A Survey of MATHEMATICS with APPLICATIONS

FIFTH EDITION

CUSTOM VERSION

ANGEL & PORTER

Addison Wesley Longman
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A Survey of Mathematics with Applications

FIFTH EDITION

Custom Version

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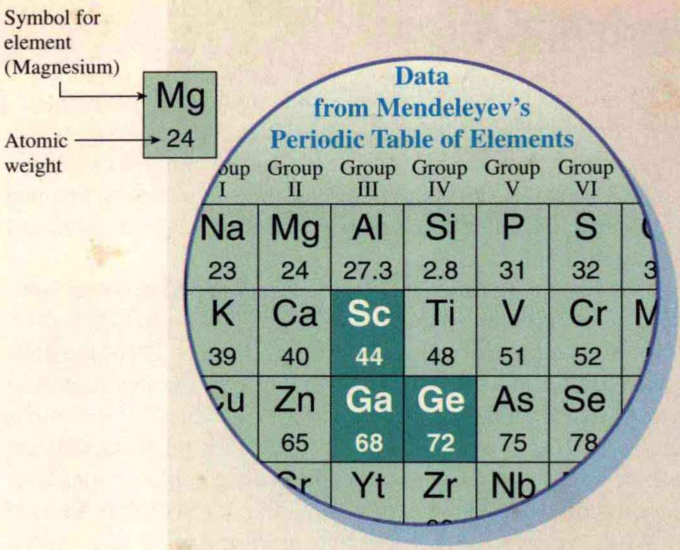


Seeing patterns often helps in solving problems. In 1869, chemists had isolated 63 of 109 of the chemical elements known today, but had not found any apparent underlying order. In that year, Russian chemist Dmitri Mendeleyev saw a pattern. He proposed a table of the elements organized by increasing atomic weight and grouped according to similar properties. To make this method work, he had to predict the existence of three then-unknown elements: scandium, gallium, and germanium.

shampoo, you will find that the 8-ounce bottle costs the least per ounce.

Sometimes solving a problem may require you to make a reasonable estimate, to look for clues, or to experiment with several possible solutions before choosing the best solution. Often the most important part of solving a problem is just understanding what question must be answered. ■

One way of determining the answer to the question "how many" is to estimate, using a small sampling of a larger group. This technique can be used to guess how many jelly beans are in a jar, or how many people are in attendance at a political rally.

1.1 INDUCTIVE REASONING

The goal of this chapter is to help you improve your reasoning and problem-solving skills. This section introduces inductive and deductive reasoning, which are used in problem solving. The next section introduces the concept of estimation. Estimation is a technique that can be used to determine if an answer obtained for a problem or from a calculation is "reasonable." Section 1.3 introduces and applies problem-solving techniques.

Before looking at some examples of inductive reasoning and problem solving, let us first review a few facts about certain numbers. The **natural numbers** or **counting numbers** are the numbers 1, 2, 3, 4, 5, 6, 7, 8, ... The three dots, called an **ellipsis**, mean that 8 is not the last number but that the numbers continue in the same manner. A word that we sometimes use is "divisible." If $a \div b$ has a remainder of zero, then *a is divisible by b*. The counting numbers that are divisible by 2 are 2, 4, 6, 8, ... These are called the *even counting numbers*. The numbers that are not divisible by 2 are 1, 3, 5, 7, 9, ... These are the *odd counting numbers*. When we refer to *odd numbers* or *even numbers*, we mean odd or even counting numbers.

Recognizing patterns is sometimes helpful in solving problems, as Examples 1 and 2 illustrate.

EXAMPLE 1

If two odd counting numbers are multiplied together, will the product always be an odd counting number?

Solution: To answer this question, we will examine the products of several pairs of odd numbers to see if there is a pattern.

$$1 \times 3 = 3 \qquad 3 \times 5 = 15 \qquad 5 \times 7 = 35$$
$$1 \times 5 = 5 \qquad 3 \times 7 = 21 \qquad 5 \times 9 = 45$$
$$1 \times 7 = 7 \qquad 3 \times 9 = 27 \qquad 5 \times 11 = 55$$
$$1 \times 9 = 9 \qquad 3 \times 11 = 33 \qquad 5 \times 13 = 65$$

None of the products is divisible by 2. Thus we might predict from these examples that the product of any two odd numbers is an odd number. ▲

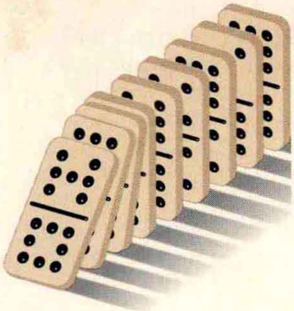
EXAMPLE 2

If an odd and an even counting number are added, will the sum be an odd or an even counting number?

