200 Puzzling Physics Problems

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200 Puzzling Physics Problems is aimed at strengthening a student's grasp of the laws of physics by applying them to situations that are practical and to problems that yield more easily to intuitive insight than to brute-force methods and complex mathematics. The problems are chosen almost exclusively from classical (i.e. non-quantum) physics, but are no easier for that. For the most part, these problems are intriguingly posed in accessible non-technical language. This requires the student to select the right framework in which to analyse the situation and to make decisions about which branches of physics are involved. The general level of sophistication needed to tackle most of the 200 problems is that of the exceptional school student, the good undergraduate or the competent graduate student. The book should be valuable to undergraduates preparing for 'general physics' papers, either on their own or in classes or seminars designed for this purpose. It is even hoped that some physics professors will find the more difficult questions challenging. By contrast, the mathematical demands made are minimal, and do not go beyond elementary calculus. This intriguing book of physics problems should prove not only instructive and challenging, but also fun.

PETER GNÄDIG graduated as a physicist from Roland Eötvös University (ELTE) in Budapest in 1971 and received his PhD in theoretical particle physics there in 1980. Currently, he is a researcher (in high energy physics) and a lecturer in the Department of Atomic Physics at ELTE. Since 1985 he has been one of the leaders of the Hungarian Olympic team taking part in the International Physics Olympiad. He is also the Physics Editor of *KÖMAL*, the 100-year-old Hungarian *Mathematical and Physical Journal of Secondary Schools*, which publishes several challenging physics problems each month, as well as one of the organisers of the formidable Hungarian Physics competition (the Eötvös Competition). Professor Gnädig has written textbooks on the theory of distributions and the use of vector-calculus in physics.

GYULA HONYEK graduated as a physicist from Eötvös University (ELTE) in Budapest in 1975 and finished his PhD studies there in 1977, after which he stayed on as a researcher and lecturer in the Department of General Physics. In 1984, following a two-year postgraduate course, he was awarded a teacher's degree in physics, and in 1985 transferred to the teacher training school at ELTE. His current post is as mentor and teacher at Radnóti Grammar School, Budapest. Since 1986 he has been one of the leaders and selectors of the Hungarian team taking part in the International Physics Olympiad. He is also a member of the editorial board of *KÖMAL*. As a co-author of a physics textbook series for Hungarian secondary schools, he has wide experience of teaching physics at all levels.

KEN RILEY read mathematics at the University of Cambridge and proceeded to a PhD there in theoretical and experimental nuclear physics. He became a research associate in elementary particle physics at Brookhaven, and then, having taken up a lectureship at the Cavendish Laboratory, Cambridge, continued this research at the Rutherford Laboratory and Stanford; in particular, he was involved in the discovery of a number of the early baryonic resonances. As well as being Senior Tutor at Clare College, where he has taught physics and mathematics for over 30 years, he has served on many committees concerned with teaching and examining of these subjects at all levels of tertiary and undergraduate education. He is also one of the authors of *Mathematical Methods for Physics and Engineering* (Cambridge University Press).

Preface

In our experience, an understanding of the laws of physics is best acquired by applying them to practical problems. Frequently, however, the problems appearing in textbooks can be solved only through long, complex calculations, which tend to be mechanical and boring, and often drudgery for students. Sometimes, even the best of these students, the ones who possess all the necessary skills, may feel that such problems are not attractive enough to them, and the tedious calculations involved do not allow their 'creativity' (genius?) to shine through.

This little book aims to demonstrate that not all physics problems are like that, and we hope that you will be intrigued by questions such as:

- How is the length of the day related to the side of the road on which traffic travels?
- Why are Fosbury floppers more successful than Western rollers?
- How far below ground must the water cavity that feeds Old Faithful be?
- How high could the tallest mountain on Mars be?
- What is the shape of the water bell in an ornamental fountain?
- How does the way a pencil falls when stood on its point depend upon friction?
- Would a motionless string reaching into the sky be evidence for UFOs?
- How does a positron move when dropped in a Faraday cage?
- What would be the high-jump record on the Moon?
- Why are nocturnal insects fatally attracted to light sources?
- How much brighter is sunlight than moonlight?
- How quickly does a fire hose unroll?

