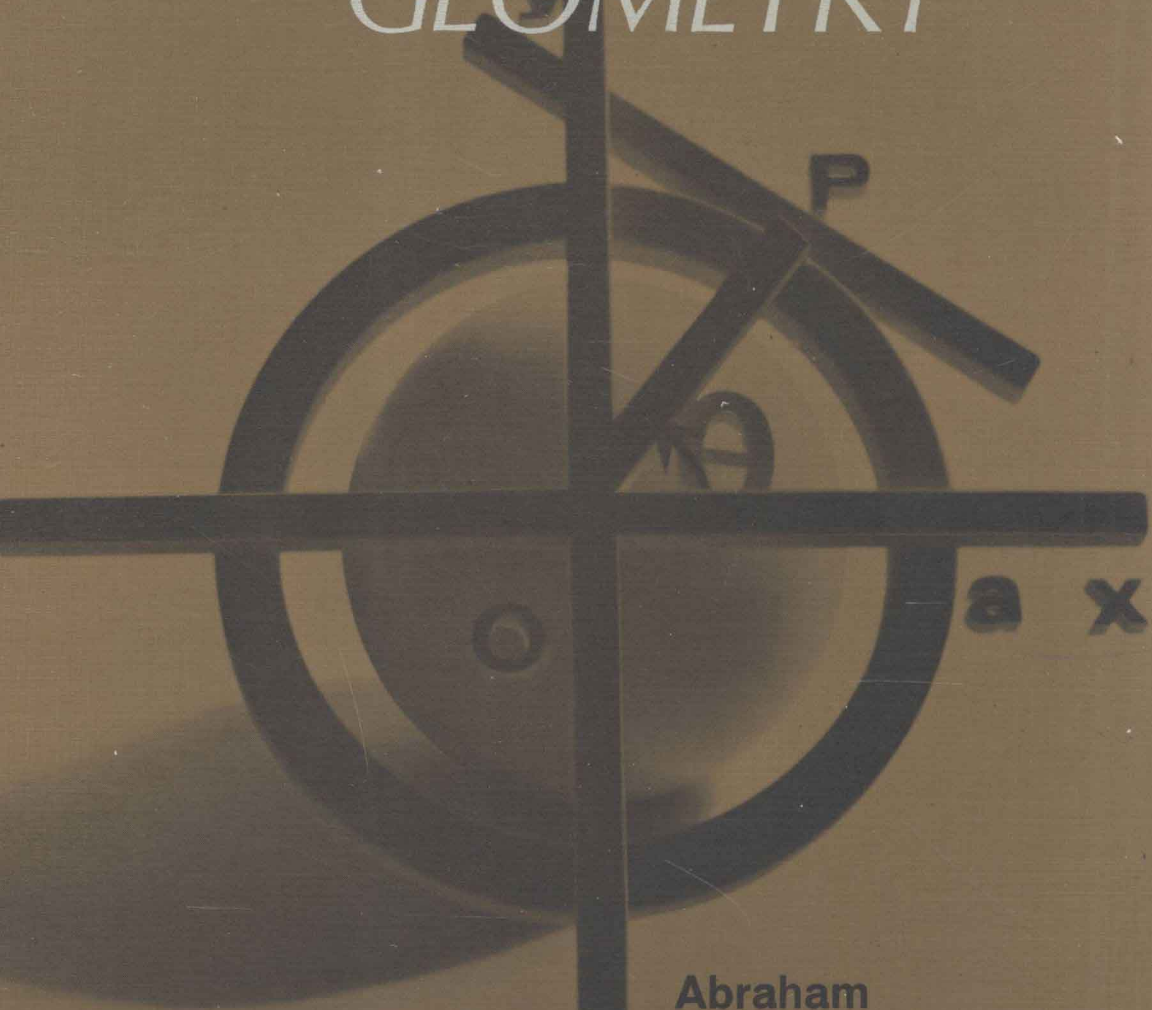
SECOND EDITION

CALCULUS and ANALYTIC GEOMETRY



Abraham Schwartz

CALCULUS AND ANALYTIC GEOMETRY

SECOND EDITION

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CALCULUS AND ANALYTIC GEOMETRY

TO MY PARENTS

PREFACE TO THE SECOND EDITION

This edition was written with the same objectives and in the same spirit as the first edition. The organization of the course has been changed only to add a chapter on ordinary differential equations at the end. That chapter discusses topics usually studied in a first course and attempts to make the student aware of questions of existence and uniqueness.

