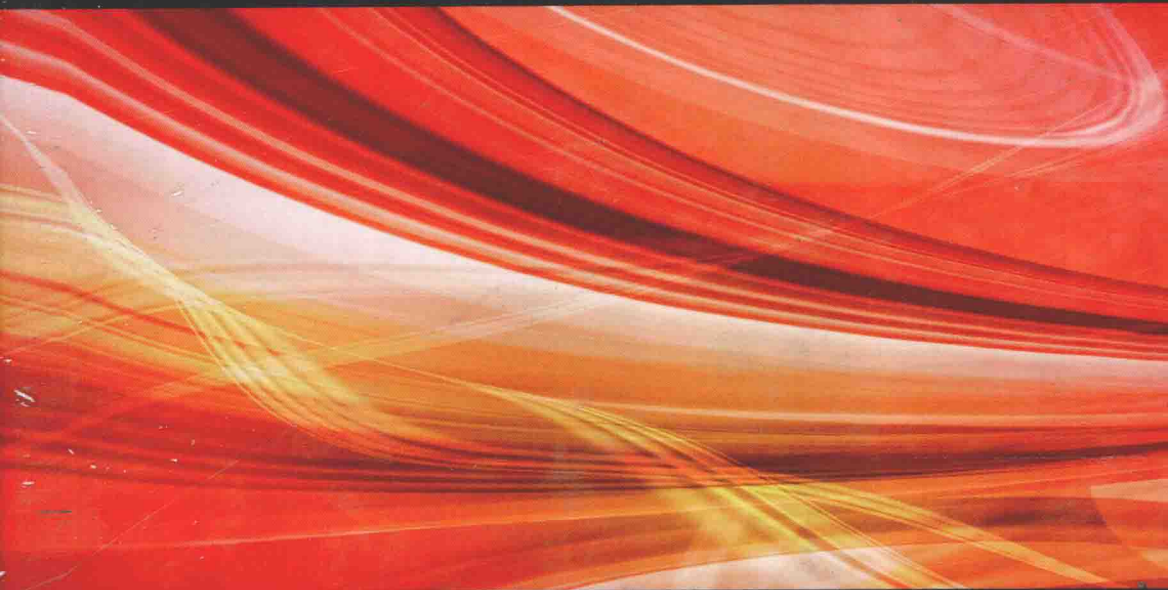


**FLUID MECHANICS SERIES**



# **Turbulent Multiphase Flows with Heat and Mass Transfer**

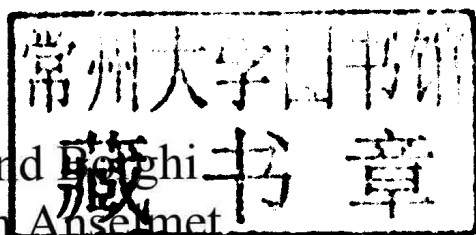
**Roland Borghi and Fabien Anselmet**

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# Turbulent Multiphase Flows with Heat and Mass Transfer

Roland Deegan  
Fabien Anselmetti



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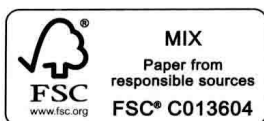
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# Introduction

## **I.1. The significance of multiphase flows and their modeling**

Many industrial systems bring into play, in one way or the other, multiphase media involving the combination of liquids and gases, non-miscible liquids, fluids and solids.

Nuclear reactors (whether they use boiling or pressurized water) possess a cooling circuit where, in certain parts of this circuit, a mixture of water and vapor circulates, with water vapor forming on contact with hot walls needing to be cooled and drops of liquid water forming on contact with cold walls needing to be heated. Numerous other thermal engineering facilities possess this type of circuit for transferring heat, in order either to use this energy elsewhere or simply to prevent the machinery from being destroyed by the heat.

