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MODERN STRUCTURAL ANALYSIS

THE MATRIX METHOD APPROACH

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The Matrix Method Approach

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Conversion of SI Units to USCS Units

Quantity	SI unit	Conversion factor	USCS unit
Area	square meter, m^2 square milimeter, mm^2	10.76391 0.001550	square foot, ft^2 square inch, in^2
Energy	joule, J megajoule, MJ joule, J	0.737561 0.277778 0.0009478	foot-pound, ft·lb kilowatthour, kWh British thermal unit, Btu
Force	newton†, N kilonewton, kN	0.22481 0.22481	pound, lb kip (1000 pounds)
Length	meter, m millimeter, mm kilometer, km	3.28084 0.03937 0.6213722	foot, ft inch, in mile, mi
Mass	kilogram, kg	0.068522	slug, lb · s^2/ft
Moment	newton-meter, N·m newton-meter, N·m kilonewton-meter, kN·m	0.73756 8.85073 0.73756	foot-pound, ft·lb inch-pound, in·lb foot-kip, ft·kip
Power	watt, W watt, W	0.737561 0.001341	foot-pound per second, ft·lb/s horsepower, hp
Stress (pressure)	pascal, Pa megapascal, MPa	0.0208854 145.04	pounds per square foot, lb/ft^2 pounds per square inch, lb/in^2
Temperature	degrees Celsius, °C	1.8°C + 32	degrees Fahrenheit, °F
Volume	cubic meter, m^3 cubic millimeter, mm^3	35.3147 61.0236×10^{-6}	cubic foot, ft^3 cubic inch, in^3

† A newton is the force required to accelerate a 1-kg mass by a constant acceleration of 1 m/s^2. A pascal is equal to 1 N/m^2.

NOTE: To convert USCS units to SI units divide by the conversion factor. For temperature, °C = (5/9)(°F − 32).

SI Prefixes

Prefix	Symbol	Multiplication factor	Prefix	Symbol	Multiplication factor
tera	T	10^{12}	milli	m	10^{-3}
giga	G	10^{9}	micro	μ	10^{-6}
mega	M	10^{6}	nano	n	10^{-9}
kilo	k	10^{3}	pico	p	10^{-12}

Modern Structural Analysis

To the memory of my mother,
Euterpe Sakis-Armenàka,
for her immense kindness and generosity

Preface

This is the second of a series of two books which cover in a unified way the most important methods for analyzing framed structures. The first book is entitled *Classical Structural Analysis: A Modern Approach,* and as its title indicates it is devoted entirely to the classical methods. These methods are best suited for analyzing relatively simple structures by hand calculations. This second book is devoted to the modern (matrix) methods for analyzing framed structures. These methods have been developed in the last 30 years and are best suited for writing programs for analyzing framed structures by computer.

These two books have been written as reference texts for practicing engineers and could be used as texts for undergraduate and graduate courses in structural analysis. The knowledge required for studying them is contained in basic books in statics, strength of materials, calculus, and elements of matrix algebra. In each section of these books, the presentation of the pertinent theory is followed by a number of solved examples which contribute to a better understanding of the theory and illustrate its application. The structures analyzed in these examples have only a few unknown quantities and thus lengthy calculations are avoided. However, these structures can be readily analyzed employing one of the more physically obvious classical methods and, consequently, the advantage of the matrix methods may not always become apparent. A number of photographs of interesting structures with a brief description and sketches of important details are presented in the two books. They bring to the attention of the reader some of the interesting structural or aesthetic features of these structures, as well as ingenious aspects of their construction.

This book is divided into six parts. Part 1 consists of Chaps. 1 to 3. In Chap. 1 certain preliminary concepts are presented, including definitions of terminology, sign conventions, and a discussion of the idealizations and assumptions made in the analysis of framed structures. In Chap. 2 the fundamental relations of the mechanics of materials theories which are pertinent to the analysis of framed structures are established, and the strong forms of the boundary-value problems for computing the components of displacement and the internal forces and moments of the elements of framed structures are formulated. In Chap. 3 the modern methods (direct stiffness and modern flexibility) for analyzing framed structures are introduced by applying them to a simple example. Our aim in this chapter is to demonstrate the salient features of these methods and to describe the important steps which must be followed when applying them, without delving into details. Moreover, we indicate some of the advantages and disadvantages of these methods.

