The *Art* of Doing SCIENCE and Engineering

Learning to Learn

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Gordon and Breach Science Publishers

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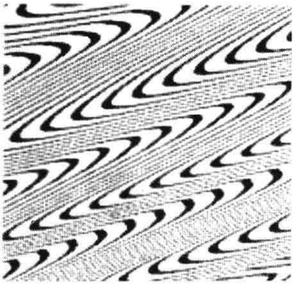

The Art of Doing Science and Engineering

PREFACE

After many years of pressure and encouragement from friends, I decided to write up the graduate course in engineering I teach at the U.S. Naval Postgraduate School in Monterey, California. At first I concentrated on all the details I thought should be tightened up, rather than leave the material as a series of somewhat disconnected lectures. In class the lectures often followed the interests of the students, and many of the later lectures were suggested topics in which they expressed an interest. Also, the lectures changed from year to year as various areas developed. Since engineering depends so heavily these days on the corresponding sciences, I often use the terms interchangeably.

After more thought I decided that since I was trying to teach "style" of thinking in science and engineering, and "style" is an art, I should therefore copy the methods of teaching used for the other arts—once the fundamentals have been learned. How to be a great painter cannot be taught in words; one learns by trying many different approaches that seem to surround the subject. Art teachers usually let the advanced student paint, and then make suggestions on how they would have done it, or what might also be tried, more or less as the points arise in the student's head—which is where the learning is supposed to occur! In this series of lectures I try to communicate to students what cannot be said in words—the essence of style in science and engineering. I have adopted a loose organization with some repetition since this often occurs in the lectures. There are, therefore, digressions and stories—with some told in two different places—all in the somewhat rambling, informal style typical of lectures.

I have used the "story" approach, often emphasizing the initial part of the discovery, because I firmly believe in Pasteur's remark, "Luck favors the prepared mind." In this way I can illustrate how the individual's preparation before encountering the problem can often lead to recognition, formulation, and solution. Great results in science and engineering are "bunched" in the same person too often for success to be a matter of random luck.

Teachers should prepare the student for the student's future, not for the teacher's past. Most teachers rarely discuss the important topic of the future of their field, and when this is pointed out they usually reply: "No one can know the future." It seems to me the difficulty of knowing the future does not absolve the teacher from seriously trying to help the student to be ready for it when it comes. It is obvious the experience of an individual is not necessarily that of a class of individuals; therefore, any one person's projection into the future is apt to be somewhat personal and will not be universally accepted. This does not justify reverting to impersonal surveys and losing the impact of the personal story.

Since my classes are almost all carefully selected navy, marine, army, air force, and coast guard students with very few civilians, and, interestingly enough, about 15% very highly selected foreign military, the students face a highly technical future—hence the importance of preparing them for *their* future and not just *our* past.

The year 2020 seems a convenient date to center the preparation for their future—a sort of 20/20 foresight, as it were. As graduate students working toward a master's degree, they have the basics well in hand. That leaves me the task of adding "style" to their education, which in practice is usually the difference between an average person and a great one. The school has allowed me great latitude in trying to teach a completely non-technical course; this course "complements" the more technical ones. As a result, my opening words, occasionally repeated, are: "There is really no technical content in the course, though I will, of course, refer to a great deal of it, and hopefully it will generally be a good review of the fundamentals of what you have learned. Do not think it is the *content* of the course—it is only illustrative material. *Style of thinking* is the center of the course."

The subtitle of this book, *Learning to Learn,* is the main solution I offer to help students cope with the rapid changes they will have to endure in their fields. The course centers around how to look at and think about knowledge, and it supplies some historical perspectives that might be useful.

This course is mainly personal experiences I have had and digested, at least to some extent. Naturally one tends to remember one's successes and forget lesser events, but I recount a number of my spectacular failures as clear examples of what to avoid. I have found that the *personal* story is far, far more effective than the *impersonal* one; hence there is necessarily an aura of "bragging" in the book that is unavoidable.

Let me repeat what I earlier indicated. Apparently an "art"—which almost by definition cannot be put into words—is probably best communicated by approaching it from many sides and doing so repeatedly, hoping thereby students will finally master enough of the art, or if you wish, style, to significantly increase their future contributions to society. A totally different description of the course is: it covers all kinds of things that could not find their proper place in the standard curriculum.

