Quantitative Methods in Accounting

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D. VAN NOSTRAND COMPANY
New York Cincinnati Toronto London Melbourne

D. Van Nostrand Company Regional Offices:
New York Cincinnati

D. Van Nostrand Company International Offices:
London Toronto Melbourne

Library of Congress Catalog Card Number: 79-66015
ISBN: 0-442-80503-9

Published by D. Van Nostrand Company
135 West 50th Street, New York, N.Y. 10020

10 9 8 7 6 5 4 3 2 1

Preface

The impact of quantitative methods on various disciplines within business administration has been dramatic. Accounting, as a discipline, has in the past remained almost immune from changes attributed to quantitative methods. During the past decade a significant change has been noted in this area, so that now almost every aspect of the accounting profession has been influenced and employs quantitative techniques. For example, consider regression analysis and its applications to accounting. Cost estimation is perhaps the most obvious accounting application, with sales forecasting and related budgeting development as other areas where regression techniques are employed. Recently, auditing regression procedures have been employed in reviewing interim financial statements and in the quality review of an audit. Regression techniques are also widely used in accounting research.

Now consider the input-output model. Accounting applications use the model for cost allocations, financial forecasting, and as a planning and control model of a multiprocess system. The financial forecasting part of the model could be potentially used in reviewing interim financial reports. Accounting journals now contain a large number of articles where quantitative methods are the basis of the presentation. Quantitative methods are making inroads in accounting education on undergraduate and graduate levels.

This book is designed for use by both undergraduate and graduate accounting students. Material throughout the text has been successfully used by the author with both levels of accounting students. With graduate students, the material in the text has served as necessary background to aid the student in reading and understanding current accounting literature. At the undergraduate level, the emphasis has been upon understanding the material so that the problems at the conclusion of each chapter can be solved.

This book contains three separate divisions. In the introductory chapter, the relationship between accounting and quantitative methods is discussed. Model building is presented as a means of applying the scientific method in decision making. A list of references relating to all areas of the discussion is provided.

Chapters two through five provide a means of reviewing the quantitative methods that are employed in the chapters dealing with accounting applications. Standard mathematics textbooks are referenced, and the problems at the conclusion of each chapter are related to accounting where appropriate. These chapters have been included because of the variation in the quantitative backgrounds of potential users.

The last nine chapters contain applications of quantitative methods to problems in accounting. In each chapter, the technique along with the underlying assumptions are introduced. Illustrative examples are then developed showing how the technique can be employed in solving accounting related problems. A list of references from accounting and management science literature is included at the end of each chapter. Classroom questions and problems are included with each chapter.

Material from the Uniform CPA Examinations and Unofficial Answers, copyright © 1959-1977 by the American Institute of Certified Public Accountants, Inc., was reprinted or adapted with permission. I wish to thank the American Institute of Certified Public Accountants and the Institute of Management Accounting for their permission to use problem materials and suggested solutions, in the text.

Thanks is expressed to various publishers and authors who gave permission to quote from their material. Each is individually acknowledged by footnotes within the text. I am grateful to the literary executor of the late Sir Ronald A. Fisher, F.R.S., to Dr. Frank Yates, F.R.S., and to Longman Group Ltd., London, for permission to reprint Table III from *Statistical Tables for Biological, Agricultural and Medical Research,* 6th ed., 1974.

Many people directly and indirectly contributed to the development of this book. Jim Martin, Eugene Kaczka, and A. Wayne Corcoran stimulated my interest in the quantitative area while I was in school. Dean H.H. Mitchell and Larry N. Killough are responsible for the environment at Virginia Polytechnic Institute and State University. It is here that I am continually stimulated through contact with students and my faculty colleagues. They have created and managed

an environment where the resources are available for one to undertake a project such as this book.

