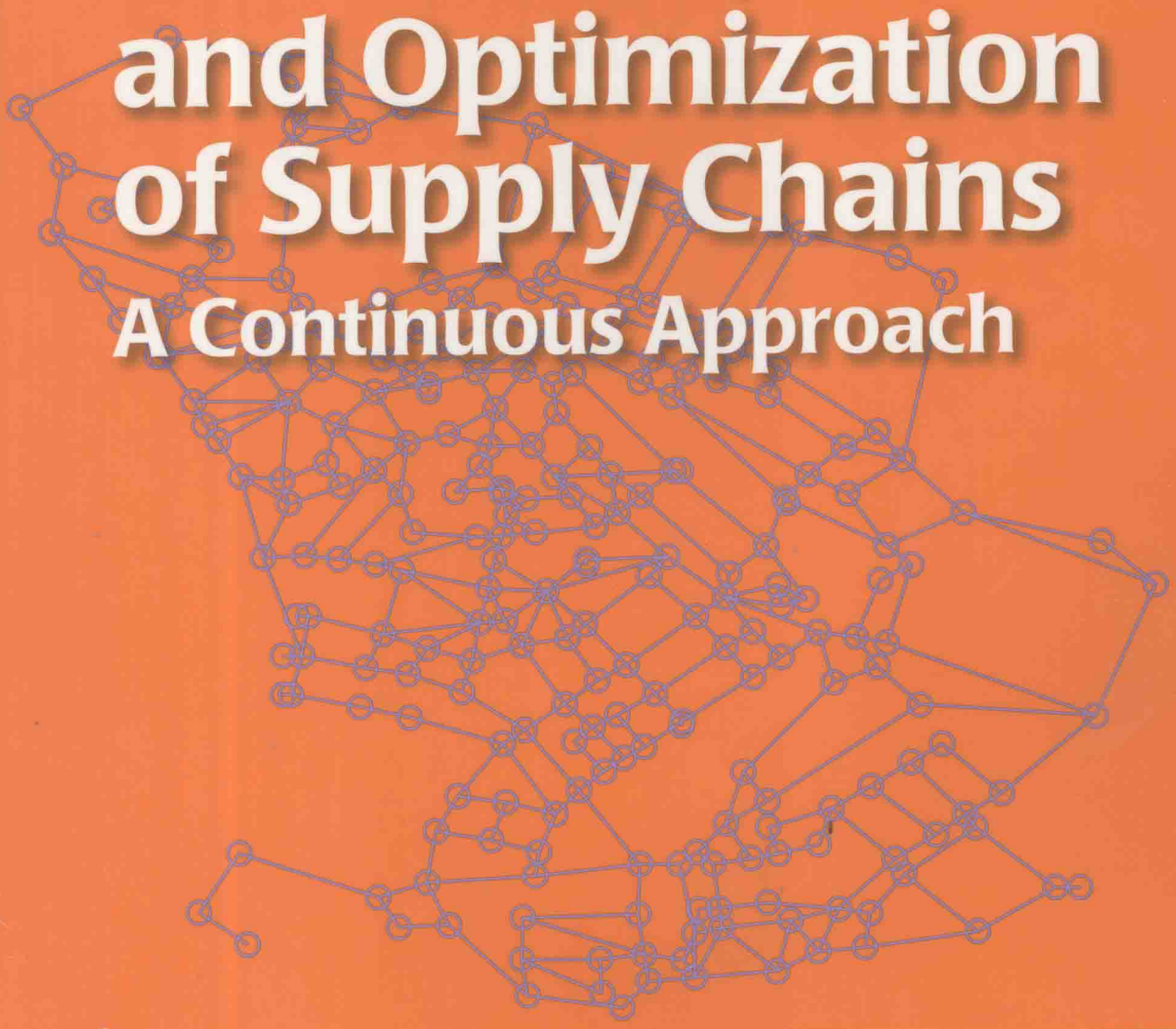
Modeling, Simulation, and Optimization of Supply Chains

A Continuous Approach



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Modeling, Simulation, and Optimization of Supply Chains

To my father Domenico
(C. D'Apice)

To my husband Joachim
(S. Göttlich)

To my son Edoardo
(B. Piccoli)

Preface

This book is devoted to primarily continuous models for a special class of supply chains often called production or supply networks. The aim is to present a mathematical description of different phenomena appearing in planning and managing supply chains. We address both the mathematical modeling as well as techniques for simulation and optimization purposes.