The extraction and transportation of oil products is done using conduits within which media are flowing with two or more phases: liquids of different densities and viscosities, gases and even solids. Problems of icing in aeronautics (on the leading edges of wings or ailerons or in Pitot tubes, etc.) also necessitate studying a humid air medium with drops of water flowing in the immediate vicinity of the wall. The short-distance transport of pulverulent materials such as wheat, sawdust and grain is done by blowing air loaded with these solid particles through ducts.

In liquid-fuel rocket engines used in space launchers, as well as in diesel engines, the combustion chamber contains a mixture of vaporizing droplets and combusting gases that give off a considerable release of heat in an astonishingly small volume. A combustible or oxidant liquid, or a mixture of both, is injected in tiny droplets into

the combustion chamber, where these drops vaporize and the vapors can burn together, in a steady regime in a rocket engine and in a periodic regime in cycles of a diesel engine.

Fuel burners in glassworks furnaces, or vapor generators in thermal power stations, also inject jets of droplets of fuel into the zone of reacting gases. They produce not only heat and burned gases, but also smoke in which very small particles of carbon are dispersed, and the control of these particles is critical; they allow high heat transfers via radiation in furnaces, but can lead to significant air pollution from chimney exhaust.

Chemical engineering uses several types of gas-liquid reactors at controlled temperature, which are intended to produce specific chemical products rather than heat. Liquid and gaseous reactants are mixed as effectively as possible in order to be able to allow various chemical reactions at the interface between phases. Many chemical reactors also use a catalyst, which is most often in the form of a solid dispersed phase, and these reactors therefore bring multiphase flows into play.

Fluidized beds are currently the most effective devices for burning coal: air is blown forcefully through a highly dense dispersed solid phase composed of particles of dolomite and small particles of coal, enabling exchanges of energy among these three phases, which then causes and maintains chemical reactions. The energy released by combustion produces water vapor by the intermediary of a heat exchanger, the tubes of which can be closer to the combustion zone. The system not only enables adequate homogeneity of the temperature field but also maintains this temperature at around 1,300 K by avoiding the overproduction of NO, while stabilizing combustion at the same time. In addition, dolomite absorbs sulfur and reduces SO<sub>2</sub> emissions. There are also recirculating fluidized beds in which solid phases are entrained by the gas phase, recovered at the exit and reinjected at the entry to perfect combustion, even in the highest mass flow rate conditions. These are also easily transposable for the combustion of different types of combustibles, ranging from gas mixtures to various types of waste. Fluidized beds are also used in chemical engineering, or simply to dry solid particles, or to manufacture various types of powders.

Multiphase media often play in nature as well. Clouds contain tiny droplets of (non-pure) water along with particles of ice or snowflakes; agriculture utilizes jets of drops or droplets for the watering or treatment of plants. The dispersion of smoke or other natural or artificial aerosols into the atmosphere or confined gaseous environments, and the possible deposit of the solid or liquid particles they contain, is a source of ongoing problems that are difficult to solve or control. Landslides, avalanches, and flows of sand or various types of debris are also natural examples of flows of multiphase media, the behavior of which is difficult to predict. Soil, even

stable soil, is always a multiphase medium containing at least one liquid phase, which is usually water, but air for backfill and ballast for railroads, and in civil engineering we must always have control over the strains of these media that is as complete as possible.

In all of these industrial or natural situations, the overall medium behaves very often like a fluid. This is obviously the case when there is no solid phase, but as soon as one fluid phase exists the medium can be in flow, at least in some parts. This makes it desirable to be able to study these media in the same way as classic fluids, for which we have nearly 100 years of accumulated knowledge and experience. Moreover, these flows almost always display a highly random nature, both in the positions of phases and because they show that velocity fluctuations develop, quite often, similarly to turbulent flows. Experience in modeling turbulent flows should, therefore, prove extremely useful. This experience is not confined solely to questions of motion; we can also represent phenomena in fluids such as heat exchange, diffusion and mixing of various constituents, and chemical reactions, all out of equilibrium. The short descriptions provided above for multiphase situations make it clear that they also include all these phenomena, and that these phenomena give them important specificities. Therefore, it is very useful to generalize the approach used for out of equilibrium continuous fluid media to these multiphase flows.