Sang Lee gave me encouragement to undertake the project. Robert F. Domosh and Sy Marchand were very helpful throughout the project. Paul Dasher and Charles Brandon reviewed the manuscript and their comments were very helpful. They helped keep the book's audience in focus and indicated many gaps and errors in the original manuscript. I only hope that I have been able to take advantage of their valuable insight.

A special note of gratitude is extended to Nadine Szymanski, Edgar Howard, Gregory Mayo, and Lien Phung for the many hours they spent in typing and correcting the manuscript. Several graduate students have influenced my thinking and therefore made a significant contribution to this undertaking. Mark Noftsinger, J. Edward Ketz, Jackson F. Gillespie, and Read Pearson were especially helpful and deserve a special word of thanks.

My family, in their own way, has sacrificed to make the completion of this project possible. My wife, Dorothy, has always been supportive throughout my time in graduate school and now in my career as an accounting educator. Our children, Elisabeth Hilma and Paul Wayne, are the very special people in our lives and to them this book is dedicated.

Contents

chapter 1

Introduction

As society has become more complex, the decision processes employed by management have become more complicated. This has resulted in the development of many new problem-solving techniques. Also, the development of the computer has permitted new applications of existing techniques to more complex problems. Indeed, a new discipline generally called decision science or management science is emerging with its foundation in the areas of economics, mathematics, psychology and sociology. Mathematical modeling is the basic tool employed in decision science. These developments have resulted in increasing application of decision science techniques to accounting.

Another trend has developed in the past twenty years that has directly influenced accounting. Viewing accounting as a social science has resulted in a shift in the philosophy of accounting research.[1] Considering accounting as a social science has had implications in both theoretical and empirical research efforts during the last twenty years. This has resulted in an increased penetration of quantitative techniques into the discipline of accounting.

In this chapter the logical connection between scientific research and quantitative techniques will be considered. Both the advantages and

[1] R.K. Mautz, "Accounting As a Social Science," *The Accounting Review* (April, 1963), pp. 317-325.

disadvantages of mathematical modeling will be presented. Then the modeling process will be demonstrated employing the cost-volume-profit model.

SCIENCE AND MATHEMATICS

A science is a body of systematic ordered knowledge whose structure can be thought of as consisting of theory on one side and data (empirical evidence) on the other. Any science is a going concern because of the interaction between the theoretical and empirical realms. The empirical side, consisting of observable data, is connected to the theoretical side by means of operational definitions. These definitions serve the purpose of defining theoretical constructs in terms of observable data.

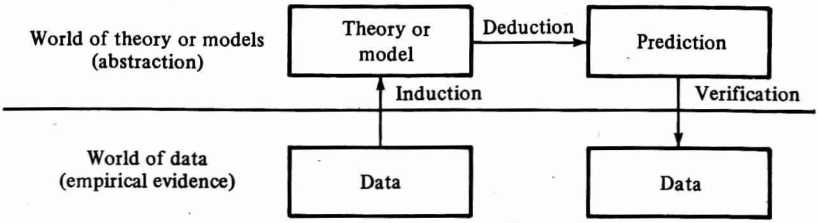

Figure 1-1 The cycle of science.

The cycle of science often starts and ends with data from the empirical realm.[2] Initially, a scientist assumes the role of an observer of data. Then employing inductive reasoning based on the observed data, the scientist formulates a theory. Based on the theoretical system, the scientist then makes predictions that are evaluated based on observable data. The process of reasoning from the general theory to a prediction is termed deduction. Theories of a scientist are tentative and he will abandon or modify a theory if observed data do not bear out the predictions. The relationships discussed here are depicted in Figure 1-1.

In describing the scientific method, Ernest Nagel concluded that "the practice of scientific method does not consist in following prescribed rules for making experimental discoveries."[3] He further states that there exists "no rules of discovery and invention in science any more than there are such rules in the arts."[4] Scientific research methodology is characterized by Kerlinger as being a "systematic, controlled, empirical, and critical investigation of hypothetical propositions about presumed relations among natural phenomena."[5] The terms

[2]This discussion is based on material contained in John G. Kemeny, *A Philosopher Looks At Science* (New York: D. Van Nostrand Company, 1959)

[3]Ernest Nagel, *The Structure of Science* (New York: Harcourt Brace & World, Inc., 1961), p. 12.