Solution: Let's look at a few examples where one number is odd and the other number is even.

$$1 + 2 = 3 \qquad 9 + 6 = 15 \qquad 23 + 18 = 41$$
$$3 + 12 = 15 \qquad 5 + 14 = 19 \qquad 81 + 32 = 113$$

None of these sums is divisible by 2. Therefore we might predict that the sum of an odd and an even number is an odd number. ▲



In Examples 1 and 2 we cannot conclude that the results are true for all counting numbers. However, from the patterns developed, we can make predictions. This type of reasoning process, arriving at a general conclusion from specific observations or examples, is called **inductive reasoning**, or **induction**.

> **Inductive reasoning** is the process of reasoning to a general conclusion through observations of specific cases.

Induction often involves observing a pattern and from that pattern predicting a conclusion. Imagine an endless row of dominoes. You knock down the first, which knocks down the second, which knocks down the third, and so on. Assuming the pattern will continue uninterrupted, you conclude that eventually all the dominoes will fall, even though you may not witness the event.

Inductive reasoning is often used by mathematicians and scientists to predict answers to complicated problems. For this reason, inductive reasoning is part of the **scientific method**. When a scientist or mathematician makes a prediction based on specific observations, it is called a **hypothesis** or **conjecture**. After looking at the products in Example 1 we might conjecture that the product of two odd counting numbers will be an odd counting number.

Examples 3 and 4 illustrate how we arrive at a conclusion using inductive reasoning.

EXAMPLE 3

What reasoning process has led to the conclusion that no two people have the same fingerprints? This conclusion has resulted in fingerprints being used in courts of law as evidence to convict persons of crimes.

Solution: In millions of tests, no two people have been found to have the same fingerprints. By induction, then, we believe that fingerprints provide a unique identification and can therefore be used in a court of law as evidence. Is it possible that sometime in the future, two people will be found who do have exactly the same fingerprints? ▲

EXAMPLE 4

Consider the conjecture "If the sum of the digits of a number is divisible by 3, then the number is divisible by 3." Test several numbers to determine whether the conjecture appears true or false.

Solution: Let's look at some numbers, the sum of whose digits are divisible by 3.

DID YOU KNOW

A LEGAL EAGLE

In 1894 Pudd'nhead Wilson, a fictional character created by Mark Twain, became the first American to use fingerprints as evidence in a court of law. When twin clients of his were charged with the murder of a judge, he successfully defended them by showing that each twin's prints were not only different from the other's but were also quite different from those found on the murder weapon. It wasn't until 1902 that fingerprints were actually used before a jury (in England) to convict one Harry Jackson of breaking into a house and stealing billiard balls. Jackson had indeed taken the loot but left a thumbprint behind. In the United States today, the FBI has more than 180 million fingerprints on file. ■

Number	Sum of the Digits	Sum of the Digits Divided by 3	Number Divided by 3
114	$1 + 1 + 4 = 6$	$6 \div 3 = 2$	$114 \div 3 = 38$
234	$2 + 3 + 4 = 9$	$9 \div 3 = 3$	$234 \div 3 = 78$
7020	$7 + 0 + 2 + 0 = 9$	$9 \div 3 = 3$	$7020 \div 3 = 2340$
2943	$2 + 9 + 4 + 3 = 18$	$18 \div 3 = 6$	$2943 \div 3 = 981$
9873	$9 + 8 + 7 + 3 = 27$	$27 \div 3 = 9$	$9873 \div 3 = 3291$

In each of the examples we find that the sum of the digits is divisible by 3 and the number itself is divisible by 3. From these specific examples we might be tempted to generalize that the conjecture "If the sum of the digits of a number is divisible by 3, then the number is divisible by 3" is true. ▲

EXAMPLE 5

Pick any number, multiply the number by 4, add 6 to the product, divide the sum by 2, and subtract 3 from the quotient. Repeat this procedure for several different numbers and then make a conjecture about the relationship between the original number and the final number.

