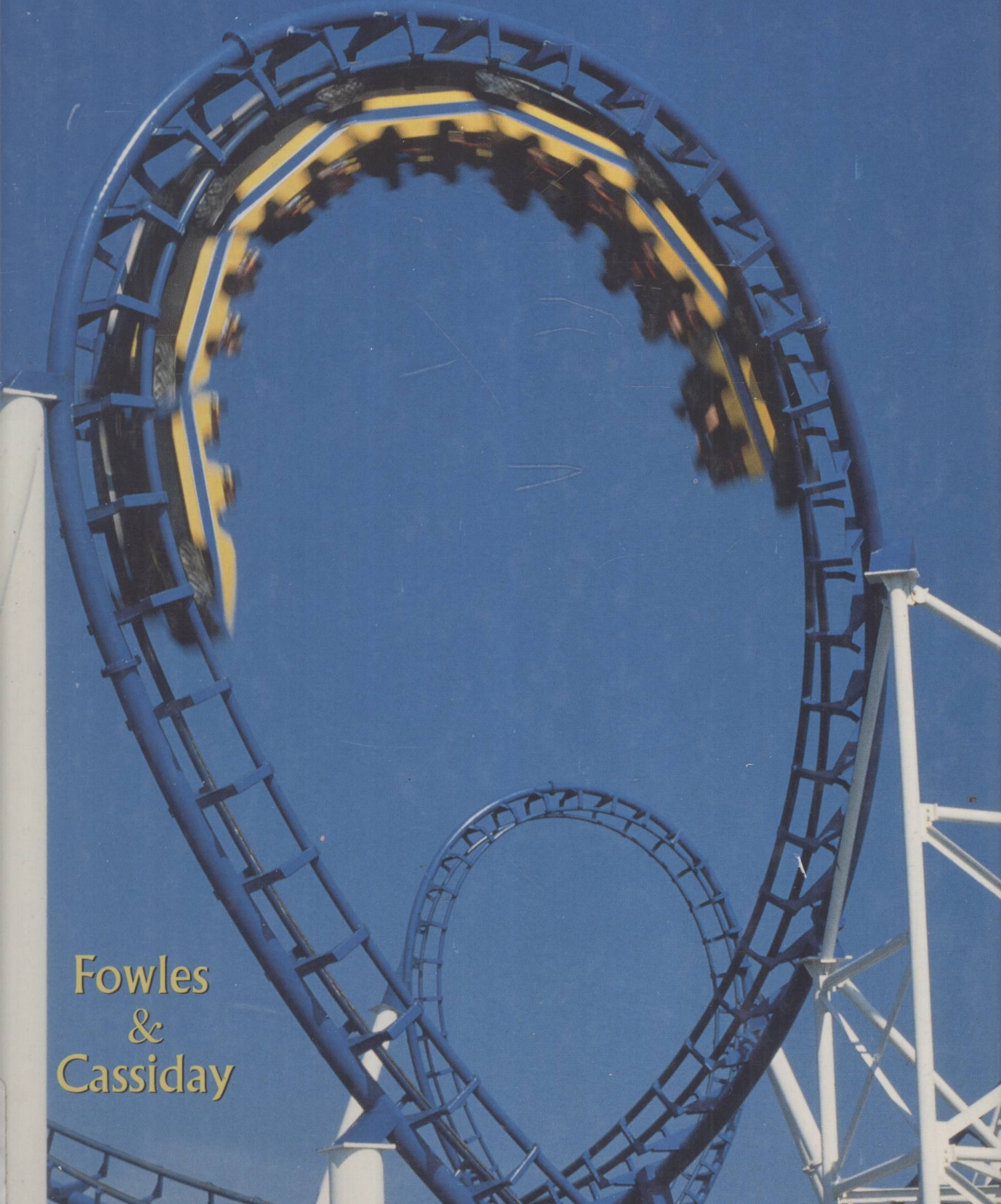
ANALYTICAL MECHANICS

sixth edition



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Analytical Mechanics

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PREFACE

This textbook is intended primarily for an undergraduate course in classical mechanics taken by students majoring in physics, physical science, or engineering. It is assumed that the student has taken a year of calculus-based general physics and a year of differential/integral calculus. It is highly recommended that a course in differential equations and matrix algebra be taken prior to or concurrently with this course in mechanics.