In modern structural analysis a framed structure is considered to be an assemblage of one-dimensional (line) elements whose ends are connected to a number of points called *nodes*. The response of a structure is determined from the response of its elements. Thus in Part 2 (Chaps. 4 to 6) the matrices and the equations which are used to describe the response of an element are established, while in Parts 3 and 4 methods for assembling and solving the equations which describe the response of a structure are presented.

In modern structural analysis the response of an element is regarded as the sum of its response when subjected to the given loads acting along its length with its ends fixed and its response when subjected only to the displacements of its ends. In the first case the response of an element is described by its fixed-end actions (see Chap. 4). In the second case the response of an element is described either by its stiffness equations (see Chap. 4) or by its flexibility equations (see Chap. 5). In Chap. 6 the transformation matrices are formed for transforming the components of internal actions and of displacements of an element from local to global and vice versa.

Part 3 consists of Chaps. 7 to 14 and is devoted to the direct stiffness or direct displacement method. This method is used in practice almost exclusively when writing programs for analyzing framed structures by computer. In this method the analysis of a structure (statically determinate or indeterminate) is formulated in terms of the components of displacements of its nodes.

In modern structural analysis the response of a structure is regarded as the sum of its response when subjected to the given loads with its nodes fixed and its response when subjected to equivalent actions on its nodes, that is, to the concentrated forces and moments which, when applied to the nodes of a structure, displace each one of them by an amount equal to the displacement of the corresponding node of the structure subjected to the given loads. Chapter 7 is devoted to the analysis of structures subjected to given loads with their nodes fixed and to the computation of the equivalent actions. The response of a structure subjected to equivalent actions on its nodes is expressed either by its stiffness equations or by its flexibility equations. In Chap. 8 methods for assembling the stiffness equations for a structure directly from the stiffness equations for its elements are presented. In Chap. 9 the boundary conditions of the structure are introduced into its stiffness equations, which are then solved to give the components of displacements of the nodes of the structure and its reactions. Subsequently, the components of displacements of the nodes of the structure are used to compute the internal actions acting on the ends of its elements. In Chap. 10 the direct stiffness method is extended to structures having skew supports or other special constraints. In Chap. 11 an effective procedure for programming the analysis of framed structures using the direct stiffness method is described.

Chapters 7 to 9, due to their introductory nature, cover only the basic procedures used in the direct stiffness method. Chapters 10 and 12 to 14 cover some of the special procedures which are essential for the efficient solution of the out-of-the-ordinary problems encountered in practice. In Chap. 12 a method is presented for condensing the stiffness equations for a structure by eliminating a number of specific unknown components of displacements of its nodes. Moreover, a method is presented for computing the approximate response of elements of complex geometry and loading. Chapter 13 is devoted to the method of substructures. In this method a structure is subdivided into parts, referred to as substructures, each substructure is analyzed, and the results are combined to obtain the components of displacements of the nodes of the structure. In Chap. 14 procedures for analyzing redesigned structures are presented, using the results of the analysis of the original structure.

Part 4 consists of Chaps. 15 to 17 and is devoted to the modern flexibility or modern force method. In this method the procedures used to analyze statically determinate structures differ from those used to analyze statically indeterminate structures. In Chap. 15 systematic procedures are presented for writing (1) the equations of equilibrium for the nodes of a structure and (2) the equations of compatibility of the components of displacements of the ends of each element of a structure with the components of displacements of the nodes, to which the element is connected. In Chap. 16 the components of displacement of statically determinate structures are computed by using the flexibility and stiffness methods. The procedures described in Chaps. 15 and 16 can be used very effectively to write programs for analyzing statically determinate structures by computer. However, such programs are of rather limited scope and, consequently, practical application. In Chap. 17 the modern flexibility method for analyzing statically indeterminate framed structures is presented. In this method the analysis of a structure is formulated in terms of some of its reactions and/or internal actions (redundants). Up to now this method has found little practical use in writing general computer programs for analyzing statically indeterminate structures. A major reason for this is that the choice of the unknown internal actions and/or reactions (redundants) is not unique and it affects the stability of the matrices which must be inverted in order to analyze a statically indeterminate structure. Thus it is possible that the modern flexibility method could find more practical applications when effective procedures are established for obtaining the optimum choice of redundants by computer.