Since the early years of the use of multiphase devices in various fields of application, a notable body of results and knowledge of an empirical or even occasionally theoretical nature has been accumulated; this has been used to develop simplified and practical approaches for the study and prediction of their characteristics. The design of industrial machinery and the interpretation and control of natural multiphase phenomena have been aided by the various separate branches of this knowledge set, each of them directly linked to a particular application. However, for the last 20 years, the desire for increasingly detailed predictions has resulted in the increasingly frequent use of the methods of continuous fluid mechanics, thus showing a certain community of approach in the different cases. Even without necessarily seeking to build theoretical representations, when we wish to understand how the various basic physical phenomena that occur on a small scale combine, we find a strong similarity of description in numerous different multiphase situations. With regard to the specific aspect of mixing, this is well emphasized in [GUY 97].

It now seems possible and interesting to attempt a unified explanation of the basic theoretical concepts used in the modeling of all multiphase media of these various applications, even though the particularities of the various different situations must explicitly be involved at a certain point. This is the aim of this book. Seeking such a unified methodology has a purpose, even a threefold purpose: first, it may provide a more complete physical understanding of each situation by bringing

together information and analyses of situations that are different but using similar phenomena. Second, it may serve as a motivation to use successfully certain modeling or study tools from one field of application in another. Finally, it may render a new field of application accessible that might seem too complex at first glance. However, it is not our objective here to develop the approach to the extent of completely addressing the issues posed by the various applications, even simply those referred to earlier. First, there are too many possible applications, but above all, it would be necessary at times to enter into more details of the modeling, and at a level that would lack interest for the non-specialist. In addition, many particular aspects of certain problems are not yet sufficiently known, and they still require critical discussions, a fact that further pushes away discussions too specific for our global purposes. We wish simply to show how a unified approach can establish a common basis of representation for all of these situations, how questions of modeling emerge, which aspects are general and which are more specific to different applications, etc. To answer these questions, the first result of this unified approach will be to make certain suggestions that are the outcome of comparisons with the issues of another type of multiphase medium.

We cited granular media and even soils above as examples of multiphase media containing solid phases. Landslides and avalanches of debris are obviously multiphase flows. Soil is generally considered as a solid medium, but it is particularly interesting to study the threshold beyond which this medium flows, completely or, most often, in part, which also poses the problem of determining the expansion of the zone which is flowing. We will show how the unified method presented here contributes a new and useful point of view for granular media. When this medium is in flow, it is easy to see how it might be seen as a particular turbulent multiphase flow. Just before the threshold of flow, the medium shifts and deforms; qualifying it as a turbulent medium is not absolutely appropriate, but the motions are sudden and random (as is the initial structure itself), even if they are of limited amplitude. However, it would be interesting to push the approach toward this limit. We will not be examining every situation here in which the medium is a deformable solid, or those in which one or more liquid or gas phases flow across a porous solid. The fields of soil mechanics and porous media mechanics are well established. It would be interesting to show how certain aspects of these studies are close to the topics we will discuss here, but space does not permit it.

## **1.2. Modeling and its related issues**

The perspective that enables a common theoretical approach begins, of course, with the acknowledgment that multiphase media are similar to continuous media when they are viewed from far enough away that their detailed structure is no longer discernable. These detailed structures, moreover, are not the objective of studies; we



are dealing simply with the phenomena that we witness and undergo at our level. However, the details of these structures have consequences for these phenomena. In these conditions, we must seek to represent the media and phenomena that occur by means of a certain number of characteristic macroscopic, or averaged, variables, and pinpoint the various types of laws they must satisfy. The first task is to adequately define the way to establish these averages. These averaged variables, like all expansive physical variables, satisfy balance equations that must then be written. The detailed structures of the medium and the small-scale phenomena that enter into play within them must be taken into account, or must be modeled, using certain adapted constitutive laws. These laws are usually found with local formulations, but this will not always be the case here; it will be necessary to create new partial differential equations, and in any case they will have to be validated in a fairly general manner through experimentation. In some cases, we will see the usefulness of suggesting several equations for the same medium but that will be valid in more or less general conditions.