[4]Ibid.

[5]Fred N. Kerlinger, *Foundations of Behavioral Research* (New York: Holt, Rinehart and Winston, Inc., 1964), p. 13.

systematic and controlled are used by Kerlinger to mean that "scientific investigation is so ordered that investigators can have critical confidence in research outcomes."[6] He further elaborates on the empirical aspect by stating that when a "scientist believes something is so, he must somehow or another put his belief to a test outside of himself."[7] In summary, the methodology of science involves a constant exchange between the abstract theoretical realm and the empirical world.

Norbert Wiener, in *The Human Use of Human Beings* observed the following: "One of the most interesting aspects of the world is that it can be considered to be made up of *patterns*. A pattern is essentially an arrangement. It is characterized by the order of the elements of which it is made rather than the intrinsic nature of the elements."[8] The basic aim of science is to find general explanations of natural events rather than attempt to explain each research problem in an *ad hoc* fashion. The scientist seeks explanations of Wiener's patterns, and these explanations are referred to as theories or models.

The terms theory and model will be used interchangeably in this discussion. However, theories are generally considered to have a broader scope than a model. The internal structure of a model is generally more rigorously specified than that of a theory. Scientific models are abstractions in two respects. A model is an abstraction in the sense that only relevant variables are considered in its formulation. The second dimension of abstraction relates to the effort of a scientist to formulate models for the general case rather than a specific problem under study. This latter form of abstraction makes possible what is often referred to as the "economy of science." By seeking general as opposed to *ad hoc* explanations, the scientist attempts to maximize the return on a research effort.

Ackoff has suggested a three tiered classification scheme for models.[9] *Iconic models* are scaled representations of a system. Such models look like what they represent since the only transformation is one of scale. Scale models of planes and trains are examples of iconic models. In an *analogue model* one property is used to represent another and a legend is necessary which specifies the transformation of properties. Graphs, charts, and slide-rules are examples of analogue models. Ackoff's final classification is called *symbolic models*. Here the properties of the system are expressed symbolically. When the symbols employed in a symbolic model represent quantities the model is a mathematical model. The relationship among velocity, distance, and acceleration are an

[6] Fred N. Kerlinger, *Foundations of Behavioral Research* (New York: Holt, Rinehart and Winston, Inc., 1964), p. 13.

[7] Ibid.

[8] Norbert Wiener, *The Human Use of Human Beings* (Boston: Houghton Mifflin Company, 1950), p. 3.

[9] Russell L. Ackoff, *Scientific Method: Optimizing Applied Research Decisions* (New York: John Wiley & Sons, Inc., 1962), p. 109.

example of a mathematical model. In accounting the cost-volume-profit model is an example of a mathematical model.

Evaluation of any scientific discipline is usually focused on the available models. Progress in a science is determined by the constant interplay between theory and observable data. When a great deal of data is available but no acceptable model, progress is impeded. A similar situtation exists when there is an elaborate model but a lack of data or the ability to measure the data. Thus, the existence of a model serves as a motivating influence either to collect data to test a model or to develop the necessary instruments to measure data.

The development of a scientific discipline usually begins with the emergence of a symbolic model where words are employed as symbols. For example, in psychology, the phenomenon of learning was initially described employing a verbal model. Much of the initial theoretical work in accounting involved developing verbal models. Verbal models are useful in both the physical and social sciences but are subject to certain inherent limitations. The first restriction of a verbal model is that it is subject to the limitations of languages. Languages are abstractions and therefore difficulties are encountered when attempting to describe phenomena in the abstract.