The problem of a continuous description of supply chains and production networks dates back to the early 60's and started with the work of [8, 30]. Significantly, the models were proposed in particular for large volume production on complex networks where a discrete description might fail. Since then, many methods and ideas have been developed concerning the modeling of different features of supply chains, including the efficient simulation and the optimization of product flows among suppliers and customers. In recent years continuous and homogeneous product flow models have been introduced, for example, in [2, 13, 23, 26, 27, 28, 29, 37, 41, 42]. These models have been built in close connection to other transport problems like vehicular traffic flow and queuing theory. Hence, this suggests that the obtained models should be given by partial differential equations for the product flow, similar to those of gas dynamics. Depending on the problem at hand, these equations are possibly accompanied by ordinary differential equations describing the load of inventories. Also some optimization techniques have been proposed in order to answer questions arising in supply chain planning [31, 50, 57].

Starting from a network formulation, we derive equations for a continuous description of homogeneous product flows. The derivation is based on first principles, but the final equations are closely related to discrete event simulations of supply chains. Additionally, we present extensions to include more realistic phenomena. Such extensions consist of systems of partial differential equations or coupled partial and ordinary differential equations. The book surveys the underlying fundamentals and provides evolved mathematical techniques for simulation and efficient optimization of the presented models.

The book is suitable for researchers and students in mathematics, physics, engineering, economics, and related disciplines. Basically, the book could also be used as lecture notes for a course in supply chain theory. Furthermore, interested readers can find many suggestions and open problems for future research.

At first, in Chapter 1, we give some elementary examples which serve as motivation for the modeling approaches following, and then we report mathematical preliminaries about conservation laws and numerical schemes for their discretization. Chapters 2–5 contain a complete and elementary description of the state-of-the-art theory of continuous supply chain models where also similarities and comparisons with already existing models are given. Optimization problems and the corresponding solution techniques are the content of Chapter 6. In particular, the interplay between discrete and continuous optimization

problems is addressed. Numerical discretization issues and computational results in Chapter 7 complete this book.

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Chapter 1

Introduction

A supply chain consists of suppliers, manufacturers, warehouses, and stores where parts are produced and distributed among different production facilities. Mathematical models are used to monitor cost-efficient distribution of parts and to measure current business processes. Naturally, depending on the scale, these models are characterized by several approaches which are either discrete (discrete event simulations) or continuous (differential equations). The main difference between these two mathematical concepts is the description of parts as individuals at discrete time instances or as a dynamic flow.

Simulations in general represent a powerful computing technique to analyze manufacturing systems while performing numerical experiments of the models. In case of discrete event simulations the evolution of the system is viewed as a sequence of significant changes in time, also called events, for each part separately. For instance, consider a supply chain consisting of numerous consecutive facilities where parts arrive, get processed, and depart when their service is completed. Informally, this means that the transportation of parts from one production step to another characterizes dynamic events that can be easily evaluated using performance measures like the number of parts in the system, the individual waiting times, and so forth. Definitely, discrete event simulations serve as the finest level of description of interacting part-based systems but with the drawback of exponentially increasing computing times for large-scale systems, i.e., up to 10^3 and more suppliers.

An alternative modeling approach which remedies the computational aspect is differential equations. In contrast to the discrete event simulation, averaged quantities in case of large quantity production predict the time evolution of parts and include the dynamics inside the different production steps. To derive accurate continuous models the overall modeling goal is to transfer as much of the detailed and complex discrete model to the continuous level. This will be achieved regarding dynamic flows, i.e., parts per time unit, instead of individual parts. Since numerical schemes for differential equations allow for fast simulation times, supply chain problems with multiple manufacturers and thousands of parts are solved very cost effectively. Figure 1.1 gives an idea of how computing times evolve when sample examples depending on 20.000 or 200.000 parts, respectively, are simulated. Keep in mind that partial differential equations, a special choice of differential equations, will always provide valid results if the size of the underlying problem is large scaled.