Of course, the situation is more complicated for a multiphase medium than for a fluid medium composed of identical molecules. First, it is necessary to be able to monitor within the medium the way in which the different phases move, i.e. to understand the composition of the medium in different places. This is similar to a situation in which the fluid is composed of different molecules. However, these different phases are also liable to have different velocities, temperature and specific agitation energies, and these variables must be clearly defined in an appropriate manner. It will, therefore, be necessary to characterize the medium, even if it is only at a macroscopic level, by more characteristic variables, some of these being new types; for example, we will see the broad utility of the mean interface area per unit of volume. It should be remembered, however, that even if we use more variables, modeling will be only an approximation; the variables we are considering will never contain the whole complexity of the media, but only the aspects that we consider are the most important, and it will be necessary in any case to model complex behaviors that remain hidden.

The methodology of modeling is, of course, similar to that of classic fluid media, which covers the detailed movements of molecules. To avoid straying too far from this example, it is helpful to consider that the characteristic scale of the multiphase structure of the medium is much smaller than the macroscopic scale in which we are interested, that is of the device containing it, as is the case for classic flows. This is the hypothesis effectively advanced for the first attempts at modeling, and it is indeed generally the case when the medium is composed of solid particles widely diluted within a continuous fluid phase. But when we are examining the fragmentation or atomization of a liquid jet into drops in a gaseous flow, the liquid core and the first ligaments or parcels of liquid are of a size comparable to the conduits within which they are moving, while the drops being produced are

expected to be much smaller. For flows in which bubbles of various sizes are transported within a liquid, or flows of an emulsion of two non-miscible liquids, there are conditions in which the bubbles, drops or inclusions are very small, but there are also others in which they extend over the entire diameter of the conduits. It is clear that these latter conditions, which cause the appearance of multiphase structural scales intermingled with the scales of the device being considered, and thus of the flow occurring, exceed the simple situation of very small heterogeneities. Even though these conditions complicate modeling, we must, nevertheless, in our theoretical description, position ourselves so as to address them. The methodology that we have chosen gives us a good perspective from which to approach this problem. The first models resulting from it, which are relatively classical in form, are not immediately well adapted to these situations but we will see how to improve them in this regard.

We have not only talked about multiphase structures but also about a certain degree of randomness; these are not well-ordered structures. The medium permanently displays disordered fluctuations of phase positions and velocity fluctuations, which very often resemble those existing in a turbulent flow. This situation originally occurs with the fact that we cannot completely control the preparation of the medium at the initial instant, or at entry of the flow field, either in terms of phases or in terms of the field of velocities. It is only when a relatively viscous liquid phase exists that this random character of preparation is not amplified during passage through the device. In the vast majority of cases, the initial disorder increases greatly during the flowing of the medium, especially when velocity is high; the spatial and temporal scales of the fluctuations grow to a certain level of saturation that ceases to be controlled solely by the initial and inlet conditions. Phenomena of amplification and saturation of fluctuations are in part the same as those that occur in single-phase turbulent flows. The multiphase character causes additional instabilities in the presence of trajectories of drops or particles (even in a Stokes flow, if more than two particles interact), and instabilities linked to the production or coalescence of drops or to collisions between solid particles enable the different inertias of the different phases to be fully at play. The resulting random and fluctuating medium can be referred to as an extended turbulent medium. It is similar to the classic single-phase turbulent medium, but shows fluctuations of new variables associated with additional phenomena and requires similar methods of modeling.