In Part 2 we establish the response of an element by solving the differential equations of equilibrium for the segments of the element and by satisfying its boundary conditions. In Parts 3 and 4 we establish the response of a framed structure by directly satisfying the requirements for equilibrium of its nodes and for compatibility of the components of displacements of the ends of its elements with the components of displacements of its nodes. In Part 5 we establish the response of an element and of a structure by using the principle of virtual work.

The principle of virtual work for a body represents an integral form of the boundary-value problem for computing the components of displacement strain and stress of this body. Integral forms of boundary-value problems have been used extensively in constructing approximate solutions for them (for example, weighted residual and finite-element methods). In Chap. 18 the principle of virtual work for a body is derived and specialized to framed structures. In Chap. 19 the finite-element method is described and applied in conjunction with the principle of virtual work to establish approximate stiffness equations and matrices of fixed-end actions for elements of framed structures. Moreover, the principle of virtual work is employed to establish exact formulas for the flexibility coefficients for certain types of elements, including tapered and curved. In Chap. 20 the principle of virtual work is employed to (1) compute a component of displacement of a point of a framed structure (method of virtual work or dummy load method) and (2) obtain the stiffness equations for a structure.

The last part of this book contains two appendices. Appendix A is a brief introduction to the concept and applications of the functions of discontinuity. Appendix B is an outline of the elements of vector analysis used in this book.

The author is deeply appreciative of and forever grateful to his valued colleague, his late wife Stella, who provided inestimable assistance and indefatigable support during the preparation of this book. Moreover, the author wishes to express his appreciation and thanks to Dr. Theodore Balderes of Grumman Aerospace Corp. for reading the entire manuscript and making many valuable suggestions, and to Professor H. H. Pan of Polytechnic University and Professors John T. Katsikadelis, Vlasis C. Koumousis, and M. Papadrakakis of the National Technical University of Athens for reading parts of the manuscript and making helpful comments.

This book was written during the time the author was professor and director of the Institute of Structural Analysis of the National Technical University of Athens. Miss Dia Troullinou typed the first draft of the manuscript and Mrs. Evgenia Kapou typed the revised manuscript. Their superior ability and patience is greatly appreciated.

Anthony E. Armenàkas

Partial List of Symbols

A	Area.
A_e	Area of element e.
$\{A\}$ or $\{\bar{A}\}$	Local or global matrix of the nodal actions, respectively, of an element of a structure subjected to given loads. For an element of a planar truss in the x_1x_2 plane $\{A\}^T = [F_1^j \quad F_1^k]$, while $\{\bar{A}\}^T = [\bar{F}_1^j \quad \bar{F}_2^j \quad \bar{F}_1^k \quad \bar{F}_2^k]$; for an element of a planar beam or frame in the x_1x_2 plane $\{A\}^T = [F_1^j \quad F_2^j \quad M_3^j \quad F_1^k \quad F_2^k \quad M_3^k]$; for an element of a space beam or frame $\{A\}^T = [F_1^j \quad F_2^j \quad F_3^j \quad M_1^j \quad M_2^j \quad M_3^j \quad F_1^k \quad F_2^k \quad F_3^k \quad M_1^k \quad M_2^k \quad M_3^k]$.
$\{A^e\}$ or $\{\bar{A}^e\}$	Local or global matrix of nodal actions, respectively, of element e of a structure ($e = 1, 2, \ldots, NE$).
$\{A^E\}$	Local matrix of nodal actions of an element of a structure subjected to the equivalent actions on its nodes.
$\{A^{Ee}\}$	Local matrix of nodal actions of element e of a structure subjected to the *equivalent actions on its nodes*.
$\{A^R\}$ or $\{\bar{A}^R\}$	Local or global matrix of nodal actions, respectively, of an element *of the restrained structure,* that is, the structure subjected to the given loads with its nodes fixed against translation and rotation.
$\{A^{Re}\}$ or $\{\bar{A}^{Re}\}$	Local or global matrix of nodal actions, respectively, of element e of the restrained structure.
$\{\hat{A}^R\}$	Global matrix of nodal actions of all the elements of the restrained structure. For a structure with NE elements $\{\hat{A}^R\}^T = [\{\bar{A}^{R1}\}^T \quad \{\bar{A}^{R2}\}^T \quad \ldots \quad \{\bar{A}^{RNE}\}^T]$.
$\{a\}$	Matrix of basic nodal actions of an element of a structure subjected to the given loads. For an element of a truss we choose $\{a\} = F_1^k$; for an element of a planar beam or frame in the x_1x_2 plane we choose $\{a\}^T = [F_1^k \quad F_2^k \quad M_3^k]$. For an