This sixth edition is the same in scope as the previous edition. New material has been introduced, and some old material has been eliminated. Great effort has been taken to clarify some of the more difficult concepts, and many figures have been added as a visualization aid. Several sections have been added that utilize the software tools *Mathcad* and *Mathematica* as part of the problem-solving methodology. The problem sets at the end of each chapter have been increased by about 25%, and many problems requiring numerical solutions have also been added. Equations and figures have been renumbered using a scheme that should make them much easier to find when referenced somewhere else in the text—or by the instructor when lecturing or discussing problem solutions.

A brief synopsis of each chapter follows. The chapters in bold font and the accompanying descriptions in italics font delineate the material that has been most extensively modified in this new edition.

- Chapter 1: A brief introduction to vector algebra; concepts of velocity and acceleration.
- Chapter 2: Newton's laws of motion; emphasis on motion in one dimension. *Introduction to numerical problem solving using Mathcad: vertical fall through a fluid.*
- **Chapter 3:** Harmonic motion, resonance, the driven oscillator. *Numerical solutions of nonlinear oscillations and chaotic motion.* Fourier techniques.
- Chapter 4: Motion of a particle in three dimensions. Introduction to the concepts of conservative forces and potential energy. *Introduction to numerical problem solving using Mathematica; projectile motion in a resistive medium.*
- Chapter 5: The analysis of motion in a noninertial frame of reference; the appearance of fictitious forces. Numerical solution of a problem involving motion in a rotating frame of reference.
- **Chapter 6:** *Expanded discussion of Newtonian gravitation; conic sections, orbits, and motion involving central forces.* Criteria for orbital stability. Rutherford scattering.

- **Chapter 7:** Systems of many particles. *The restricted three-body problem; numerical solution of three-body orbital motion; the Lagrangian points.* Conservation laws and particle collisions. *Rocket motion.*
- Chapter 8: Rotation of a body about a fixed axis; laminar motion of a rigid body; moments of inertia.
- **Chapter 9:** Expanded discussion of rotation of a rigid body in three dimensions. *Increased emphasis on the use of matrices and tensors to describe rotational motion. Numerical solutions of problems involving the rotation of bodies with different principal moments of inertia.* Analysis of gyroscopic motion.
- **Chapter 10:** Expanded discussion of Lagrangian mechanics. The use of both Hamilton's and *D'Alembert's principles to derive Lagrange's equations of motion. The method of Lagrange multipliers to solve problems with forces of constraint. The concept of generalized forces.* Hamiltonian mechanics and conservation principles.
- **Chapter 11:** Expanded discussion of coupled oscillators with *increased emphasis on matrix techniques. Expanded discussion of normal coordinates and normal modes of oscillation. Expanded discussion on solving the eigenvalue problem.* Discussion of the loaded string and wave motion.

Worked examples abound. They are typically found at the end of each section in the text. The problem set found at the end of each chapter contains problems that can be solved analytically. It is followed by a computer problem set, containing problems that are to be solved numerically, using either *Mathcad* or *Mathematica* (or any other software tool available to the student). Appendix I presents two sample worksheets that were generated in *Mathcad* and *Mathematica* in order to solve two specific numerical problems presented in the text. Answers to a few selected odd-numbered problems are given at the end of the book. A list of units, physical constants, mathematical aids, formulae, and discussions is also presented in the appendices.

A solutions manual is available to instructors upon adoption of the text. Saunders College Publishing may provide complimentary instructional aids and supplements or supplement packages to those adopters qualified under our adoption policy. Please contact your sales representative for more information. If as an adopter or potential user you receive supplements you do not need, please return them to your sales representative or send them to:

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July 1998

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FUNDAMENTAL CONCEPTS: VECTORS

"Let no one unversed in geometry enter these portals."

