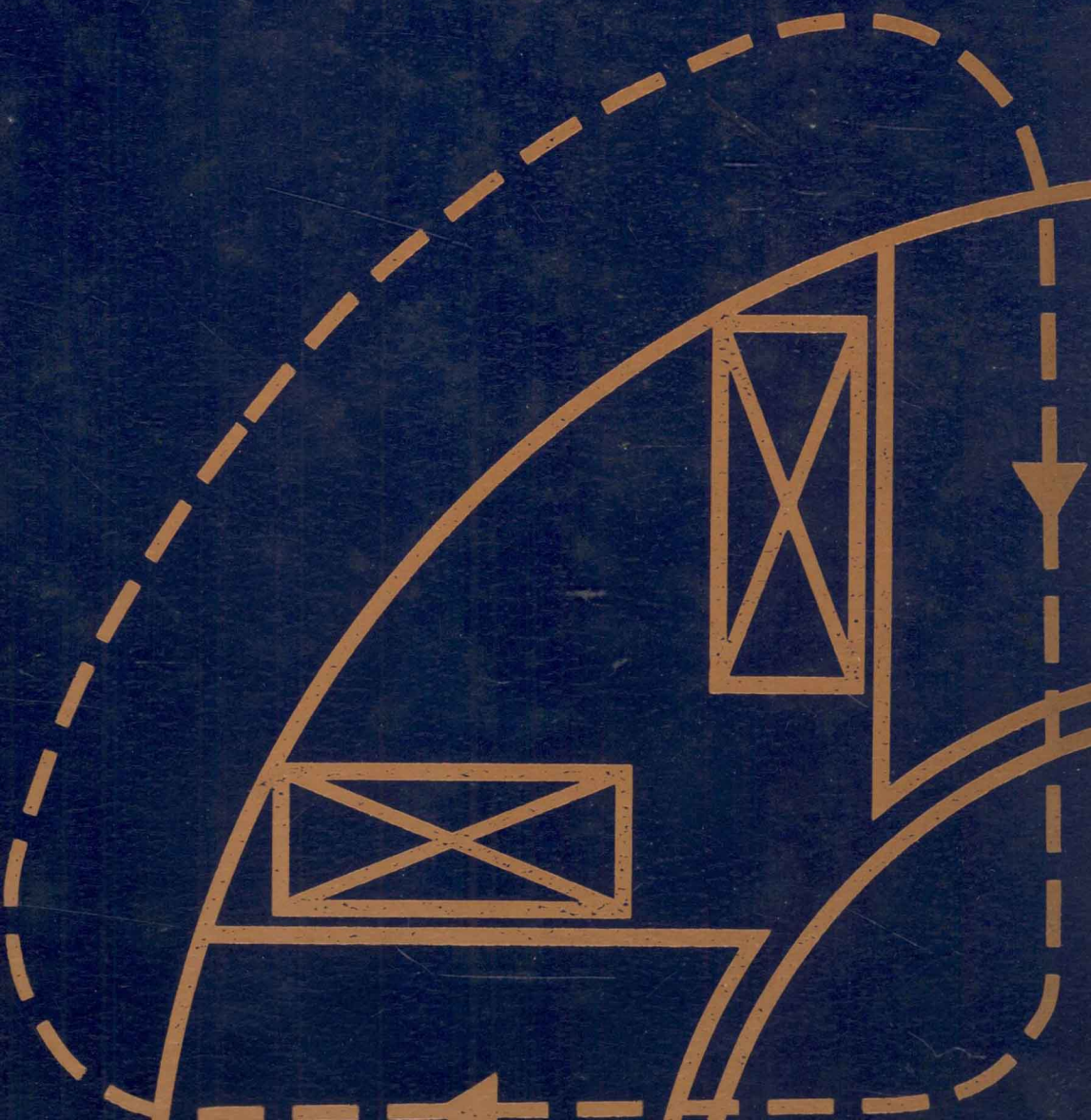


# Electric Machines and Drives

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Gordon R. Slemon



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Gordon R. Slemon

*University of Toronto*

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# **Electric Machines and Drives**

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# Preface

In writing this book, I have had two basic objectives in mind: to provide the sort of introduction to electric machines that I feel every student of electrical engineering should have, and to provide a more in-depth treatment of electric machines and drives for those who may wish to know more about the subject.

Chapters 1 through 6 address the first objective by providing a selection of topics considered suitable for a one-term course. During this introductory course the student should encounter a number of basic concepts:

- How ferromagnetic materials behave and why they are basic components of most useful electric machines.
- How a transformer works, first ideally, and then taking into account some of its imperfections.
- What are the basic useful mechanisms for producing force and torque from electric energy sources and how these mechanisms can be exploited to produce useful types of electric machines such as commutator, induction, and synchronous.
- How all types of electric machines are inherently capable of converting energy from a mechanical source to electric form as generators and also of converting electric energy to mechanical form in a motor or drive.