	element of a space beam or frame we choose $\{a\}^T = [F_1^k \;\; F_2^k \;\; F_3^k \;\; M_1^k \;\; M_2^k \;\; M_3^k]$.
$\{a^E\}$	Matrix of basic nodal actions of an element of a structure subjected to equivalent actions on its nodes.
$\{a^R\}$	Matrix of basic nodal actions of an element of the restrained structure, that is, of the structure subjected to the given loads with its nodes fixed against translation and rotation.
$\{a^{Re}\}$	Matrix of basic nodal actions of element e of the restrained structure.
$\{\hat{a}^E\}$	Matrix of basic nodal actions of all the elements of a structure subjected to equivalent actions on its nodes. For a structure with NE elements $\{\hat{a}^E\}^T = [\{a^{E1}\}^T \;\; \{a^{E2}\}^T \;\; \ldots \;\; \{a^{ENE}\}^T]$.
$\{\hat{a}^{EM}\}$	Matrix of the basic nodal actions of all the elements of the model of a structure subjected to equivalent actions on its nodes.
$\{a_x^{EM}\}$	Matrix of the chosen redundant basic nodal actions of the model of a structure subjected to equivalent actions on its nodes.
$\{a_0^{EM}\}$	Matrix of the basic nodal actions of the model for a structure subjected to equivalent actions on its nodes which are not included in the matrix $\{a_x^{EM}\}$.
$[B]$	Equilibrium matrix for a structure subjected to equivalent actions on its nodes. It is defined by $\{P^{EF}\} = [B]\{\hat{a}^E\}$.
$[\hat{B}]$	Equilibrium matrix for a structure subjected to equivalent actions on its nodes. It is defined by $\{\hat{P}^E\} = [\hat{B}] = \begin{Bmatrix} \{\hat{a}^E\} \\ -\{R\} \end{Bmatrix}$.
$[b] = [B]^{-1}$	This matrix exists only for statically determinate structures.
$[C]$	Compatibility matrix for a structure subjected to equivalent actions on its nodes. It is defined by $[\hat{d}] = [C]\{\Delta^F\}$.
$[\hat{C}]$	Compatibility matrix for a structure subjected to equivalent actions on its nodes. It is defined as $\begin{Bmatrix} \{\hat{d}\} \\ \{\Delta^S\} \end{Bmatrix} = [\hat{C}][\hat{\Delta}]$.
$[c] = [C]^{-1}$	This matrix exists only for statically determinate structures.