The casual reader should not be put off by the mathematics; it is only "window dressing" used to illustrate and connect up with earlier learned material. Usually the underlying ideas can be grasped from the words alone.

It is customary to thank various people and institutions for help in producing a book. Thanks obviously go to AT&T Bell Laboratories, Murray Hill, New Jersey, and to the U.S. Naval Postgraduate School, especially the Department of Electrical and Computer Engineering, for making this book possible.

INTRODUCTION

This book is concerned more with the future and less with the past of science and engineering. Of course future predictions are uncertain and usually based on the past; but the past is also much more uncertain—or even falsely reported—than is usually recognized. Thus we are forced to *imagine* what the future will probably be. This course has been called "Hamming on Hamming" since it draws heavily on my own past experiences, observations, and wide reading.

There is a great deal of mathematics in the early part because almost surely the future of science and engineering will be more mathematical than the past, and also I need to establish the nature of the foundations of our beliefs and their uncertainties. Only then can I show the weaknesses of our current beliefs and indicate future directions to be considered.

If you find the mathematics difficult, skip those early parts. Later sections will be understandable provided you are willing to forgo the deep insights mathematics gives into the weaknesses of our current beliefs. General results are always stated in words, so the content will still be there but in a slightly diluted form.

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Orientation

The purpose of this course is to prepare you for your technical future. There is really no technical content in the course, though I will, of course, refer to a great deal of it, and hopefully it will generally be a good review of the fundamentals you have learned. Do not think the technical content is the course – it is only illustrative material. Style of thinking is the center of the course. I am concerned with educating and not training you.

I will examine, criticize, and display styles of thinking. To illustrate the points of style I will often use technical knowledge most of you know, but, again, it will be, I hope, in the form of a useful review which concentrates on the fundamentals. You should regard this as a course which complements the many technical courses you have learned. Many of the things I will talk about are things which I believe you ought to know but which simply do not fit into courses in the standard curriculum. The course exists because the department of Electrical and Computer Engineering of the Naval Postgraduate School recognizes the need for both a general education and the specialized technical training your future demands.

The course is concerned with "style", and almost by definition style cannot be taught in the normal manner by using words. I can only approach the topic through particular examples, which I hope are well within your grasp, though the examples come mainly from my 30 years in the mathematics department of the Research Division of Bell Telephone Laboratories (before it was broken up). It also comes from years of study of the work of others.

The belief anything can be "talked about" in words was certainly held by the early Greek philosophers, Socrates (469–399), Plato (427–347), and Aristotle (384–322). This attitude ignored the current *mystery cults* of the time who asserted you had to "experience" some things which could not be communicated in words. Examples might be the gods, truth, justice, the arts, beauty, and love. Your scientific training has emphasized the role of words, along with a strong belief in *reductionism,* hence to emphasize the possible limitations of language I shall take up the topic in several places in the book. I have already said "style" is such a topic.

I have found to be effective in this course, I must use mainly first hand knowledge, which implies I break a standard taboo and talk about myself in the first person, instead of the traditional impersonal way of science. You must forgive me in this matter, as there seems to be no other approach which will be as effective. If I do not use direct experience then the material will probably sound to you like merely pious words and have little impact on your minds, and it is your minds I must change if I am to be effective.

This talking about first person experiences will give a flavor of "bragging", though I include a number of my serious errors to partially balance things. Vicarious learning from the experiences of others saves making errors yourself, but I regard the study of successes as being basically more important than the study of failures. As I will several times say, there are so many ways of being wrong and so few of being right, studying successes is more efficient, and furthermore when your turn comes you will know how to succeed rather than how to fail!

I am, as it were, only a coach. I cannot run the mile for you; at best I can discuss styles and criticize yours. You know you must run the mile if the athletics course is to be of benefit to you – hence *you* must think carefully about what you hear or read in this book if it is to be effective in changing you – which must obviously be the purpose of any course. Again, you will get out of this course only as much as you put in, and if you put in little effort beyond sitting in the class or reading the book, then it is simply a waste of your time. *You* must also mull things over, compare what I say with your own experiences, talk with others, and make some of the points part of your way of doing things.