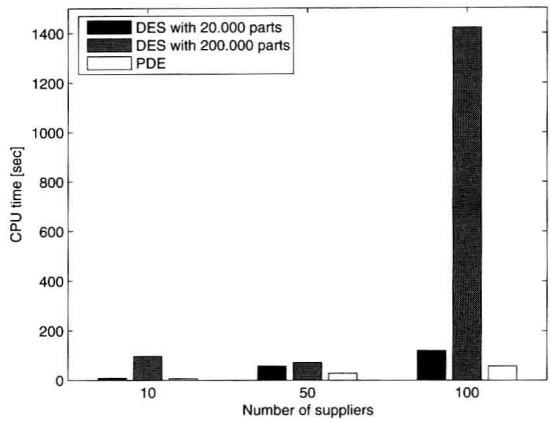

Figure 1.1. *The efficiency of using partial differential equations (PDE) in comparison with discrete event simulations (DES) is depicted. Three different scenarios of different complexity varying the number of suppliers and parts as well are simulated over* 400 *time periods.*

In many applications the simulation and prediction of production systems is only one important purpose. The formulation of optimization problems for supply chain management is an immediate consequence of performing successful simulations. Nowadays, there exist a variety of sophisticated models and adapted mathematical methods to find the optimal load balance on the interconnections between different entities. Typically, such problems are mainly tackled by methods of discrete optimization, linear and mixed-integer programming methods or models based on differential equations. Simpler models based on linear programs often neglect the time dependence of processes as well as nonlinear cost effects; however, they are applicable to large-scale networks. On the other hand, advanced optimization procedures recover the whole dynamics of the system while adjusting parameters in the models in an optimal way. Concerning manufacturing problems this may include the question of optimal processing parameters, minimizing inventories to reduce costs or to ensure fully loaded production lines.

Mathematically, supply chain optimization problems are given by either the minimization or maximization of an objective function (alternatively cost functional) satisfying flow conservation and capacity constraints. A relevant example would be the optimal routing of parts through a network such that inventory costs are minimized. This situation normally occurs whenever it is possible to distribute parts among several capacity-limited suppliers; cf. Figure 1.2.

Here, the rate $A^{1,2}$ denotes the percentage of parts coming from supplier e^1 continuing on supplier e^2, and equivalently $A^{2,6}$ determines the percentage of parts coming from supplier e^2 continuing on supplier e^6. The network allows for different capacity restrictions of suppliers and the possibility to store surplus material in queues q^e.

A bottleneck situation emerges from supplier e^6, which means that this capacity is the lowest in the complete network. The visualization of all objective function values

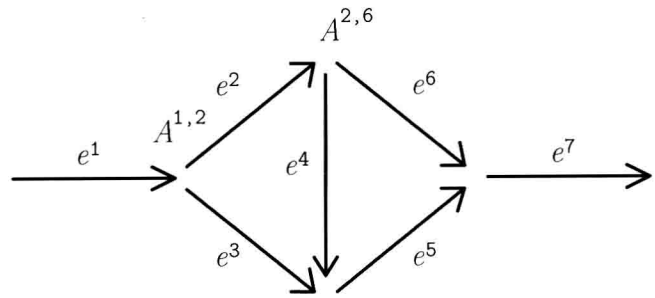

Figure 1.2. *A network consisting of seven suppliers and exactly two critical points $A^{1,2}$ and $A^{2,6}$ where the dispatch of parts obviously may be influenced.*