$\{D\}$ or $\{\bar{D}\}$	Local or global matrix of nodal displacements, respectively, of an element of a structure subjected to given loads. It is identical to that of an element of the structure subjected to equivalent actions on its nodes. For an element of a truss $\{D\}^T = [u_1^j \;\; u_1^k]$, while $\{\bar{D}\}^T = [\bar{u}_1^j \;\; \bar{u}_2^j \;\; \bar{u}_1^k \;\; \bar{u}_2^k]$; for an element of a planar beam or frame $\{D\}^T = [u_1^j \;\; u_2^j \;\; \theta_3^j \;\; u_1^k \;\; u_2^k \;\; \theta_3^k]$; for an element of a space beam or frame $\{D\}^T = [u_1^j \;\; u_2^j \;\; u_3^j \;\; \theta_1^j \;\; \theta_2^j \;\; \theta_3^j \;\; u_1^j \;\; u_2^j \;\; u_3^j \;\; \theta_1^j \;\; \theta_2^j \;\; \theta_3^j]$.
$\{D^e\}$ or $\{\bar{D}^e\}$	Local or global matrix of nodal displacements, respectively, of element e of a structure subjected to given loads or to the equivalent actions on its nodes.
$\{d\}$	Matrix of the basic deformation parameters of an element of a structure.
$\{d^e\}$	Matrix of the basic deformation parameters of element e of a structure.
$\{\hat{d}\}$	Matrix of the basic deformation parameters of all the elements of a structure. For a structure with NE elements $\{\hat{d}\}^T = [\{d^1\}^T \;\; \{d^2\}^T \;\; \ldots \;\; \{d^{NE}\}^T]$.
E	Modulus of elasticity.
E_e	Modulus of elasticity of element e.
F_i^q or $\bar{F}_i^q$ ($i = 1, 2, 3; q = j$ or k)	Local or global component in the x_i or $\bar{x}_i$ direction, respectively, of the internal force acting at the end q ($q = j$ or k) of an element of a structure subjected to given loads.
F_i^{eq} or $\bar{F}_i^{eq}$ ($i = 1, 2, 3; q = j$ or k)	Local or global component in the x_i or $\bar{x}_i$ direction, respectively, of the internal force acting at the end q ($q = j$ or k) of element e of a structure subjected to given loads.
F_i^{Eq} or $\bar{F}_i^{Eq}$ ($i = 1, 2, 3; q = j$ or k)	Local or global component in the x_i or $\bar{x}_i$ direction, respectively, of the internal force acting at the end q ($q = j$ or k) of an element of a structure subjected to equivalent actions on its nodes.
F_i^{Rq} or $\bar{F}_i^{Rq}$ ($i = 1, 2, 3; q = j$ or k)	Local or global component in the x_i or $\bar{x}_i$ direction of the internal force, respectively, acting at the end q ($q = j$ or k) of an element of the restrained structure, that is, the structure subjected to the given loads with its nodes fixed against translation and rotation.
F_i^{Req} or $\bar{F}_i^{Req}$ ($i = 1, 2, 3; q = j$ or k)	Local or global component in the x_i or $\bar{x}_i$ direction of the internal force, respectively, acting at the end q ($q = j$ or k) of an element e

	of the restrained structure, that is, the structure subjected to the given loads with its nodes fixed against translation and rotation.
$[f]$	Flexibility matrix for an element of a structure.
$[f^e]$	Flexibility matrix for element e of a structure.
$[\hat{f}]$	Flexibility matrix of all the elements of a structure defined by relation (16.9).
G	Shear modulus.
I_i $(i = 2, 3)$	Moment of inertia of the cross section of an element about its x_i $(i = 2, 3)$ principal centroidal axis.
$I_i^{(e)}$ $(i = 2, 3)$	Moment of inertia of the cross section of element e about its x_i $(i = 2, 3)$ principal centroidal axis.
J	Polar moment of inertia of the cross section of an element.
K	Torsional constant of the cross section of an element.
$[K]$ or $[\bar{K}]$	Local or global stiffness matrix for an element of a structure, respectively.
$[K^e]$ or $[\bar{K}^e]$	Local or global stiffness matrix for element e of a structure, respectively.
$[k]$	Basic local stiffness matrix for an element of a structure. It relates its basic nodal actions to its basic deformation parameters $\{a\} = \{k\}\{d\}$.
$[k^e]$	Basic local stiffness matrix for element e of a structure defined by relations (16.12).
L	Length.
L_e	Length of element e.
$\mathcal{M}_i^{(A)}$ or $\mathcal{M}_i^{(3)}$	Local component in the x_i $(i = 1, 2, 3)$ direction of the concentrated external moment acting on point A or 3 of an element of a structure, respectively.
$\boldsymbol{\mathcal{M}}^{(A)}$ or $\boldsymbol{\mathcal{M}}^{(3)}$	Concentrated external moment vector acting on point A or 3 of a structure, respectively.
$M_i(x_i)$	Local component in the x_i $(i = 1, 2, 3)$ direction of the internal moment of an element.
M_i^q or $\bar{M}_i^q$ $(i = 1, 2, 3;\ q = j$ or $k)$	Local or global component in the x_i or $\bar{x}_i$ direction, respectively, of the internal moment acting at the end q $(q = j$ or $k)$ of an element.
M_i^{eq} or $\bar{M}_i^{eq}$ $(i = 1, 2, 3;\ q = j$ or $k)$	Local or global component in the x_i or $\bar{x}_i$ direction, respectively, of the internal moment acting at the end q $(q = j$ or $k)$ of element e.

$m_i(x_1)$ $(i = 1, 2, 3)$ — Component in the x_i direction of the external distributed moment acting on an element. It is given in units of moment per unit length of the element.

$\mathbf{m}(x_i)$ — External distributed moment vector acting on an element.

N or $N(x_1)$ — Axial component of internal force in an element.