Plato's inscription over his academy in Athens

1.1 | INTRODUCTION

The science of classical mechanics deals with the motion of objects through absolute *space* and *time* in the Newtonian sense. Although central to the development of classical mechanics, the concepts of space and time would remain arguable for more than two and a half centuries following the publication of Sir Isaac Newton's *Philosophie naturalis principia mathematica* in 1687. As Newton put it in the first pages of the *Principia*, "Absolute, true and mathematical time, of itself, and from its own nature, flows equably, without relation to anything external, and by another name is called duration. Absolute space, in its own nature, without relation to anything external, remains always similar and immovable."

Ernst Mach (1838–1916), who was to have immeasurable influence on Albert Einstein, questioned the validity of these two Newtonian concepts in *The Science of Mechanics: A Critical and Historical Account of Its Development* (1907). There he claimed that Newton had acted contrary to his expressed intention of "framing no hypotheses," that is, accepting as fundamental premises of a scientific theory nothing that could not be inferred directly from "observable phenomena" or induced from them by argument.

Indeed, although Newton was on the verge of overtly expressing this intent in Book III of the *Principia* as the fifth and last rule of his *Regulae Philosophandi* (rules of reasoning in philosophy), it is significant that he refrained from doing so.

Throughout his scientific career he exposed and rejected many hypotheses as false; he tolerated many as merely harmless; he put to use those that were verifiable. But he encountered a class of hypotheses that, neither "demonstrable from the phenomena nor following from them by argument based on induction," proved impossible to avoid. His concepts of space and time fell in this class. The acceptance of such hypotheses as fundamental was an embarrassing necessity; hence, he hesitated to adopt the frame-no-hypotheses rule. Newton certainly could be excused this sin of omission. After all, the adoption of these hypotheses and others of similar ilk (such as the "force" of gravitation) led to an elegant and comprehensive view of the world the likes of which had never been seen.

Not until the late 18th and early 19th centuries would experiments in electricity and magnetism yield observable phenomena that could be understood only from the vantage point of a new space–time paradigm arising from Albert Einstein's special relativity. Hermann Minkowski introduced this new paradigm in a semipopular lecture in Cologne, Germany in 1908 as follows:

> Gentlemen! The views of space and time which I wish to lay before you have sprung from the soil of experimental physics and therein lies their strength. They are radical. From now on, space by itself and time by itself are doomed to fade away into the shadows, and only a kind of union between the two will preserve an independent reality.

Thus, even though his own concepts of space and time were superceded, Newton most certainly would have taken great delight in seeing the emergence of a new space–time concept based upon observed "phenomena," which vindicated his unwritten frame-no-hypotheses rule.

1.2 MEASURE OF SPACE AND TIME. UNITS[1]

In this text we shall assume that space and time are described strictly in the Newtonian sense. Three-dimensional space is Euclidian, and positions of points in that space are specified by a set of three numbers (x, y, z) relative to the origin $(0, 0, 0)$ of a rectangular Cartesian coordinate system. A length is the spatial separation of two points relative to some standard length.

Time is measured relative to the duration of reoccurrences of a given configuration of a cyclical system—say, a pendulum swinging to and fro, an Earth rotating about its axis, or electromagnetic waves from a cesium atom vibrating inside a metallic cavity. The time of occurrence of any event is specified by a number t, which represents the number of reoccurrences of a given configuration of a chosen cyclical standard. For example, if 1 vibration of a standard physical pendulum is used to define 1 s, then to say that some event occurred at $t = 2.3$ s means that the standard pendulum executed 2.3 vibrations after its "start" at $t = 0$, when the event occurred.

[1] A delightful account of the history of the standardization of units can be found in H. A. Klein, *The Science of Measurement—A Historical Survey,* Dover Publ., Mineola, 1988, ISBN 0-486-25839-4 (pbk).