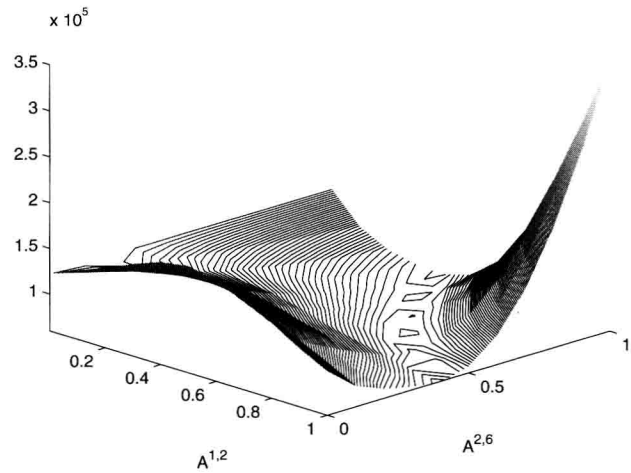

Figure 1.3. *Objective function values evaluated at all possible combinations of $A^{1,2}$ and $A^{2,6}$. As can be seen in the plot, the minimal objective value is reached at $A^{1,2} = 1$ and $A^{2,6} = 0.4$.*

in Figure 1.3 shows a rather naive way to choose the best value from a set of available alternatives. Nevertheless, we observe that the objective function has a unique minimum and steep gradients as $A^{2,6} \to 1$, due to the capacity bottleneck of supplier e^6, which implies an increasing inventory if filled by parts from supplier e^2. More evolved mathematical optimization methods are, for example, mixed-integer problems including dynamics which are consistent with the underlying detailed description by partial differential equations and/or particle/discrete event models. For the optimization of continuous supply chain models one could proceed in a different way. In the differential equation framework a natural approach would be to use an optimization procedure based on the Lagrange principle where the original constrained model is reformulated as an unconstrained one.

Altogether, the objective of this book will be to highlight the different ways of modeling supply chains and to emphasize their application for simulation and optimization purposes as well. In particular, similarities of the models will be discussed, and evolved mathematical methods governing the dynamic behavior of processes will be developed and analyzed. A major focus is the description of the underlying dynamics using ordinary and partial differential equations which may fit best in the context of mass production. Moreover, simulation and optimization procedures for these dynamical networks can be derived in a straightforward way combining already existing and innovative procedures.

Chapter 2

Mathematical Preliminaries

2.1 Introduction to Conservation Laws

Some of the models for supply chains we present in this book are based either on scalar or systems of hyperbolic conservation laws. A complete theory of hyperbolic systems of conservation laws is beyond the aim of the book. The theory of conservation laws is extensively described in many books, such as Bressan [10], Dafermos [21], Smoller [70], or Holden and Risebro [54]. Important results have been established in the past years and briefly revised some important results: the general solution to the Riemann problem for a strictly hyperbolic system of conservation laws was first obtained by Lax. The first proof of global existence for weak entropic solution appeared in the seminal paper by Glimm [34]. It is based on a construction of approximate solutions generated by Riemann problems with a randomly restarting procedure. For the scalar case, there is another proof, based on piecewise constant approximations, for the existence of an entropy admissible solution. This method is due to Dafermos [21]. The wave-front tracking method was first introduced by Di Perna, and then it was extended by Bressan [10]. Uniqueness and Lipschitz continuous dependence of solutions to scalar conservation laws (in many space variables) were first obtained by Kruzkov using the special entropies. The first proof of uniqueness for systems was obtained in 1996 by Bressan, Crasta, and Piccoli [11]. The proof was much simplified using the Bressan–Liu–Yang functionals; see [12].

In this section we give some basic preliminaries about systems of conservation laws.

Conservation Laws

A system of conservation laws in one space dimension can be written in the form

$$u_t + f(u)_x = 0, \tag{2.1}$$

where $u : [0, +\infty[\times \mathbb{R} \to \mathbb{R}^n$ is the "conserved quantity" and $f : \mathbb{R}^n \to \mathbb{R}^n$ is the flux.

Indeed, if we integrate (2.1) on an arbitrary space interval $[a,b]$, then

$$\frac{d}{dt}\int_a^b u(t,x)dx = -\int_a^b f(u(t,x))_x dx = f(u(t,a)) - f(u(t,b)),$$

and so the number of u in any interval $[a,b]$ varies according to the quantity of u entering and exiting at $x=a$ and $x=b$.