- How do you arrange two magnets so that the mutual couples they experience are not equal and opposite?
- How long would it take to defrost an 8-tonne Siberian mammoth?
- What perils face titanium-eating little green men who devour their own planet?
- What is the direction of the electric field due to an uniformly charged rod?
- What is the catch in an energy-generating capacitor?
- What is the equivalent resistance of an n-dimensional cube of resistors?
- What factors determine the period of a sand-glass egg timer?
- How does a unipolar dynamo work?
- How 'deep' is an electron lying in a box?

These, and some 180 others, are problems that can be solved elegantly by an appropriate choice of variables or coordinates, an unusual way of thinking, or some 'clever' idea or analogy. When such an inspiration or eureka moment occurs, the solution often follows after only a few lines of calculation or brief mental reasoning, and the student feels justifiably pleased with him- or herself.

Logic in itself is not sufficient. Nobody can guess these creative approaches without knowing and understanding the basic laws of physics. Accordingly, we would not encourage anybody to tackle these problems without first having studied the subject in some depth. Although successful solutions to the problems posed are clearly the principal goal, we should add that success is not to be measured by this alone. Whatever help you, the reader, may seek, and whatever stage you reach in the solution to a problem, it will hopefully bring you both enlightenment and delight. We are sure that some solutions will lead you to say 'how clever', others to say 'how nice', and yet others to say 'how obvious or heavy-handed'! Our aim is to show you as many useful 'tricks' as possible in order to enlarge your problem-solving arsenal. We wish you to use this book with delight and profit, and if you come across further similar 'puzzling' physics problems, we would ask you to share them with others (and send them to the authors).

The book contains 200 interesting problems collected by the authors over the course of many years. Some were invented by us, others are from the Hungarian 'Secondary School Mathematics and Physics Papers' which span more than 100 years. Problems and ideas from various Hungarian and international physics contests, as well as the Cambridge Colleges' entrance examination, have also been used, often after rewording. We have also been

guided by the suggestions and remarks of our colleagues. It is impossible to determine the original authors of most of the physics problems appearing in the international 'ideas-market'. Nevertheless, some of the inventors of the most puzzling problems deserve our special thanks. They include Tibor Bíró, László Holics, Frederick Károlyházy, George Marx, Ervin Szegedi and István Varga. We thank them and the other people, known and unknown, who have authored, elaborated and improved upon 'puzzling' physics problems.

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How to use this book

The following chapter contains the *problems*. They do not follow each other in any particular thematic order, but more or less in *order of difficulty*, or in groups requiring *similar methods of solution*. In any case, some of the problems could not be unambiguously labelled as belonging to, say, mechanics or thermodynamics or electromagnetics. Nature's secrets are not revealed according to the titles of the sections in a text book, but rather draw on ideas from various areas and usually in a complex manner. It is part of our task to find out what type of problem we are facing. However, *for information*, the reader can find a list of topics, and the problems that more or less belong to these topics, on the following page. Some problems are listed under more than one heading. The symbols and numerical values of the principal physical constants are then given, together with astronomical data and some properties of material.

The majority of the problems are not easy; some of them are definitely difficult. You, the reader, are naturally encouraged to try to solve the problems on your own and, obviously, if you do, you will get the greatest pleasure. If you are unable to achieve this, you should not give up, but turn to the relevant page of the short *hints* chapter. In most cases this will help, though it will not give the complete solution, and the details still have to be worked out. Once you have done this and want to check your result (or if you have completely given up and only want to see the *solution*), the last chapter should be consulted.

Problems whose solutions require similar reasoning usually follow each other. But if a particular problem relates to another elsewhere in this book, you will find a cross-reference in the relevant hint or solution. Those requiring especially difficult reasoning or mathematically complicated calculations are marked by one or two *asterisks*.

Some problems are included whose solutions raise further questions that are beyond the scope of this book. Points or issues worth further consideration are indicated at the end of the respective solutions, but the answers are not given.

Thematic order of the problems

Kinematics: 1, 3, 5, 36, 37, 38*, 40, 41, 64, 65*, 66, 84*, 86*.

Dynamics: 2, 7, 8, 12, 13, 24, 32*, 33, 34, 35, 37, 38*, 39*, 70*, 73*, 77, 78*, 79*, 80**, 82, 83, 85**, 90, 154*, 183*, 184*, 186*, 193*, 194.