- How an adequate analytical model, usually in the form of an equivalent electric circuit, can be developed for a device.
- That engineers, recognizing that all models are approximate representations of reality, choose a model which is no more complex than necessary for the application at hand.

I have assumed that students taking an introductory electric machines course will have some prior knowledge of electric and magnetic field concepts, but without much depth in ferromagnetics, and also some knowledge of elementary electric circuit analysis.

It is my conviction that the essence of engineering is design. Accordingly, the end objective of each phase of preparatory study should be to increase the student's capability to design useful devices and systems to meet the needs of society. The educational route to this objective follows the sequence of aspects: an understanding of the physical processes, the derivation of approximate models, the use of analytical techniques and, finally, design.

Throughout the first six chapters I have attempted to develop a sound physical understanding of the energy conversion processes utilized in magnetic devices, transformers, and machines. To emphasize the importance of this approach, consider the force tending to close the air gap in a ferromagnetic core. This force may be calculated by use of the principles of energy conservation and virtual displacement. While this method is both analytically simple and powerful, it provides little appreciation of the origin of the force or its area of action. The insight arising from physical visualization is an important ingredient in arriving at an appropriate model of the device. Use of a purely mathematical approach without adequate attention to the physical model can frequently lead to serious error.

Modeling is an art which will develop as knowledge and experience grow. Considerable emphasis has been given in the book to the freedom to choose a model which is just adequate to meet the present need for performance prediction. This choice cannot be made in the abstract, but requires an assessment of the actual numerical parameter values. Insofar as possible, without incurring undue complexity, the governing parameters of each device have been related directly to its dimensions and to the properties of its materials to assist in developing the engineering judgment basic to modeling.

Chapter 1 begins with a review of magnetic field concepts. It then introduces the basic modeling ideas of equivalent magnetic and electric circuits. An understanding of ferromagnetic materials is based on the visualization of magnetic domains and how the orientation boundaries of the domain can be moved to produce intense fields around closed paths including those with air gaps. A treatment of permanent-magnet materials is integrated into this introduction in view of the increasing importance of permanent-magnet devices and machines.

Chapter 2 deals with the understanding and basic modeling of a two-winding transformer, followed by a few of its most important operating properties. The

important parameter of leakage inductance is treated from a physical rather than a coupled-circuit point of view.

Basic principles of electromechanical energy conversion machines are introduced in Chapter 3. Again, an attempt has been made to develop a physical understanding of the various machine types before launching into the more detailed discussion of these machines in the following chapters.

Commutator or direct-current machines have been considered first in Chapter 4, not because of their importance relative to the induction and synchronous machines of Chapters 5 and 6, but rather because they are so widely used in electric-machine laboratories in experiments on all types of rotating machines. Where this laboratory use is not of significance, the study of the dominant categories of induction and synchronous machines can be undertaken immediately after Chapter 3.

Since a majority of all electric machines are of the induction type, particular emphasis has been given to the concept of the rotating magnetic field and its interaction with currents induced in a squirrel-cage rotor. The elements of space-vector notation are introduced as a compact and convenient means of representing such rotating fields. The features of a double-cage or deep-bar rotor are included in the introductory phase of study because they are characteristic of essentially all induction motors encountered in practice.

The treatment of synchronous machines in Chapter 6 emphasizes the basic nature of such machines as a source of alternating current, in contrast with the more conventional modeling as a voltage source. This approach builds naturally on the previous modeling of induction machines. A discussion of permanent-magnet machines and their use in electronically switched drives is included.

Chapters 7 through 11 may form the basis for several selections of material for a second course for those students who choose to pursue further the fascinating and important specialty of electric machines and drive systems. Alternatively, these chapters may be useful for later reference when the practicing engineer discovers a need to know somewhat more than was covered in the brief introductory course.

Magnetic systems and transformers are revisited in Chapter 7, which presents more concepts on the modeling of ferromagnetic devices, some design concepts to aid users in appreciating operating limitations, and a number of magnetically based devices and transformers frequently encountered in practice. Instructors may wish to incorporate selected parts of this chapter into the introductory course syllabus if time permits.

A rapidly increasing number of electric drives require sources of variable voltage, current, and/or frequency. Most of such sources are produced through the use of semiconductor switches. Chapter 8 presents a brief introduction to the idealized structure and behavior of the basic power-semiconductor systems in common use as converters.

Chapter 9 presents a treatment of electric drives using commutator machines. While the trend in variable-speed drives is now away from these d-c



motors, they are still practically significant. Also, they provide a simple and useful appreciation of the transient behavior of electromechanical systems for speed and position control.