All this sounds simple enough, but a substantial difficulty has been swept under the rug: Just what are the standard units? The choice of standards has usually been made more for political reasons than for scientific ones. For example, to say that a person is 6 feet tall is to say that the distance between the top of his head and the bottom of his foot is six times the length of something, which is taken to be the standard unit of 1 foot. In an earlier era that standard might have been the length of an actual human foot or something that approximated that length, as per the writing of Leonardo da Vinci on the views of the Roman architect–engineer Vitruvius Pollio (first century B.C.E.):

> . . . Vitruvius declares that Nature has thus arranged the measurements of a man: four fingers make 1 palm and 4 palms make 1 foot; six palms make 1 cubit; 4 cubits make once a man's height; 4 cubits make a pace, and 24 palms make a man's height . . .

Clearly, the adoption of such a standard does not make for an accurately reproducible measure. An early homemaker might be excused her fit of anger upon being "short-footed" when purchasing a bolt of cloth measured to a length normalized to the foot of the current short-statured king.

The Unit of Length

The French Revolution, which ended with the Napoleanic *coup d'etat* of 1799, gave birth to (among other things) an extremely significant plan for reform in measurement. The product of that reform, the metric system, expanded in 1960 into the Système International d'Unités (SI).

In 1791, toward the end of the first French National Assembly, Charles Maurice de Talleyrand-Perigord (1754–1838) proposed that a task of weight and measure reform be undertaken by a "blue ribbon" panel with members selected from the French Academy of Sciences. This problem was not trivial. Metrologically, as well as politically, France was still absurdly divided, confused, and complicated. A given unit of length recognized in Paris was about 4% longer than that in Bordeaux, 2% longer than that in Marseilles, and 2% shorter than that in Lille. The Academy of Sciences panel was to change all that. Great Britain and the United States refused invitations to take part in the process of unit standardization. Thus was born the antipathy of English-speaking countries toward the metric system.

The panel chose 10 as the numerical base for all measure. The fundamental unit of length was taken to be one ten-millionth of a quadrant, or a quarter of a full meridian. A surveying operation, extending from Dunkirk on the English Channel to a site near Barcelona on the Mediterranean coast of Spain (a length equivalent to 10 degrees of latitude or one ninth of a quadrant), was carried out to determine this fundamental unit of length accurately. Ultimately, this monumental trek, which took from 1792 until 1799, changed the standard meter—estimated from previous, less ambitious surveys—by less than 0.3 mm, or about 3 parts in 10,000. We now know that this result, too, was in error by a similar factor. The length of a standard quadrant of meridian is 10,002,288.3 m, a little over 2 parts in 10,000 greater than the quadrant length established by the Dunkirk–Barcelona expedition.

Interestingly enough, in 1799, the year in which the Dunkirk–Barcelona survey was completed, the national legislature of France ratified new standards, among them the meter. The standard meter was now taken to be the distance between two fine scratches made on a bar of a dense alloy of platinum and iridium shaped in an X-like cross section to minimize sagging and distortion. The United States has two copies of this bar, numbers 21 and 27, stored at the Bureau of Standards in Gaithersburg, MD, just outside Washington, DC. Measurements based on this standard are accurate to about 1 part in 10^6. Thus, an object (a bar of platinum), rather than the concepts that led to it, was established as the standard meter. The Earth might alter its circumference if it so chose, but the standard meter would remain safe forever in a vault in Sevres, just outside Paris, France. This standard persisted until the 1960s.

The 11th General Conference of Weights and Measures, meeting in 1960, chose a reddish-orange radiation produced by atoms of krypton-86 as the next standard of length, with the meter defined in the following way:

> The meter is the length equal to 1,650,763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2\,p^{10}$ and $5\,d^5$ of the krypton-86 atom.

Krypton is all around us; it makes up about 1 part per million of the Earth's present atmosphere. Atmospheric krypton has an atomic weight of 83.8, being a mixture of six different isotopes that range in weight from 78 to 86. Krypton-86 composes about 60% of these. Thus, the meter was defined in terms of the "majority kind" of krypton. Standard lamps contained no more than 1% of the other isotopes. Measurements based on this standard were accurate to about 1 part in 10^8.