Solution: Let's go through this one together.

Pick a number:	say, 5
Multiply the number by 4:	$4 \times 5 = 20$
Add 6 to the product:	$20 + 6 = 26$
Divide the sum by 2:	$26 \div 2 = 13$
Subtract 3 from the quotient:	$13 - 3 = 10$

Note that we started with the number 5 and finished with the number 10. If you start with the number 2, you will end with the number 4. Starting with 3 would result in a final number of 6, 4 would result in 8, and so on. On the basis of these few examples many of you would conjecture that when you follow the given procedure, the number you end with will always be twice the original number. ▲

The result reached by inductive reasoning is often correct for the specific cases studied but not correct for all cases. History has shown that not all conclusions arrived at by inductive reasoning are correct. For example, Aristotle (384–322 B.C.) reasoned inductively that heavy objects fall at a faster rate than light objects. About 2000 years later, Galileo dropped two pieces of metal—one 10 times heavier than the other—from the Leaning Tower of Pisa in Italy. He found that both hit the ground at exactly the same moment, so they must have traveled at the same rate.

When forming a general conclusion using inductive reasoning, you should test it with several special cases to see whether the conclusion appears correct. If a special case is found that satisfies the conditions of the conjecture but produces a different result, such a case is called a **counterexample**. This case proves that the conjecture is false because only one exception is needed to show that a con-

DID YOU KNOW

AN EXPERIMENT REVISITED

Apollo 15 Astronaut David Scott used the moon as his laboratory to show that a heavy object (a hammer) does indeed fall at the same rate as a light object (a feather). Had Galileo dropped a hammer and feather from the tower of Pisa, the hammer would have fallen more quickly to the ground, and he still would have concluded that a heavy object falls faster than a lighter one. If it is not the object's mass that is affecting the outcome, then what is it? The answer is air resistance or friction: The earth has an atmosphere; the moon does not. ■

clusion is not valid. Galileo's counterexample disproved Aristotle's conjecture. If a counterexample cannot be found, the conjecture is neither proven nor disproven.

A second type of reasoning process is called **deductive reasoning**, or **deduction**. Mathematicians use deductive reasoning to *prove* conjectures true or false.

> **Deductive reasoning** is the process of reasoning to a specific conclusion from a general statement.

EXAMPLE 6

Prove, using deductive reasoning, that the procedure in Example 5 will always result in twice the original number selected.

Solution: To use deductive reasoning we begin with the *general* case rather than specific examples. In Example 5, specific cases were used. Let's select the letter n to represent *any number*.

Pick any number:	n
Multiply the number by 4:	$4n$ (4n means 4 times n)
Add 6 to the product:	$4n + 6$
Divide the sum by 2:	$\frac{4n+6}{2} = \frac{\overset{2}{\cancel{4}}n}{\underset{1}{\cancel{2}}} + \frac{\overset{3}{\cancel{6}}}{\underset{1}{\cancel{2}}} = 2n + 3$
Subtract 3 from the quotient:	$2n + 3 - 3 = 2n$

Note that, for any number n selected, the result is $2n$, or twice the original number selected. ▲

In Example 5 you may have conjectured, using specific examples and inductive reasoning, that the result would be twice the original number selected. In Example 6 we proved, using deductive reasoning, that the result will always be twice the original number selected.

Section 1.1 Exercises

1. a) List the natural numbers.
b) What is another name for the natural numbers?

2. a) What does it mean to say, "a is divisible by b," where a and b represent natural numbers?
b) List three natural numbers that are divisible by 5.
c) List three natural numbers that are divisible by 12.

In Exercises 3–6, explain your answer in a sentence or sentences.

3. What is a conjecture?

4. What is inductive reasoning?

5. What is deductive reasoning?

6. What is a counterexample?

7. In the 1950s doctors noticed that many of their lung cancer patients were also cigarette smokers. Doctors reasoned that cigarette smoking increased a person's chance of getting lung cancer. What type of reasoning did the doctors use? Explain.

8. Bill tells his son that if he continues to drive over the speed limit he will eventually get a traffic ticket. What type of reasoning is Bill using? Explain.

In Exercises 9–12, use inductive reasoning to predict the next line in the pattern.