Induction machines are revisited in Chapter 10 but this time with a view to developing an understanding of transient as well as steady-state performance. While induction machines will continue to dominate constant-speed drive applications, they will also be used extensively in variable-speed drive systems incorporating variable-frequency electric supply from power-semiconductor converters. In such applications, the transient behavior is of increased relevance. The transient analysis is based on space-vector concepts, which are particularly convenient for induction machines because of their cylindrical symmetry.

The discussion of synchronous machines in Chapter 11 includes both the important class of synchronous generators used in electric supply systems, and synchronous motors which are increasingly being used in controlled-speed electric drives. Again, analytical emphasis is on transient modeling and performance. Various types of permanent-magnet machines are given special attention because of their major and increasing application in drives.

Selected material from Chapters 7 through 11 has been used as a basis for a senior-undergraduate elective course on controlled electric drives at the University of Toronto. Also, a graduate course on modeling of electric machines has been based on a selection from the same chapters.

Most of the models developed in this book are in the form of equivalent circuits. A major reason for this choice is that nonlinear parameters can be represented directly in circuit form in relation to the controlling variables. Very little space has been given to methods of analyzing these models since it can be assumed that parallel courses in electric circuits, differential equations, computer programming, and systems analysis will have provided an adequate range of analytical techniques. Where appropriate, some of these techniques have been used to derive typical operating characteristics.

Most of the models derived in this book are appropriate for the solution of both dynamic and steady-state performance. They can therefore be integrated into system representations. Transient solutions for some simple situations have been included in the book. For more complex situations, the analytical and simulation methods developed in companion courses on control systems may be employed.

A substantial number of problems have been included at the end of each chapter. In most instances, the pertinent section of the book is indicated at the end of the problem statement. These problems draw on all the concepts of understanding, modeling, analysis, and design. Answers for most of the problems have been included in a separate section to reassure the reader of progress. To the instructor who wishes to use any of these problems as test or quiz assignments, I would suggest that a new set of parameter values be used.

A significant proportion of the problems relate to design. For these, some engineering judgment may have to be exercised in arriving at an appropriate approximation and in choosing materials, configurations, and dimensions. Where



answers are given to such problems, they should be regarded as typical rather than definitive. Many engineering applications have been introduced in the problem sections only, partly to demonstrate that, with a good grounding in basic concepts, a very wide range of engineering systems can be understood, analyzed, and devised.

Lists of the principal symbols used in the book have been printed on the endpapers. Each vector and phasor quantity has been identified through the use of a normal letter with an arrow above the symbol rather than the common use of boldface type. I consider the latter practice unfortunate since instructors and students cannot write in boldface. In expressing data, the International System (SI) units and notation have been employed. Conversion factors to other unit systems are given in an appendix.

Substantial sections of the book have been derived in revised form from the book *Electric Machines*, by Alan Straughen and myself, published in 1980 by Addison-Wesley Publishing Company. As I acknowledge this, I would add that it has been a pleasure for me to work with this excellent publisher again. Also, a number of sections and problems have been incorporated into the text in revised form from one of my earlier books, *Magnetoelectric Devices—Transducers, Transformers, and Machines*, published in 1966 and now out of print. I am grateful to the publishers of that book, John Wiley and Sons, for permitting me to use this material in the present work.

I am grateful to my colleagues and graduate students in the Power Devices and Systems Group at the University of Toronto for their many helpful suggestions and criticisms. Finally, I wish to acknowledge my deep gratitude to my wife, Margaret, not only for typing the manuscript, but mainly for her continued support during the gestation period of this book.

Toronto

G.R.S.

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# Magnetic Systems

This book is concerned with a wide range of electromagnetic devices that are used for conversion of electric energy to or from mechanical energy or, in some instances, to transform electric energy to a more readily usable form. The machine systems considered include (1) generators for the production of electric power from hydraulic, steam, or gas turbines, (2) transformers to transform electric energy from its most convenient generation voltage to a voltage level suitable for transmission and then to transform it to voltages appropriate for distribution and use, (3) motors to convert electric energy to mechanical form, (4) electronic converters that change the available electric power to other desired forms with controllable voltage, current, or frequency, and (5) drive systems consisting of motors and converters capable of controlling the speed or position of mechanical loads.

Most electric machines convert energy by use of a magnetic field as an intermediary. Therefore, it is appropriate to begin with a discussion of the concepts, materials, and structures that are involved in producing magnetic fields in the required form, location, and intensity. In particular, the means of directing and controlling magnetic fields, in much the same way as electric currents are directed by electrical conductors, will be examined.