In addition to the averaged variables we have already mentioned, we will need to define and study the variances of fluctuations of these variables, and even their probability density functions, and find the balance equations they satisfy. In principle, these equations do not directly involve the physical nature of the phenomena that regulate these fluctuations; therefore, they can be used much more widely than for turbulent flows. The closure hypotheses of these equations will

necessarily take into account the nature of the phenomena involved, and it is here that classic hypotheses for turbulence will have to be adapted. Moreover, it will also be necessary to take into account the fact that the small scales of fluctuations, which still retain the particularities of each type of medium being considered, may in this case retain an influence more often than in classic turbulent flows, where the so-called high Reynolds number situations are very common. Obviously, it will be necessary to complicate the representations of these fluctuating flows and the equations to which they lead, that is to build more complex models, but it will be interesting to be able to do this with the same spirit, and perhaps the same tools, as for classical turbulent flows.

We have also stated that various physical or chemical phenomena may occur, particularly, in the internal structure, giving it its specific utility. Momentum exchanges occur between various constituents via the intermediary of forces; exchanges of heat and mass occur as well (phase changes), most often coupled together. Chemical phenomena can also be used beneficially inside a given phase or at the interface between two phases. The approach we are seeking must be general enough to adapt to these diverse physical phenomena.

Part 1 of this book deals with the definition and description of the *basic approach*, which can be presented simply as the generalization to several phases concerning the usual approach used in the mechanics of continuous media to the thermomechanics of fluids in turbulent flow. Its objective is to provide a context of theoretical representation, its hypotheses, its possibilities and its limits, and to identify the different types of equations necessary to make this approach useful, i.e. to draw from it predictions concerning specific flows. The general description includes four aspects: the theoretical description of a piecewise continuous flow; the definition of an averaged or filtered description, usable in practice, and it is statistical averages that will first be used here; the writing of balance equations for averaged quantities that can be chosen to represent the medium in its evolution; and the definition of the necessary constitutive laws, which include equations of state and laws of irreversible processes, and the problem of choosing these. These basic developments are the subject of Chapters 1–4.

Part 2 of the book begins by discussing the principles of modeling the effect of irreversible processes that are active within the flows. There are, in Chapter 5, the irreversible processes linked to exchanges between phases, masses, momentum or energy. We will show the significance of defining and studying exchanges in the form of exchange fluxes per unit of interface area. It is not possible to calculate these exchange fluxes without understanding the detail of the phenomena occurring on a small scale in the immediate vicinity of the interfaces, and it is necessary to suggest approximate models. The specificities of the different multiphase flows must explicitly be involved in this; we will not attempt to examine each interesting

case in detail, or even a single interesting case, but will study a certain number of specific *local exchange configurations* and discuss how they can serve to construct the desired models.

Equally important, there are also additional dispersion fluxes inside a phase, for momentum or energy, that are due to fluctuations, often turbulent in nature, the modeling of which is discussed in Chapter 6. We will see that they may be modeled by generalizing approaches that are usual in single-phase turbulent flows, but there remains at least one major problem to be studied: the influence of the presence of phases on the spectrum of the characteristic length scales. We will also discuss simplification in which the mass of one phase is considered simply as diffusing in the overall multiphase mixture itself, in a manner analogous to a mixture of truly miscible gaseous or liquid components, without examining its own convection velocity in detail.

In Chapter 7, we then focus on the mean interface area per unit of volume, which plays a critical role in all of these questions related to exchanges between phases. The generalization of the approach for filtered averages, such as the large eddy simulation, is discussed in Chapter 8. This approach is, particularly, indicated for more precise study of the presence of large scales of multiphase or dynamic structures, which are avoided in statistical approaches. There is still a lot of work to be done in these areas. To date, models and their approximations have been presented and discussed in relation to the available experimental data. Chapter 9 reconsiders most of these questions with the general viewpoint of thermodynamics of irreversible processes, which will require a generalization of the classic thermodynamic approach. Classically used approximations will be reinforced for the most part, but there will also be interesting openings. Chapters 10 and 11 focus on the experimental methods and results available to date, and on the new aspects to be studied further, after a brief description of the current and new experimental methods.