Since the subject matter is "style", I will use the comparison with teaching painting. Having learned the fundamentals of painting, you then study under a master you accept as being a great painter; but you know you must forge your own style out of the elements of various earlier painters plus your native abilities. You must also adapt your style to fit the future, since merely copying the past will not be enough if you aspire to future greatness – a matter I assume, and will talk about often in the book. I will show you my style as best I can, but, again, you must take those elements of it which seem to fit you, and you must finally create your own style. Either you will be a leader, or a follower, and my goal is for you to be a leader. You cannot adopt every trait I discuss in what I have observed in myself and others; you must select and adapt, and make them your own if the course is to be effective.

Even more difficult than what to select is that what is a successful style in one age may not be appropriate to the next age! My predecessors at Bell Telephone Laboratories used one style; four of us who came in all at about the same time, and had about the same chronological age, found our own styles and as a result we rather completely transformed the overall style of the Mathematics Department, as well as many parts of the whole Laboratories. We privately called ourselves "The four young Turks", and many years later I found top management had called us the same!

I return to the topic of education. You all recognize there is a significant difference between *education* and *training*.

> Education is what, when, and why to do things,
> Training is how to do it.

Either one without the other is not of much use. You need to know both what to do and how to do it. I have already compared mental and physical training and said to a great extent in both you get out of it what you put into it – all the coach can do is suggest styles and criticize a bit now and then. Because of the usual size of these classes, or because you are reading the book, there can be little direct criticism of your thinking by me, and you simply have to do it internally and between yourselves in conversations, and apply the things I say to your own experiences. You might think education should precede training, but the kind of educating I am trying to do must be based on your past experiences and technical knowledge. Hence this inversion of what might seem to be reasonable. In a real sense I am engaged in "meta-education", the topic of the course is education itself and hence our discussions must rise above it – "meta-education", just as metaphysics was supposed to be above physics in Aristotle's time (actually "follow", "transcend" is the translation of "meta").

This book is aimed at your future, and we must examine what is likely to be the state of technology (Science and Engineering) at the time of your greatest contributions. It is well known that since about Isaac Newton's time (1642–1727) knowledge of the type we are concerned with has about doubled every 17 years. First, this may be measured by the books published (a classic observation is libraries must double their holdings every 17 years if they are to maintain their relative position). Second, when I went to Bell Telephone Laboratories in 1946 they were trying to decrease the size of the staff from WW-II size down to about 5500. Yet during the 30 years I was there I observed a fairly steady doubling of the number of employees every 17 years, regardless of the administration having hiring freezes now and then, and such things. Third, the growth of the number of scientists generally has similarly been exponential, and it is said currently almost 90% of the scientists who ever lived are now alive! It is hard to believe in your future there will be a dramatic decrease in these expected rates of growth, hence you face, even more than I did, the constant need to learn new things.

Here I make a digression to illustrate what is often called "back of the envelop calculations". I have frequently observed great scientists and engineers do this much more often than "the run of the mill" people, hence it requires illustration. I will take the above two statements, knowledge doubles every 17 years, and 90% of the scientists who ever lived are now alive, and ask to what extent they are compatible. The model of the growth of knowledge and the growth of scientists assumed are both exponential, with the growth of knowledge being proportional to the number of scientists alive. We begin by assuming the number scientists at any time t is

$$y(t) = a\exp\{bt\}$$

and the amount of knowledge produced annually has a constant k of proportionality to the number of scientists alive. Assuming we begin at minus infinity in time (the error is small and you can adjust it to Newton's time if you wish), we have the formula

$$\frac{1}{2}=\frac{\int_{-\infty}^{t-17}kae^{bt}\,dt}{\int_{-\infty}^{t}kae^{bt}\,dt}$$

$$=\frac{(ka/b)e^{b(t-17)}}{(ka/b)e^{bt}}=e^{-17b}=\frac{1}{2}$$

hence we know b. Now to the other statement. If we allow the lifetime of a scientist to be 55 years (it seems likely that the statement meant living and not practicing, but excluding childhood) then we have

$$\frac{\int_{t-55}^{t}ae^{bt}\,dt}{\int_{-\infty}^{t}ae^{bt}\,dt}=\frac{e^{bt}-e^{\{bt-b(55)\}}}{e^{bt}}=1-e^{-55b}$$

$$=1-(\tfrac{1}{2})^{55/17}=1-0.106\ldots=0.894\ldots$$

which is very close to 90%.