$P_i^{(A)}$ or $P_i^{(3)}$ $(i = 1, 2, 3)$ — Local components in the x_i direction of the concentrated external force acting on point A or 3 of an element of a structure, respectively.

$\mathbf{P}^{(A)}$ — Concentrated external force vector acting on point A of a structure.

$\{\hat{P}^E\}$ — Matrix of the global components of the equivalent actions acting on all the nodes of a structure.

$\{P^{EF}\}$ — Matrix of the global components of the equivalent actions acting on the nodes of a structure, which are not directly absorbed by its supports.

$\{P^{ES}\}$ — Matrix of the global components of the equivalent actions acting on the nodes of a structure, which are directly absorbed by its supports.

$\{\hat{P}^G\}$ — Matrix of the global components of the given actions acting on the nodes of a structure.

$\mathbf{p}$ — Distributed external forces acting on an element. It is given in units of force per unit of length of the element.

p_i or $p_i(x_1)$ $(i = 1, 2, 3)$ — Local component in the x_i direction of the distributed external forces acting on an element.

$R_i^{(n)}$ — Global component in the $\bar{x}_i$ direction of the reaction at support n of a structure.

$\bar{S}_i^{(n)}$ — Global component in the $\bar{x}_i$ direction of the restraining action acting on node n of a structure.

$\mathbf{u}$ or $\mathbf{u}(x_1)$ — Translation vector of the points of an element.

$u_i(x_1)$ $(i = 1, 2, 3)$ — Local component in the x_1 direction of the translation vector of the points of an element.

$u_i^e(x_1)$ $(i = 1, 2, 3)$ — Local component in the x_1 direction of the translation vector of the points of element e of a structure.

$u_i^{(A)}$ or $u_i^{(3)}$ $(i = 1, 2, 3)$ — Local component in the x_i direction of the translation vector of point A or point 3 of an element of a structure, respectively.

$u_i^E(x_i)$ $(i = 1, 2, 3)$	Local component in the x_i direction of the translation vector of the points of an element of a structure subjected to equivalent actions.
$u_i^R(x_i)$ $(i = 1, 2, 3)$	Local component in the x_i direction of the translation vector of the points of an element of the restrained structure.
u_i^q ($q = j$ or k; $i = 1, 2, 3$)	Local component in the x_i direction of the translation of the end q ($q = j$ or k) of an element of a structure subjected either to the given loads or to the equivalent actions.
u_i^{eq} ($q = j$ or k; $i = 1, 2, 3$)	Local component in the x_i direction of the translation of the end q ($q = j$ or k) of element e of a structure subjected either to the given loads or to the equivalent actions.
$\{u\}$	Matrix of the basic components of displacement of an element. For an axial deformation element $\{u\} = u_1(x_1)$. For a general planar element $\{u\}^T = [u_1(x_1) \quad u_2(x_1)]$. For a general space element $\{u\}^T = [u_1(x_1) \quad u_2(x_1) \quad u_3(x_1) \quad \theta_1(x_1)]$.
$\hat{u}_i(x_1, x_2, x_3)$ $(i = 1, 2, 3)$	Component of displacement in the x_i direction of the particles of a deformable body.
$T_k^{(+)}, T_k^{(-)}$	Temperature of the points of a cross section of an element of the structure where the positive and negative x_k $(k = 2, 3)$ axis, respectively, intersects the perimeter of its cross section.
T_0	Uniform temperature at the stress-free state of a structure, that is, the temperature at which the structure was constructed.
ΔT_c	Change of temperature at the centroid of the cross sections of an element of a structure. It could be a function of x_1.
$[T]$	Matrix which transforms the matrix of basic nodal actions to the matrix of nodal actions of an element of a structure subjected to equivalent actions on its nodes $\{A^E\} = [T]\{a^E\}$.
x_i $(i = 1, 2, 3)$	Local cartesian coordinate of a point of an element. The coordinate x_i is measured along the axis of the element from its end j.
x_i^e $(i = 1, 2, 3)$	Local cartesian coordinate of a point of element e.
$\bar{x}_i$ $(i = 1, 2, 3)$	Global cartesian coordinate of the points of a structure.
α	Coefficient of linear thermal expansion.
θ_i^q $(i = 1, 2, 3)$	Component of rotation about the x_i axis of the end q ($q = j$ or k) of an element of a structure.