Part 3 analyzes in more detail the *granular flows and media*, a type of multiphase media that has generated recent interest. In Chapter 12, fluidized beds, which are widely used in industry, are considered as a typical example, and we give a summary of the various models proposed and show how the developments in the preceding chapters are useful to explain them, as well as to emphasize the improvements that are necessary. In Chapters 13–15, we discuss an improvement for dense granular media in which lasting contacts between grains must be taken into account. There is, of course, a regime of flow in such a media, where only short collisions are occurring, but this is a limited case, when the flows are relatively rapid. At the other limit of small displacements, the media can be studied like elastic solid media, and between these two limits one or more other regimes of motion may exist due to the fact that the contacts between two or more grains may last for more

or less time, small deformations and displacements of grains take place, and sudden sliding is possible locally. The multiphase approach is still possible, but it is necessary to consider that each grain is a different solid phase for which we know the constitutive laws, and the search for constitutive laws for a solid phase that would be the ensemble of all of the grains is the center of the modeling issue. These situations are at the limit of the domain of soil mechanics, and we will not go deep into this area. We will confine ourselves to demonstrate how the approach can lead to a form of model in which the stress tensor of the medium is given by a constitutive law in the form of a differential balance equation, like a kind of viscoelastic medium with a threshold.

Part 4 examines the improvement of the representation of all these media and adds the possibility of better understanding the fluctuations around mean values. The fact that we have defined mean variables from the beginning does not actually mean that it is impossible to understand the fluctuations around them. This is possible, first, by simply studying the variances of the fluctuations of variables, which has already been discussed in the preceding chapters for velocity fluctuations. This will now be extended to enable knowledge of the *probability density functions, even joint probability density functions, of the various variables in certain phases*, which is an interesting improvement in several situations. When relatively rapid chemical reactions occur in a gas in the vicinity of liquid droplets that supply one of the reactants, it is absolutely necessary to take into account the strong and fluctuating heterogeneities of composition and temperature that are found in the gas, precisely due to the exchange of chemical species between phases and the chemical reactions locally occurring. In problems where the vaporization of an ensemble of drops takes place, the distribution of temperatures of different drops, if wide enough, can have a significant effect on the mean rate of vaporization. Finally, for the dispersion of particles or droplets, a better understanding of the distribution of fluctuations of velocities and diameters of droplets is very useful.

In Chapter 16, we examine the modeling of the probability density of compositions for the gas phase of a two-phase medium in combustion. Chapter 17 discusses the possibility of better understanding the temperature fluctuations of the liquid phase or of drops or solid particles, which can have significant consequences for phase changes; finally, in Chapter 18, we study the possibility of better understanding the distribution of velocity fluctuations and parcel sizes of a phase. To obtain a better understanding of the distribution of sizes of a phase, we must define different categories of parcels in this phase, with each one behaving like a phase itself, and the treatment of all these virtual phases can thus become quite unwieldy. For situations in which the phases are always highly dispersed, the classically used stochastic Lagrangian–Eulerian simulation method is more appropriate. We will show how this approach connects to the Eulerian method discussed in this book.

Since the field contains problems that are still confined largely to the research level (not only in the last part), we have been careful throughout to emphasize the main subjects in which modeling still has critical flaws and that must be the subject of the most promising effort.

*Literature on the domain*, less general and more focused on one of the applicative aspects, provides additional information and interesting developments. In particular, we will refer more than once to the following works: [SOO 89, DEL 81, NIG 91, OES 06, CRO 01, JAK 08, PRA 06, AND 11].

Finally, the basics of the thermo mechanical description of continuous media used here were taken from [BOR 08a] and [BOR 08b].

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