9. $1 \times 10 + 2 = 12$
$12 \times 10 + 3 = 123$
$123 \times 10 + 4 = 1234$

10. $1 = 1$
$1 + 2 = 3$
$1 + 2 + 3 = 6$
$1 + 2 + 3 + 4 = 10$
$1 + 2 + 3 + 4 + 5 = 15$

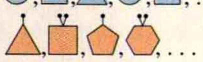
11. $11 \times 11 = 121$
$11 \times 12 = 132$
$11 \times 13 = 143$

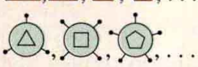
12.

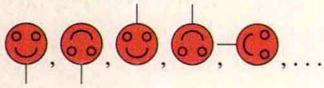
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

In Exercises 13–16, draw the next figure in the pattern (or sequence).

13. 

14. 

15. 

16. 

In Exercises 17–26, use inductive reasoning to predict the next three numbers in the pattern (or sequence).

17. 1, 4, 7, 10, . . .

18. 12, 10, 8, 6, . . .

19. 5, 3, 1, −1, −3, . . .

20. 1, −1, 1, −1, 1, . . .

21. 2, −6, 18, −54, . . .

22. 1, 1/2, 1/4, 1/8, . . .

23. 1, 4, 9, 16, 25, . . .

24. 1, 1, 2, 3, 5, 8, 13, 21, . . .

25. 0, 3, 8, 15, 24, 35, 48, 63, . . .

26. $5, -\frac{10}{3}, \frac{20}{9}, -\frac{40}{27}, \ldots$

In Exercises 27 and 28, look for a pattern in the first three products and use it to find the fourth product.

27. $9 \times 9 = 81$ $99 \times 9 = 891$ $999 \times 9 = 8991$ $9999 \times 9 = ?$

28. $9 \times 9 = 81$ $909 \times 9 = 8181$ $90909 \times 9 = 818181$ $9090909 \times 9 = ?$

29. Study the following entries and use the pattern they exhibit to complete the last two rows.
$1 + 3 = 4 \text{ or } 2^2$
$1 + 3 + 5 = 9 \text{ or } 3^2$
$1 + 3 + 5 + 7 = 16 \text{ or } 4^2$
$1 + 3 + 5 + 7 + 9 = ?$
$1 + 3 + 5 + 7 + 9 + 11 = ?$

30. Consider the number 142,857 and its first four multiples:

$142857 \times 1 = 142857$ $142857 \times 2 = 285714$ $142857 \times 3 = 428571$ $142857 \times 4 = 571428$

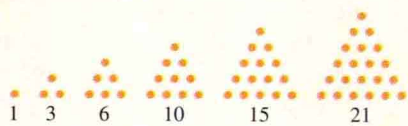
a) Observe the digits in the product and use inductive reasoning to make a conjecture about the digits that will appear in the product $142,857 \times 5$.

b) Multiply 142,857 by 5 to see whether your conjecture appears to be correct.

c) Can you make a more general conjecture about the digits in the product of a multiplication problem where 142,857 is multiplied by a one-digit positive number?

d) Multiply 142,857 by the digits 6 through 8 and see whether your conjecture appears to be correct.

31. The ancient Greeks labeled certain numbers as **triangular numbers**. The numbers 1, 3, 6, 10, 15, 21, and so on are triangular numbers.

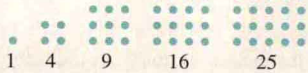


a) Can you determine the next two triangular numbers?

b) Describe a procedure to determine the next five triangular numbers without drawing the figures.

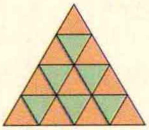
c) Is 72 a triangular number? Explain how you determined your answer.

32. Just as there are triangular numbers, there are also **square numbers**. The numbers 1, 4, 9, 16, 25, and so on are square numbers.



a) Determine the next three square numbers.
b) Describe a procedure to determine the next five square numbers without drawing the figures.
c) Is 72 a square number? Explain how you determined your answer.

33. Four rows of a triangular figure are shown.

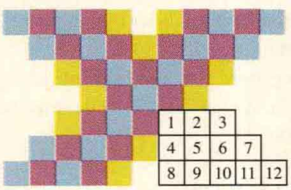


a) If you added six additional rows to the bottom of this triangle, using the same pattern displayed, how many triangles would appear in the 10th row?
b) If the triangles in all 10 rows were added, how many triangles would appear in the entire figure?

34. Find the letter that is the 118th entry in the following sequence. Explain how you determined your answer.

Y, R, Y, R, R, Y, R, R, R, Y, R, R, R, R, Y, R, R, . . .