We always assume f to be smooth; thus, if u is a smooth function, then (2.1) can be rewritten in the quasi-linear form

$$u_t + A(u)u_x = 0, \tag{2.2}$$

where $A(u)$ is the Jacobian matrix of f at u.

Definition 2.1. *The system* (2.2) *is said to be hyperbolic if, for every* $u \in \mathbb{R}^n$, *all the eigenvalues of the matrix* $A(u)$ *are real. Moreover,* (2.2) *is said to be strictly hyperbolic if it is hyperbolic and if, for every* $u \in \mathbb{R}^n$, *the eigenvalues of the matrix* $A(u)$ *are all distinct.*

Remark 2.1.1. *It is clear that* (2.1) *and* (2.2) *are completely equivalent for smooth solutions. If instead u has a jump, the quasi-linear equation* (2.2) *is in general not well defined, since there is a product of a discontinuous function* $A(u)$ *with a Dirac measure. A notion of solution in the case of discontinuous functions is given in this section.*

If $n=1$ so u takes values in $\mathbb{R}$ and $f:\mathbb{R}\to\mathbb{R}$, then (2.1) is a single equation. In this case we say that (2.1) is a scalar equation. If $n>1$, then (2.1) is a system of n equations of conservation laws. Indeed if $u=(u_1,\dots,u_n)$ and $f=(f_1,\dots,f_n)$, then (2.1) can be written in the form

$$\begin{cases} \partial_t u_1 + \partial_x f_1(u) = 0, \\ \vdots \\ \partial_t u_n + \partial_x f_n(u) = 0. \end{cases}$$

Weak Solutions

A standard fact for the nonlinear system (2.1) is that classical solutions may not exist for some positive time, even if the initial datum is smooth. Let us consider, for example, the scalar Burgers equation

$$u_t + uu_x = 0,$$

with the initial condition $u(0,x)=u_0(x)=\frac{1}{1+x^2}$. One shows that the solution $u(t,x)$ to this Cauchy problem must be constant along the lines

$$t \mapsto \left(t, x + \frac{t}{1+x^2}\right).$$

For t sufficiently small ($t<\frac{8}{\sqrt{27}}$) these lines do not intersect together, and so the solution is classical, but at $t=\frac{8}{\sqrt{27}}$ the characteristics intersect together and a classical solution, i.e., differentiable solution, cannot exist for $t\geq\frac{8}{\sqrt{27}}$; see Figure 2.1.

Hence we must deal with weak solutions.

Definition 2.2. *Fix* $u_0 \in L^1_{loc}(\mathbb{R};\mathbb{R}^n)$ *and* $T>0$. *A function* $u:[0,T]\times\mathbb{R}\to\mathbb{R}^n$ *is a weak solution to the Cauchy problem*

$$\begin{cases} u_t + f(u)_x = 0, \\ u(0,x) = u_0(x) \end{cases} \tag{2.3}$$

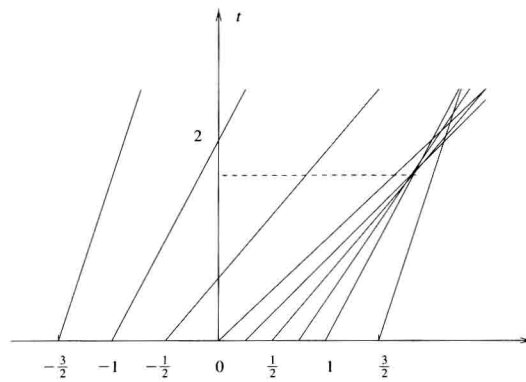

Figure 2.1. *The characteristic curves for the Burgers equation in the (t,x)-plane.*

if u is continuous as a function from $[0,T]$ into L^1_{loc} and if, for every C^1 function ψ with compact support contained in the set $]-\infty,T[\times\mathbb{R}$, it holds that

$$\int_0^T \int_{\mathbb{R}} \{u\cdot\psi_t + f(u)\cdot\psi_x\}\,dxdt + \int_{\mathbb{R}} u_0(x)\cdot\psi(0,x)dx = 0. \tag{2.4}$$

A weak solution u to (2.3) satisfies

$$u(0,x) = u_0(x) \quad \text{for a.e. } x\in\mathbb{R}.$$

This is a consequence of the fact that u is continuous as a function from $[0,T]$ to L^1_{loc} and of (2.4). We summarize further properties of weak solutions.