Typically the first back of the envelop calculations use, as we did, definite numbers where one has a feel for things, and then we repeat the calculations with parameters so you can adjust things to fit the data better and understand the general case. Let the doubling period be D, and the lifetime of a scientist be L. The first equation now becomes

$$\tfrac{1}{2}=e^{bD},$$

and the second becomes:

$$\tfrac{9}{10}=1-e^{bL}=1-(\tfrac{1}{2})^{L/D},$$

$$(\tfrac{1}{2})^{L/D}=\tfrac{1}{10},$$

$$\frac{L}{D}=\frac{\log 10}{\log 2}=\frac{1}{0.30103}=3.3219\ldots$$

With $D=17$ years we have $17\times 3.3219=56.47\ldots$ years for the lifetime of a scientist, which is close to the 55 we assumed. We can play with ratio of L/D until we find a slightly closer fit to the data (which was approximate, though I believe more in the 17 years for doubling than I do in the 90%). Back of the envelop computing indicates the two remarks are reasonably compatible. Notice the relationship applies for all time so long as the assumed simple relationships hold.

The reason back of the envelop calculations are widely used by great scientists is clearly revealed – you get a good feeling for the truth or falsity of what was claimed, as well as realize which factors you were inclined not to think about, such as exactly what was meant by the lifetime of a scientist. Having done the calculation you are much more likely to retain the results in your mind. Furthermore, such calculations keep the ability to model situations fresh and ready for more important applications as they arise. Thus I recommend when you hear quantitative remarks such as the above you turn to a quick modeling to see if you believe what is being said, especially when given in the public media like the press and TV. Very often you find what is being said is nonsense, either no definite statement is made which you can model, or if you can set up the model then the results of the model do not agree with what was said. I found it very valuable at the physics table I used to eat with; I sometimes cleared up misconceptions at the time they were being formed, thus advancing matters significantly.

Added to the problem of the growth of new knowledge is the obsolescence of old knowledge. It is claimed by many the half-life of the technical knowledge you just learned in school is about 15 years – in 15 years half of it will be obsolete (either we have gone in other directions or have replaced it with new material). For example, having taught myself a bit about vacuum tubes (because at Bell Telephone Laboratories they were at that time obviously important) I soon found myself helping, in the form of computing, the development of transistors – which obsoleted my just learned knowledge!

To bring the meaning of this doubling down to your own life, suppose you have a child when you are x years old. That child will face, when it is in college, about y times the amount you faced.

y factor of increase	x years
2	17
3	27
4	34
5	39
6	44
7	48
8	51

This doubling is not just in theorems of mathematics and technical results, but in musical recordings of Beethoven's Ninth, of where to go skiing, of TV programs to watch or not to watch. If you were at times awed by the mass of knowledge you faced when you went to college, or even now, think of your children's troubles when they are there! The technical knowledge involved in your life will quadruple in 34 years, and many of you will then be near the high point of your career. Pick your estimated years to retirement and then look in the left-hand column for the probable factor of increase over the present current knowledge when you finally quit!

What is my answer to this dilemma? One answer is you must concentrate on fundamentals, at least what *you think* at the time are fundamentals, and also develop the ability to learn new fields of knowledge when they arise so you will not be left behind, as so many good engineers are in the long run. In the position I found myself in at the Laboratories, where I was the only one locally who seemed (at least to me) to have a firm grasp on computing, I was forced to learn numerical analysis, computers, pretty much all of the physical sciences at least enough to cope with the many different computing problems which arose and whose solution could benefit the Labs, as well as a lot of the social and some the biological sciences. Thus I am a veteran of learning enough to get along without at the same time devoting all my effort to learning new topics and thereby not contributing my share to the total effort of the organization. The early days of learning had to be done while I was developing and running a computing center. You will face similar problems in your career as it progresses, and, at times, face problems which seem to overwhelm you.

How are you to recognize "fundamentals"? One test is they have lasted a long time. Another test is from the fundamentals all the rest of the field can be derived by using the standard methods in the field.

I need to discuss science vs. engineering. Put glibly:

> In science if you know what you are doing you should not be doing it.
>
> In engineering if you do not know what you are doing you should not be doing it.

Of course, you seldom, if ever, see either pure state. All of engineering involves some creativity to cover the parts not known, and almost all of science includes some practical engineering to translate