The illustrative examples and the exercises of the first edition seem to have been quite successful, and they have been changed in relatively few places, usually for the purpose of adding some easier exercises. Many three-dimensional figures have been improved in detail, but they are still as simple as possible so that the student can hope to copy from them and thus to improve his own sketches. There have been hundreds of minor changes in exposition in places where experience has shown that particular passages were not quite as clear to the student as had been expected. It is believed that the definition of function in Chapter 1 has been made more rigorous, with subsequent improvements throughout the book, and that the first treatment of integration in Chapter 2 has also been improved.

More instructors and students than I can mention have made encouraging, useful, and significant comments, and I am grateful to all of them. I would especially like to acknowledge here the help I received from Joseph d' Atri, Sidney Neuman, and Bennington Gill, and the comments of J. L. Baker, H. J. Cohen, Gerald Freilich, Edwin Goldfarb, Alvin Hausner, Solomon Hurwitz, Frank Kocher, Henry Malin, T. O. Moore, John Shaw, Fritz Steinhardt, and Fred Supnick.

A.S.

Englewood, New Jersey
February 1967

PREFACE TO THE FIRST EDITION

Three principles guided the writing of this book. First, the student must be able to read his textbook and learn from it. The exposition therefore is detailed and carefully motivated, and there are many illustrative examples. Second, the student should be led to reason carefully and to write precisely. Definitions and theorems are carefully stated, and the theorems are proved with as much rigor as was deemed feasible for a first-year college course. Where no proof is given this is indicated and, if possible, the difficulty pointed out. Third, not only are the ideas of the course important, but also the ability to apply them to specific situations. The author tried to maintain a reasonable balance between theory on the one hand and technique, drill, and application on the other.

The book assumes training on the secondary school level in trigonometry and advanced algebra; there are, however, articles on inequalities and absolute values in the appendix and review articles on the trigonometric functions and the logarithmic function in the text itself.

The book starts with chapters on the differential and integral calculus which rest on an intuitive basis rather than an abstract basis. These chapters are also intended to give the student enough technique and experience with applications to start him in a good physics course. Next, a long chapter on plane analytic geometry introduces vector analysis early and uses the calculus tool previously prepared in a significant role. There follow chapters on the trigonometric, logarithmic, and exponential functions, and a chapter on formal integration.

By this time the student has perhaps gained a certain mathematical maturity and it is wise to examine more closely the underlying assumptions made earlier in the book. No two instructors will place the same relative emphasis on the various ideas of Chapter 7; indeed, only Part 1 of Section 7.5 is indispensable for the reading of the later chapters. Even though most students will find this more abstract theory difficult, the author firmly believes that it should be attempted.

Chapter 8 returns to more applications of the calculus tool, and polar coordinates are studied in Chapter 9. Chapter 10 is devoted to solid analytic geometry, the vector analysis again playing a prominent role. Here, for the first time in this text, the student has to deal with three-dimensional configurations, and an attempt is made to help him with his sketches *before* he is called upon to use them in various theories and applications.

There then follow chapters on the calculus of functions of two or more variables and on applications which use three-dimensional geometry. The last chapter is devoted to infinite series.

The ideas and techniques considered are not of equal difficulty and significance and the sections are not all of equal length. It is not intended that a class consider every part of every section at a uniform rate of one section per lesson. Different instructors will wish to emphasize different points and many of the longer sections are subdivided to allow the instructor more flexibility in making his assignments.

A special effort has been made to supply original exercises and many more are included than any one class can use. These have been carefully graded so that the instructor can select assignments according to the emphasis he wishes to place on the ideas covered. More difficult exercises are included in most sections to motivate the better student and to teach him something significant and new. The answers for many exercises are given in the exercises themselves. For those more formal exercises, which are subdivided into many parts, answers are usually given in the answer section at the back of the book for the alternate parts — (a), (c), (e), etc. The other answers given in the back of the book are for the odd-numbered exercises.