Gravitation: 15, 16, 17, 18, 32*, 81**, 87, 88, 109, 110*, 111*, 112*, 116, 134*.

Mechanical energy: 6, 7, 17, 18, 32*, 51, 107.

Collisions: 20, 45, 46, 47, 48, 71, 72*, 93, 94, 144*, 194, 195.

Mechanics of rigid bodies: 39*, 42**, 58, 60*, 61**, 94, 95*, 96, 97*, 98, 99**.

Statics: 9, 10*, 11, 14*, 25, 26, 43, 44, 67, 68, 69*.

Ropes, chains: 4, 67, 81**, 100, 101*, 102**, 103*, 104*, 105**, 106*, 108**.

Liquids, gases: 19, 27, 28, 49*, 50, 70*, 73*, 74, 75*, 91*, 115**, 143, 200.

Surface tension: 29, 62, 63, 129, 130*, 131*, 132**, 143, 199*.

Thermodynamics: 20, 21*, 133, 135**, 136, 145, 146*, 147, 148.

Phase transitions: 134*, 137*, 138, 140*, 141*.

Optics: 52, 53, 54, 55, 56, 125*, 126, 127, 128*.

Electrostatics: 41, 90, 91*, 92, 113*, 114, 117*, 118, 121, 122, 123*, 124*, 149, 150, 151*, 152, 155, 156, 157, 183*, 192*, 193*.

Magnetostatics: 89**, 119, 120**, 153*, 154*, 172, 186*.

Electric currents: 22, 23, 158, 159, 160*, 161, 162*, 163*, 164*, 165, 169, 170*, 172.

Electromagnetism: 30, 31, 166, 167, 168*, 171*, 173*, 174*, 175*, 176, 177, 178*, 179, 180, 181*, 182*, 184*, 185*, 186*, 187*.

Atoms and particles: 93, 188, 189*, 190*, 191, 194, 195, 196, 197*, 198*.

Dimensional analysis, scaling, estimations: 15, 57, 58, 59*, 76*, 77, 126, 139, 142, 185*, 199*.

*,** A single or double asterisk indicates those problems that require especially difficult reasoning or mathematically complicated calculations.

Physical constants

Gravitational constant, G	$6.673 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Speed of light (in vacuum), c	$2.998 \times 10^8\,\mathrm{m\,s^{-1}}$
Elementary charge, e	$1.602 \times 10^{-19}\,\mathrm{C}$
Electron mass, m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
Proton mass, m_p	$1.673 \times 10^{-27}\,\mathrm{kg}$
Boltzmann constant, k	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Planck constant, h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
Avogadro constant, N_A	$6.022 \times 10^{23}\,\mathrm{mol^{-1}}$
Gas constant, R	$8.315\,\mathrm{J\,mol^{-1}K^{-1}}$
Permittivity of free space, ε_0	$8.854 \times 10^{-12}\,\mathrm{C\,V^{-1}\,m^{-1}}$
Coulomb constant, $k = 1/4\pi\varepsilon_0$	$8.987 \times 10^9\,\mathrm{V\,m\,C^{-1}}$
Permeability of free space, μ_0	$4\pi \times 10^{-7}\,\mathrm{V\,s^2\,C^{-1}\,m^{-1}}$

Some astronomical data

Mean radius of the Earth, R	$6371\,\mathrm{km}$
Sun–Earth distance (Astronomical Unit, AU)	$1.49 \times 10^8\,\mathrm{km}$
Mean density of the Earth, ρ	$5520\,\mathrm{kg\,m^{-3}}$
Free-fall acceleration at the Earth's surface, g	$9.81\,\mathrm{m\,s^{-2}}$

Some physical properties

Surface tension of water, γ	$0.073\,\mathrm{N\,m^{-1}}$
Heat of vaporisation of water, L	$2256\,\mathrm{kJ\,kg^{-1}} = 40.6\,\mathrm{kJ\,mol^{-1}}$
Tensile strength of steel, σ	500–$2000\,\mathrm{MPa}$

Densities[a], ρ $(\mathrm{kg\,m^{-3}})$

Hydrogen	0.0899	Titanium	4510
Helium	0.1786	Iron	7860
Air	1.293	Mercury	13 550
Water (at 4 °C)	1000	Platinum	21 450

[a] Densities quoted in normal state.

