition. To find the sum of two numbers we simply add them. *Difference* implies subtraction, *product* implies multiplication, and *quotient* implies division. Notice also that there is more than one way to write the product or quotient of two numbers.

Grouping Symbols

Parentheses (and) and brackets [and] are the symbols used for grouping numbers together. (Occasionally braces { and } are also used for grouping, although they are usually reserved for set notation as we shall see.)

The following examples illustrate the relationship of the symbols for comparing, operating, and grouping with the English language.

▼ Examples

Mathematical expression	*English equivalent*
1. $4 + 1 = 5$	The sum of 4 and 1 is 5.
2. $8 - 1 < 10$	The difference of 8 and 1 is less than 10.
3. $2(3 + 4) = 14$	Twice the sum of 3 and 4 is 14.
4. $3(7 - 5) \neq 10$	Three times the difference of 7 and 5 is *not* equal to 10. ▲

The symbols for comparing, operating, and grouping are to mathematics what punctuation symbols are to English. These symbols are the punctuation symbols for mathematics.

Consider the following sentence.

Paul said John is tall.

It can have two different meanings depending on how it is punctuated.

1. "Paul," said John, "is tall."
2. Paul said, "John is tall."

Without the punctuation we do not know which way the sentence is intended. It is ambiguous without punctuation.

Let's take a look at a similar situation in mathematics. Consider the following mathematical statement.

$$2 \cdot 7 + 5$$

If we add the 7 and 5 first and then multiply by 2, we get an answer of 24. On the other hand, if we multiply the 2 and the 7 first and then add 5, we are left with 19. We have a problem that seems to have two different answers, depending on whether we add first or multiply first. We would like to avoid this type of situation. That is, every problem like $2 \cdot 7 + 5$ should have only one answer. In order to do this, we have developed the following rule for the order of operation.

Order of Operation

When evaluating a mathematical expression we will perform the operations in the following order:

1. Do what is in the parentheses first, if you can. (In some cases it is not possible to do what is in the parentheses, as is the case when one of the quantities is a number and the other is a variable.)

2. Then perform all multiplications and divisions left to right. (In the same direction you read.)

3. Perform all additions and subtractions left to right.

Examples

5. $2 \cdot 7 + 5 = 14 + 5 = 19$ — Multiply $2 \cdot 7$ first. (Multiplication before addition or subtraction.)

6. $5 + 8 \cdot 2 = 5 + 16 = 21$ — Multiply $8 \cdot 2$ first. (Multiplication before addition or subtraction.)

7. $2 \cdot 7 + 3(6 + 4) = 2 \cdot 7 + 3 \cdot 10$ — Do what is in the parentheses first.
$= 14 + 30$ — Multiply left to right.
$= 44$ — Add.

Problem Set 1.1

Write an equivalent statement in English. Include the words sum, difference, product, and quotient when possible.

1. $7 + 8 = 15$
2. $6 + 3 = 9$
3. $7 < 10$
4. $12 < 15$
5. $8 + 2 \neq 6$
6. $10 - 5 \neq 15$
7. $21 > 20$
8. $32 > 22$
9. $x + 1 = 5$
10. $y + 2 = 10$
11. $x < y$
12. $r < s$
13. $x + 2 = y + 3$
14. $x - 5 = y + 7$
15. $x - 1 < 2x$
16. $x + 3 > 3x$
17. $2x + 1 = 10$
18. $3x - 2 = 5$
19. $4(x + 1) = 6$
20. $5(x - 3) = 2$

Mark the following statements true or false.

21. $16 < 17$
22. $18 < 15$
23. $10 = 19$
24. $11 \neq 21$
25. $3 + 2 < 5$
26. $5 + 1 > 6$
27. $11 \not< 10$
28. $9 \not< 8$
29. $3 \cdot 6 < 2 \cdot 4 + 1$
30. $4 \cdot 1 > 3 \cdot 2 - 4$

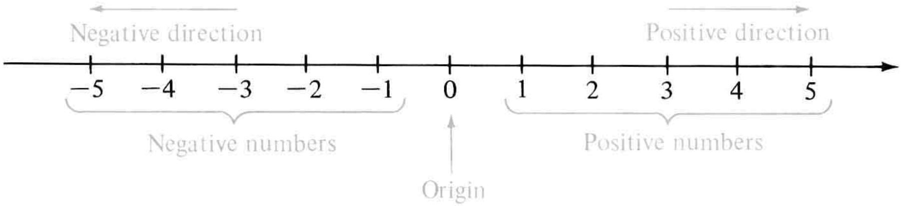
Use the rule for order of operation to simplify each problem as much as possible.

31. $2 \cdot 3 + 5$
32. $8 \cdot 7 + 1$
33. $5 + 2 \cdot 6$
34. $8 + 9 \cdot 4$
35. $3 \cdot 7 + 5 \cdot 2$
36. $8 \cdot 6 + 4 \cdot 3$

37. $7 \cdot 4 - 8 \cdot 1$
38. $6 \cdot 3 - 5 \cdot 2$
39. $19 - 2 \cdot 3 + 4$
40. $8 - 1(5) - 2$
41. $9 + (3 \cdot 2 + 5)$
42. $14 - (3 \cdot 5 - 2)$
43. $20 + 2(8 - 5) + 1$
44. $10 + 3(7 + 1) + 2$
45. $5 + 2(3 \cdot 4 - 1) + 8$
46. $11 - 2(5 \cdot 3 - 10) + 2$

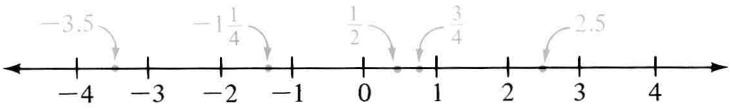
1.2 Real Numbers

In this section we will get the idea of what real numbers are. In order to do this we will draw what is called the *real number line*. We first draw a straight line and label a convenient point on the line with 0. Then we mark off equally spaced distances in both directions from 0. Label the points to the right of 0 with the numbers 1, 2, 3, . . . (the dots mean "and so on"). The points to the left of 0 we label in order $-1, -2, -3, \ldots$. Here is what it looks like.



The numbers increase in value going from left to right. If we "move" to the right, we are moving in the positive direction. If we move to the left, we are moving in the negative direction.

Drawing and labeling the number line in this way allows us to associate numbers with points on a line. For every number there is one point on the line, and for every point on the line there is a unique number. There are points associated with the numbers -3.5, $-1\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 2.5.



Definition The number associated with a point on the real number line is called the coordinate of that point.

In the above example the numbers $\frac{1}{2}$, $\frac{3}{4}$, 2.5, -3.5, and $-1\frac{1}{4}$ are the coordinates of the points they represent. The numbers that can be represented with points on the real number line are called real numbers. Real numbers include whole numbers, fractions, decimals, and other numbers that are not as familiar to us at this moment. We will look at