It is difficult to trace the development of one's own ideas about a course of study, but I am keenly aware of how much I have been influenced by my colleagues and students, both at the Pennsylvania State University and at the City College of New York.

It is a pleasure to thank Professors Burton W. Jones, Melvin Henriksen, and Andrew J. Terzuoli, who read the manuscript and made valuable suggestions, and Mrs. Olga Skelley, who typed the manuscript.

Englewood, New Jersey
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A. S.

CALCULUS AND ANALYTIC GEOMETRY

CONTENTS

1

The Derivative

1.1 The Definition of a Function

Science studies correspondences between sets of numbers. Thus there may be a relationship between the viscosity η of a certain oil and the temperature T; perhaps with each T number selected from a certain suitable temperature interval we can associate a corresponding viscosity number η. The number N of bacteria in a culture changes as the time t changes; perhaps one can explain how to compute an N number for each t number selected from a certain suitable time interval. The current i flowing in a circuit might depend on the resistance R of the circuit. The volume V of a sphere is related to its radius r.

In describing such correspondences the mathematician uses the word "function."

DEFINITION 1

Function; domain; range. We are given a set of numbers, which we shall call the domain D, and instructions for associating a number y with each number x of D. The set of all numbers y associated with numbers x of D shall be called the range R. The correspondence thus created between the sets D and R shall be called a function.

Example 1. The formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, associates one number V with each number $r \geq 0$.* For instance, corresponding to $r = 3$ we have $V = \frac{4}{3}\pi\, 3^3 = 36\pi$. The domain D for this function is the set $D : r \geq 0$; the instructions for associating a V number with each r number of D are furnished by the formula; the range is the set $R : V \geq 0$, because any V number of this range can be achieved by choosing a suitable r number of D.

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* To review inequalities, see Appendix 1.

Example 2. Consider the algebraic instructions furnished by $y = \sqrt{25 - x^2}$.* Here one real number y is associated with each number x of the interval $-5 \leq x \leq 5$. For instance, with $x = 4$ we associate $y = 3$. The domain D for this function is the interval $-5 \leq x \leq 5$, the instructions for associating a y number with each x number of D are furnished by the formula, and the range R is the set $0 \leq y \leq 5$.

Example 3. Consider $y = \sqrt{x^2}$ for all real x. The instructions for finding the number y associated with a particular x call for squaring the x number and then taking the positive square root. For instance, when $x = -2$ we compute $y = \sqrt{(-2)^2} = \sqrt{4} = 2$. These instructions could also have been written

$$y = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0, \end{cases}$$

or as $y = |x|$.† The domain D for this function is the set of all real numbers; the range R is the set of all nonnegative numbers.

Example 4. Consider

$$C = \begin{cases} 35 \text{ for } 0 \leq d < 1/5 \\ 40 \text{ for } 1/5 \leq d < 2/5 \\ 45 \text{ for } 2/5 \leq d < 3/5 \\ 50 \text{ for } 3/5 \leq d < 4/5 \\ 55 \text{ for } 4/5 \leq d < 1. \end{cases}$$

This set of instructions associates a specific number C with each number d of the set $0 \leq d < 1$. Thus, for $d = .24$ we are instructed to take $C = 40$ and for $d = .60$ we are instructed to take $C = 50$. The domain D for this function is the set $0 \leq d < 1$; the range R is the set of five numbers $\{35, 40, 45, 50, 55\}$. The number C might represent the cost of a taxi ride in cents and d the distance traveled in miles.

Example 5. Consider

$$y = \begin{cases} \dfrac{1}{1 + 2^{1/x}} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$$

Here the domain D is the set of all real numbers and we have instructions for associating a number y with any real x. For $x = 1/3$, for instance, the first line of the instructions tells us to take

$$y = \frac{1}{1 + 2^3} = \frac{1}{9};$$

* Recall that the radical notation conventions require that the positive square root be taken here. If both positive and negative square roots are intended, as in the familiar quadratic formula, then both signs must be written. The roots of $ax^2 + bx + c = 0$ are $x = (-b \pm \sqrt{b^2 - 4ac})/2a$.

† The absolute value symbol, $|\ |$, is considered in Appendix 2.