Optical Refractive Indices[b], n

Water	1.33	Glass	1.5–1.8
Ice	1.31	Diamond	2.42

[b] At $\lambda = 590\,\mathrm{nm}$.

Contents

Problems

P1 Three small snails are each at a vertex of an equilateral triangle of side 60 cm. The first sets out towards the second, the second towards the third and the third towards the first, with a uniform speed of 5 cm min^{-1}. During their motion each of them always heads towards its respective target snail. How much time has elapsed, and what distance do the snails cover, before they meet? What is the equation of their paths? If the snails are considered as point-masses, how many times does each circle their ultimate meeting point?

P2 A small object is at rest on the edge of a horizontal table. It is pushed in such a way that it falls off the other side of the table, which is 1 m wide, after 2 s. Does the object have wheels?

P3 A boat can travel at a speed of 3 m s^{-1} on still water. A boatman wants to cross a river whilst covering the shortest possible distance. In what direction should he row with respect to the bank if the speed of the water is (i) 2 m s^{-1}, (ii) 4 m s^{-1}? Assume that the speed of the water is the same everywhere.

P4 A long, thin, pliable carpet is laid on the floor. One end of the carpet is bent back and then pulled backwards with constant unit velocity, just above the part of the carpet which is still at rest on the floor.
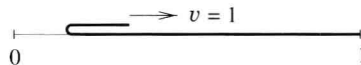

Find the speed of the centre of mass of the moving part. What is the minimum force needed to pull the moving part, if the carpet has unit length and unit mass?

P5   Four snails travel in uniform, rectilinear motion on a very large plane surface. The directions of their paths are random, (but not parallel, i.e. any two snails could meet), but no more than two snail paths can cross at any one point. Five of the $(4 \times 3)/2 = 6$ possible encounters have already occurred. Can we state with certainty that the sixth encounter will also occur?

P6   Two 20-g flatworms climb over a very thin wall, 10 cm high. One of the worms is 20 cm long, the other is wider and only 10 cm long. Which of them has done more work against gravity when half of it is over the top of the wall? What is the ratio of the amounts of work done by the two worms?

P7   A man of height $h_0 = 2$ m is bungee jumping from a platform situated a height $h = 25$ m above a lake. One end of an elastic rope is attached to his foot and the other end is fixed to the platform. He starts falling from rest in a vertical position.
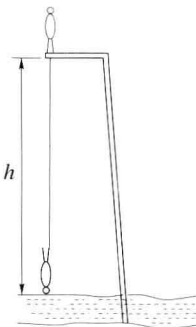
The length and elastic properties of the rope are chosen so that his speed will have been reduced to zero at the instant when his head reaches the surface of the water. Ultimately the jumper is hanging from the rope, with his head 8 m above the water.

(i) Find the unstretched length of the rope.
(ii) Find the maximum speed and acceleration achieved during the jump.

P8   An iceberg is in the form of an upright regular pyramid of which 10 m shows above the water surface. Ignoring any induced motion of the water, find the period of small vertical oscillations of the berg. The density of ice is 900 kg m^{-3}.

P9   The suspension springs of all four wheels of a car are identical. By how much does the body of the car (considered rigid) rise above each of the wheels when its right front wheel is parked on an 8-cm-high pavement? Does the result change when the car is parked with both right wheels on

the pavement? Does the result depend on the number and positions of the people sitting in the car?

P10* In Victor Hugo's novel *les Misérables*, the main character Jean Valjean, an escaped prisoner, was noted for his ability to climb up the corner formed by the intersection of two vertical perpendicular walls. Find the minimum force with which he had to push on the walls whilst climbing. What is the minimum coefficient of static friction required for him to be able to perform such a feat?

P11 A sphere, made of two non-identical homogeneous hemispheres stuck together, is placed on a plane inclined at an angle of 30° to the horizontal. Can the sphere remain in equilibrium on the inclined plane?

P12 A small, elastic ball is dropped vertically onto a long plane inclined at an angle α to the horizontal. Is it true that the distances between consecutive bouncing points grow as in an arithmetic progression? Assume that collisions are perfectly elastic and that air resistance can be neglected.