Since 1983 the meter standard has been specified in terms of the velocity of light. A meter is the distance light travels in 1/299,792,458 s in a vacuum. In other words, the velocity of light is defined to be 299,792,458 m/s. Clearly, this makes the standard of length dependent on the standard of time.

The Unit of Time

Astronomical motions provide us with three great "natural" time units: the day, the month, and the year. The day is based on the Earth's spin, the month on the moon's orbital motion about the Earth, and the year on the Earth's orbital motion about the Sun. Why do we have ratios of 60 : 1 and 24 : 1 connecting the day, hour, minute, and second? These relationships were born about 6000 years ago on the flat alluvial plains of Mesopotamia (now Iraq), where civilization and city-states first appeared on Earth. The Mesopotamian number system was based on 60, not on 10 as ours is. It seems likely that the ancient Mesopotamians were more influenced by the 360 days in a year, the 30 days in a month, and the 12 months in a year than by the number of fingers on their hands. It was in such an environment that sky watching and measurement of stellar positions first became precise and continuous. The movements of heavenly bodies across the sky were converted to clocks.

The second, the basic unit of time in SI, began as an arbitrary fraction (1/86,400) of a mean solar day ($24 \times 60 \times 60 = 86{,}400$). The trouble with astronomical clocks, though, is that they do not remain constant. The mean solar day is lengthening, and the lunar month, or time between consecutive full phases, is shortening. In 1956 a new second was defined to be 1/31,556,926 of one particular and carefully measured mean solar year, that of 1900. That second would not last for long! In 1967 it was redefined again, in terms of a specified number of oscillations of a cesium atomic clock.

A cesium atomic clock consists of a beam of cesium-133 atoms moving through an evacuated metal cavity and absorbing and emitting microwaves of a characteristic resonant frequency, 9,192,631,770 Hertz (Hz), or about 10^{10} cycles per second. This absorption and emission process occurs when a given cesium atom changes its atomic configuration and, in the process, either gains or loses a specific amount of energy in the form of microwave radiation. The two differing energy configurations correspond to situations where the spins of the cesium nucleus and that of its single outer-shell electron are either opposed (lowest energy state) or aligned (highest energy state). This kind of a "spin-flip" atomic transition is called a *hyperfine transition.* The energy difference and, hence, the resonant frequency are precisely determined by the invariable structure of the cesium atom. It does not differ from one atom to another. A properly adjusted and maintained cesium clock can keep time with a stability of about 1 part in 10^{12}. Thus, in one year, its deviation from the right time should be no more than about 30 μs (30×10^{-6} s). When two different cesium clocks are compared, it is found that they maintain agreement to about 1 part in 10^{10}.

It was inevitable then that in 1967, because of such stability and reproducibility, the 13th General Conference on Weights and Measures would substitute the cesium-133 atom for any and all of the heavenly bodies as the primary basis for the unit of time. The conference established the new basis with the following historic words:

> The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the cesium-133 atom.

So, just as the meter is no longer bound to the surface of the Earth, the second is no longer derived from the "ticking" of the heavens.

The Unit of Mass

This chapter began with the statement that the science of mechanics deals with the motion of objects. The final concept, with its accompanying unit, needed to specify completely any physical quantity.[2] That concept is *mass,* and the *kilogram* is its basic unit. This primary standard, too, is stored in a vault in Sevres, France, with secondaries owned and kept by most major governments of the world. Note that the units of length and time are based on atomic standards. They are *universally* reproducible and virtually indestructible. Unfortunately, the unit of mass is not yet quite so robust.

A concept involving mass, which we shall have occasion to use throughout this text, is that of the *particle,* or point mass, an entity that possesses mass but no spatial extent. Clearly, the particle is a nonexistent idealization. Nonetheless, the concept serves as a useful approximation of physical objects in a certain context, namely, in a situation where the dimension of the object is small compared to the dimensions of its environment. Examples include a bug on a phonograph record, a baseball in flight, and the Earth in orbit around the Sun.

[2] The concept of mass will be treated in Chapter 2.