35. The pattern shown here is taken from a quilt design known as a Triple Irish Chain. Complete the color pattern by indicating the color assigned to each square.



36. Pick a number, multiply the number by 2, add 4 to the product, divide the sum by 2, and subtract 2 from the quotient. See Example 5.
a) What is the relationship between the number you started with and the final number?
b) Arbitrarily select some different numbers and repeat the process, recording the original number and the result.
c) Can you make a conjecture about the relationship between the original number and the final number?
d) Prove, using deductive reasoning, the conjecture you made in part (c). See Example 6.

37. Pick any number and multiply the number by 6. Add 3 to the product. Divide the sum by 3, and subtract 1 from the quotient.
a) What is the relationship between the number you started with and the final answer?
b) Arbitrarily select some different numbers and repeat the process, recording the original number and the results.
c) Can you make a conjecture about the relationship between the original number and the final number?
d) Try to prove, using deductive reasoning, the conjecture you made in part (c).

38. Pick any number and add 1 to it. Find the sum of the new number and the original number. Add 9 to the sum. Divide the sum by 2 and subtract the original number from the quotient.
a) What is the final number?
b) Arbitrarily select some different numbers and repeat the process. Record the results.
c) Can you make a conjecture about the final number?
d) Try to prove, using deductive reasoning, the conjecture you made in part (c).

39. Pick a number, add 5 to the number, divide the sum by 5, subtract 1 from the quotient, and multiply the result by 5.
a) What is the relationship between the number you started with and the final number?
b) Arbitrarily select some different numbers and repeat the process, recording the original number and the result.
c) Can you make a conjecture about the relationship between the original number and the final number?
d) Try to prove, using deductive reasoning, the conjecture you made in part (c).

40. Pick a number and add 10 to the number. Divide the sum by 5. Multiply this quotient by 5. Subtract 10 from the product. Then subtract your original number.
a) What is the result?
b) Arbitrarily select some different numbers and repeat the process, recording the original number and the result.
c) Can you make a conjecture regarding the result when this process is followed?
d) Try to prove, using deductive reasoning, the conjecture you made in part (c).

In Exercises 41–47, find a counterexample to show that each of the statements is incorrect.

41. The product of any two counting numbers is divisible by 2.

42. Every counting number greater than 5 is the sum of either two or three consecutive counting numbers. For example, $9 = 4 + 5$ and $17 = 9 + 8$.

43. The product of a number multiplied by itself is even.

44. The product of 2 two-digit numbers is a three-digit number.

45. The sum of 3 two-digit numbers is a three-digit number.

46. The sum of any two odd numbers is divisible by 3.

47. When a counting number is added to 3 and the sum is divided by 2, the quotient will be an even number.

48. a) Construct a triangle and measure the three interior angles with a protractor. What is the sum of the measures?
b) Construct three other triangles, measure the angles, and record the sums. Are your answers the same?
c) Make a conjecture about the sum of the measures of the three interior angles of a triangle.

49. a) Construct a quadrilateral (a four-sided figure) and measure the four interior angles with a protractor. What is the sum of their angle measures?
b) Construct three other quadrilaterals, measure the angles, and record the sums. Are your answers the same?
c) Make a conjecture about the sum of the measures of the four interior angles of a quadrilateral.

50. a) Select one- and two-digit numbers and multiply each by 99. Record your results.
b) Find the sum of the digits in each of your products in part (a).
c) Make a conjecture about the sum of the digits when a one- or two-digit number is multiplied by 99.

51. a) Calculate the squares of 15, 25, 35, and 45 and record the results. (Note that the square of 15 is $15 \times 15 = 225$.) Examine the products and see whether you can establish a pattern in relation to the numbers being multiplied.
b) Make a conjecture about how to mentally calculate the square of any number whose unit digit is 5.
c) Calculate the squares of 55, 75, 95, and 105 using your conjecture.

Problem Solving/Group Activities

52. Complete the following square of numbers. Explain how you determined your answer.

1	2	3	4
2	5	10	17
3	10	25	52
4	17	52	?

53. Find the next three numbers in the sequence.

1, 8, 11, 88, 101, 111, 181, 1001, 1111, . . .

54. The following numbers are swogs.

12347, 70523, 56123, 90341, 16325, 34127

The following numbers are not swogs.