P13 A small hamster is put into a circular wheel-cage, which has a frictionless central pivot. A horizontal platform is fixed to the wheel below the pivot. Initially, the hamster is at rest at one end of the platform.
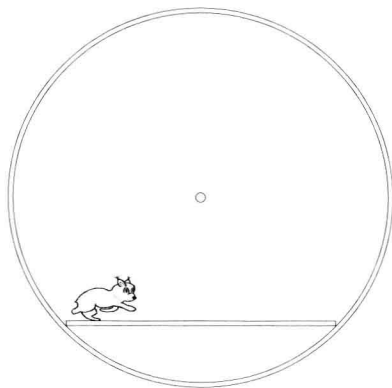

When the platform is released the hamster starts running, but, because of the hamster's motion, the platform and wheel remain *stationary*. Determine how the hamster moves.

P14* A bicycle is supported so that it is prevented from falling sideways but can move forwards or backwards; its pedals are in their highest and lowest positions. A student crouches beside the bicycle and applies a horizontal force, directed towards the back wheel, to the lower pedal.

(i) Which way does the bicycle move?

  (ii) Does the chain-wheel rotate in the same or opposite sense as the rear wheel?

  (iii) Which way does the lower pedal move relative to the ground?

P15 If the solar system were proportionally reduced so that the average distance between the Sun and the Earth were 1 m, how long would a year be? Take the density of matter to be unchanged.
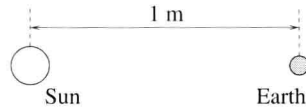
P16 If the mass of each of the members of a binary star were the same as that of the Sun, and their distance apart were equal to the Sun–Earth distance, what would be their period of revolution?

P17 (i) What is the minimum launch speed required to put a satellite into a circular orbit?

  (ii) How many times higher is the energy required to launch a satellite into a polar orbit than that necessary to put it into an Equatorial one?

  (iii) What initial speed must a space probe have if it is to leave the gravitational field of the Earth?

  (iv) Which requires a higher initial energy for the space probe – leaving the solar system or hitting the Sun?

P18 A rocket is intended to leave the Earth's gravitational field. The fuel in its main engine is a little less than the amount that is necessary, and an auxiliary engine, only capable of operating for a short time, has to be used as well. When is it best to switch on the auxiliary engine: at take-off, or when the rocket has nearly stopped with respect to the Earth, or does it not matter?

P19 A steel ball with a volume of 1 cm^3 is sinking at a speed of 1 cm s^{-1} in a closed jar filled with honey. What is the momentum of the honey if its density is 2 g cm^{-3}?
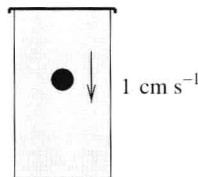
P20 A gas of temperature T is enclosed in a container whose walls are (initially) at temperature T_1. Does the gas exert a higher pressure on the walls of the container when $T_1 < T$ or when $T_1 > T$?

P21* Consider two identical iron spheres, one of which lies on a thermally insulating plate, whilst the other hangs from an insulating thread.
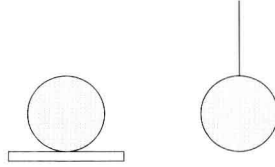
Equal amounts of heat are given to the two spheres. Which will have the higher temperature?

P22 Two (non-physics) students, A and B, living in neighbouring college rooms, decided to economise by connecting their ceiling lights in series. They agreed that each would install a 100-W bulb in their own rooms and that they would pay equal shares of the electricity bill. However, both decided to try to get better lighting at the other's expense; A installed a 200-W bulb and B installed a 50-W bulb. Which student subsequently failed the end-of-term examinations?

P23 If a battery of voltage V is connected across terminals I of the black box shown in the figure, a voltmeter connected to terminals II gives a reading of $V/2$; while if the battery is connected to terminals II, a voltmeter across terminals I reads V.

The black box contains only passive circuit elements. What are they?

P24 A bucket of water is suspended from a fixed point by a rope. The bucket is set in motion and the system swings as a pendulum. However, the bucket leaks and the water slowly flows out of the bottom of it. How does the period of the swinging motion change as the water is lost?

P25 An empty cylindrical beaker of mass 100 g, radius 30 mm and negligible wall thickness, has its centre of gravity 100 mm above its base. To what depth should it be filled with water so as to make it as stable as possible?