Definition 2.3. *A function $u = u(t,x)$ has an approximate jump discontinuity at the point (τ,ξ) if there exist vectors $u^-,u^+\in\mathbb{R}^n$ and $\lambda\in\mathbb{R}$ such that*

$$\lim_{r\to 0^+}\frac{1}{r^2}\int_{-r}^{r}\int_{-r}^{r}\|u(\tau+t,\xi+x)-U(t,x)\|dxdt = 0,$$

where

$$U(t,x) := \begin{cases} u^-, & \text{if } x<\lambda t,\\ u^+, & \text{if } x>\lambda t. \end{cases} \tag{2.5}$$

The function U is called a shock travelling wave.

Theorem 2.4. *Consider a bounded weak solution u to* (2.1) *with an approximate jump discontinuity at (τ,ξ). Then*

$$\lambda(u^+-u^-) = f(u^+)-f(u^-). \tag{2.6}$$

A proof can be found in [10].

Equation (2.6), called the Rankine–Hugoniot condition, gives a condition on discontinuities of weak solutions of (2.1) relating the right and left states with the "speed" λ of the "shock." In the scalar case (2.6) is a single equation and, for arbitrary $u^- \neq u^+$, we have

$$\lambda = \frac{f(u^+) - f(u^-)}{u^+ - u^-}.$$

For an $n \times n$ system of conservation laws, (2.6) is a system of n scalar equations.

It is known that weak solutions are in general not unique: let u_0 be the function defined by

$$u_0(x) := \begin{cases} 1, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases}$$

For every $0 < \alpha < 1$, the function $u_\alpha : [0, +\infty[\times \mathbb{R} \to \mathbb{R}$ defined by

$$u_\alpha(t,x) := \begin{cases} 0, & \text{if } x < \frac{\alpha t}{2}, \\ \alpha, & \text{if } \frac{\alpha t}{2} \leq x < \frac{(1+\alpha)t}{2}, \\ 1, & \text{if } x \geq \frac{(1+\alpha)t}{2} \end{cases}$$

is a weak solution to the Burgers equation for all values of α. Therefore the notion of weak solution must be supplemented with admissibility conditions, motivated by physical considerations.

Entropy Admissible Solutions

A first admissibility criterion, coming from physical considerations (see Dafermos [21]), is that of the entropy admissibility condition.

Definition 2.5. *A C^1 function $\eta : \mathbb{R}^n \to \mathbb{R}$ is an entropy for* (2.1) *if it is convex and there exists a C^1 function $q : \mathbb{R}^n \to \mathbb{R}$ such that*

$$D\eta(u) \cdot Df(u) = Dq(u) \tag{2.7}$$

for every $u \in \mathbb{R}^n$. The function q is said to be an entropy flux for η. The pair (η, q) is said to be an entropy–entropy flux pair for (2.1).

Definition 2.6. *A weak solution $u = u(t,x)$ to the Cauchy problem*

$$\begin{cases} u_t + f(u)_x = 0, \\ u(0,x) = u_0(x) \end{cases} \tag{2.8}$$

is said to be entropy admissible if, for every C^1 function $\phi \geq 0$ with compact support in $[0, T[\times \mathbb{R}$ and for every entropy–entropy flux pair (η, q), it holds that

$$\int_0^T \int_{\mathbb{R}} \{\eta(u)\phi_t + q(u)\phi_x\} \, dx dt \geq 0. \tag{2.9}$$

Existence of entropies is in general not granted. However, in the scalar case there is a family of convex entropies introduced first by Kruzkov.