1573, 12345, 953, 56213, 56132, 34325

a) Describe the common characteristics of numbers that are swogs.
b) Which two of the following numbers are swogs?

43217, 54323, 52307, 16235, 36521

55. In this section we defined *conjecture*. The following is *Ulm's conjecture*. Pick any positive integer. If the integer is even, divide it by 2. If the integer is odd, multiply it by 3 and add 1. If you continue this process with each answer obtained, the result will eventually be 1. For example, suppose that we begin with the number 5 and carry out the process.

Pick 5	Multiply 5 by 3 and add 1; obtain 16.
16	Divide by 2; obtain 8
8	Divide by 2; obtain 4
4	Divide by 2; obtain 2
2	Divide by 2; obtain 1
1	

a) Select three positive even integers greater than 10 and show that Ulm's conjecture holds for each number.
b) Select three positive odd integers greater than 10 and show that Ulm's conjecture holds for each number.

Research Activities

56. a) Using newspapers, magazines, and other sources, find examples of conclusions arrived at by inductive reasoning.
b) Explain how inductive reasoning was used in arriving at the conclusion.

57. When a jury decides the guilt or innocence of a defendant, do the jurors collectively use primarily inductive reasoning, deductive reasoning, or an equal amount of each? Write a brief report supporting your answer.

1.2 ESTIMATION

An important step in solving mathematical problems—or, in fact, *any* problem—is to make sure that the answer you've arrived at makes sense. One technique for determining whether an answer is reasonable is to estimate. **Estimation** is the process of arriving at an approximate answer to a question. This section demonstrates several estimation methods.

To estimate, or approximate, an answer we often round off numbers as illustrated in the following examples. The symbol $\approx$ means *is approximately equal to*.

EXAMPLE 1

Estimate the cost of 22 poster boards at \$0.89 each.

Solution: We may round off the amounts as follows to obtain an estimate.

$$\begin{array}{rcr} 22 & \rightarrow & 20 \\ \times\ 0.89 & \rightarrow & \times\ 0.90 \\ \hline & & 18.00 \end{array}$$

Thus the 22 poster boards will cost approximately \$18.00, written ≈ \$18. ▲

In Example 1 we could have rounded the 22 to 20 and the \$0.89 to \$1.00, which would result in an estimate of \$20. The true cost is \$0.89 × 22, or \$19.58. *Estimates are not meant to give exact values for answers but are a means of determining whether your answer is reasonable.* If you calculated an answer of \$19.58 and then did a quick estimate to check it, you would know that the answer is reasonable because it is close to your estimated answer.

EXAMPLE 2

At a local supermarket Amy purchased milk for \$2.39, lettuce for \$0.89, bread for \$0.99, hot dogs for \$2.15, ground beef for \$4.76, bananas for \$0.49, and a green onion for \$0.40. The total bill was \$16.08. Use estimation to determine whether this amount is reasonable.

Solution: The most expensive item is \$4.76 and the least expensive is \$0.40. How should we estimate? We will estimate two different ways. First, we will round the cost of each item to the nearest 10 cents. Then we will round the cost to the nearest dollar. Rounding to the nearest 10 cents is more accurate. However, to determine whether the total bill is reasonable we may need to round only to the nearest dollar.

	Rounding to the Nearest 10 Cents		**Rounding to the Nearest Dollar**	
Milk	\$2.39 →	\$2.40	\$2.39 →	\$2.00
Lettuce	0.89 →	0.90	0.89 →	1.00
Bread	0.99 →	1.00	0.99 →	1.00
Hot dogs	2.15 →	2.20	2.15 →	2.00
Ground beef	4.76 →	4.80	4.76 →	5.00
Bananas	0.49 →	0.50	0.49 →	0.00
Onion	0.40 →	0.40	0.40 →	0.00
		\$12.20		\$11.00

Using either estimate, we find that the bill of \$16.08 is quite high. Therefore Amy should check the bill carefully before paying it. Adding the prices of all seven items gives the true cost of \$12.07. ▲

EXAMPLE 3

The number of bushels of grapes produced at a vineyard are 62,408 Cabernet Sauvignon, 118,916 French Colombard, 106,490 Chenin Blanc, 5960 Charbono, and 12,104 Chardonnay. Select the best estimates of the total number of bushels produced by the vineyard.

a) 500,000 **b)** 30,000 **c)** 300,000 **d)** 5,000,000