## I.1

### Magnetic Field Concepts

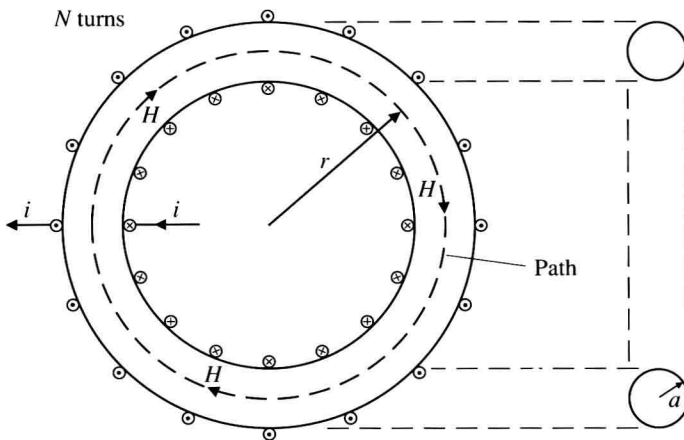
Magnetic fields are produced by (1) electric currents, (2) permanent magnets, or (3) electric fields that are changing with time. The first two are the subject of this chapter; the last can be ignored in the context of this book because it is significant only for high-frequency systems that radiate electromagnetic energy.

Let us start with Ampere's Circuital Law, which relates the intensity of a magnetic field to the current that produces it. The law may be expressed in the form

$$\oint \vec{H} \cdot d\vec{\ell} = \int_A \vec{J} \cdot d\vec{A} \quad \text{A} \quad (1.1)$$

It is important that expressions such as this be read with a view to visualizing the physical field. Therefore, let us first consider a system that produces a relatively uniform magnetic field within a confined space.

Figure 1.1 shows cross sections of a coil of  $N$  turns of conductor uniformly wound around a torus made of any nonferromagnetic material. When a current  $i$  is passed through the coil, a magnetic field with an intensity denoted by the vector quantity  $\vec{H}$  is produced within the torus. Consider the closed path shown within the torus at radius  $r$ . Because of the circular symmetry, the magnitude of the field intensity must be the same at all points along the path. According to Eq. (1.1), the product of the magnetic field intensity directed along this path and the length of the closed path is equal to the sum of all the electric current passing through the area enclosed by the path. Each of the  $N$  turns



**Figure 1.1** Coil wound on toroidal core showing magnetic field intensity  $\vec{H}$ .

of the coil penetrates this enclosed surface. Thus, the right-hand side of Eq. (1.1) is equal to  $Ni$  or, more generally, to the product of the conductor current density  $J$  and the total cross-sectional area of the  $N$  conductors. Thus, at radius  $r$ ,

$$H_r(2\pi r) = Ni \quad \text{A} \quad (1.2)$$

or

$$H_r = \frac{Ni}{2\pi r} \quad \text{A/m} \quad (1.3)$$

Application of the circuital law for a circular path at any radius that is not within the torus shows no net enclosed current. Therefore, the magnetic field is effectively confined to the volume within the torus.

Examination of Eq. (1.3) shows that the field intensity will be greatest along a path at the inside edge of the torus and least at the outer side of the torus. If the cross-sectional radius  $a$  of the torus is small relative to the radius of the torus, the value of  $H$  at the average value of path radius  $r$  may be used as a good approximation to the field over the whole cross section.

For our purposes, the most important property of a magnetic field is described by its magnetic flux density, denoted by a vector quantity  $\vec{B}$ . For free space, and practically for all nonferromagnetic materials, this quantity is related to the magnetic field intensity vector by a constant  $\mu_0$  having a value of

$$\mu_0 = 4\pi \times 10^{-7} \quad \text{T} \cdot \text{m/A} \quad (1.4)$$

so that

$$\vec{B} = \mu_0 \vec{H} \quad \text{T} \quad (1.5)$$

the unit of flux density being the tesla (T).

Just as current density  $\vec{J}$  integrated over the cross-sectional area of a conductor gives the current  $i$ , the integration of the flux density over the cross-sectional area of the magnetic field gives the magnetic flux  $\Phi$  measured in webers (Wb). Thus, for the torus of Fig. 1.1,

$$\Phi = \bar{B}A \quad \text{Wb} \quad (1.6)$$

where, for the torus of Fig. 1.1,

$$A = \pi a^2 \quad \text{m}^2 \quad (1.7)$$

and  $\bar{B}$  is the average magnitude of the flux density directed perpendicular to the area  $A$ . Magnetic flux is a scalar quantity and is continuous around a closed path, similar to the way electric current is continuous in a conducting path.

The next concept that is required is that an electric current that is changing with time sets up in the space both within and around the conductor an electric field of intensity  $\vec{E}$  which opposes the change in current. Figure 1.2 shows this electric field around a cross section of the toroidal coil of Fig. 1.1. Usually we are interested in the voltage that is induced either in the turns of the toroi-