In discussing the construction of a model, J. H. Woodger pointed out another limitation of a verbal model when he wrote: "The importance of logical form in the organization of scientific theories is obscured and easily overlooked when they are expressed in a natural language."[10] A model formulated in verbal terms does not lend itself to manipulation. As a result, it does not allow formulation of concepts in a rigorous form so that terms such as cause, effect, determination, and proof meet minimum generally accepted logical criteria.[11] Finally, a theory presented in verbal terms does not often contain exact concept specifications that can be defined operationally. Words are often too vague or ambiguous to convey the precise meaning necessary for prediction based on a model. Any lack of prediction impedes exchanges between the theoretical and empirical realms necessary for a science to be a going concern.

The language of mathematics is most useful in constructing models. Emphasis in mathematics is on the hypothetical state of things and it is a science concerned with abstract and external concepts. Model construction in mathematical terms makes possible a clear distinction between mathematical notation and its physical interpretation.[12] This aids in the interpretation of the mathematical system through empirical analysis and the logical truth contained in the abstract theoretical system.

[10] J.H. Woodger, *The Technique of Theory Construction* (Chicago: The University of Chicago Press, 1947), p. 65.

[11] Ibid., p. 66.

[12] Charles H. Griffin and Thomas H. Williams, "A Comparative Analysis of Accounting and Mathematics," *The Accounting Review* (July, 1962), pp. 410-414.

Since it is virtually impossible to generalize in the absence of abstraction, and since generality is an objective of constructing a model, then mathematics as an abstract language provides a valuable means for developing generalizable models. The ultimate generalization, as Woodger noted, only possesses a logical form, and it is this form that determines the relation of the premise to consequence. Mathematics lends itself to complete generalization but still retains its logical form.

To balance the discussion, several shortcomings of mathematical models should be considered. Kaplan has pointed out several limitations of mathematical models.[13] He states that an overemphasis on symbols and rigor are constraints that must be recognized when mathematical models are employed. Some people possess an unconscious belief in the magic of symbols and lose sight of the fact that the sole purpose of the model is not manipulation according to the rules of some calculus. The same can be said for an undue emphasis on exactness and rigor. This is a fault of the model builder and may result in a model that calls for measures that cannot be obtained.

Another shortcoming of mathematical models as suggested by Kaplan to be identified here is that of oversimplification. Since models are abstractions, they will by definition always be simpler than the system being modeled. The criterion suggested by Kaplan in evaluating the degree of simplification depends on the factors taken into account when arriving at a particular degree of simplification. This standard for evaluating models, according to Kaplan, should not be viewed in the singular direction of "over" simplification. The question is whether the model builder has "over" or "under" simplified.

Indeed some of the greatest achievements of science, such as the laws of gravity and energy are simple yet precise relationships. Simplification in a direction to obtain a more elegant model or one that is easy to manipulate ignores the basic question. According to Kaplan, the crucial point is that nothing has been ignored that is important or has been incorporated but has no relationship to the purposes of the model.

The limits of the mathematical abilities of model builders will directly influence the output of a research effort. In addition the availability of research resources must be taken into account when evaluating output. Time, money, and manpower are limited and may dictate simplification procedures so that a project can be completed. Models in any discipline are evaluated based upon their logical structure, predictive ability, ability to permit someone to control a system, and research that is generated based on the model. These criteria are consistent with the maintenance of interaction between the theoretical and empirical realms of science.

In the next section the accountant's linear break-even model is developed. Assumptions and limitations of the model are considered. The linearity

[13]Abraham Kaplan, *The Conduct of Inquiry* (San Francisco: Chandler Publishing Company, 1964), pp. 275-288.

assumption is relaxed and the non-linear model is developed. This model is more difficult to manipulate, but it does provide additional insight concerning the system being represented. Elementary calculus is employed with the non-linear model. If the student encounters difficulty with this material, a review of some material in the second chapter might be helpful.

THE MODELING PROCESS: AN EXAMPLE

This section contains an example of the modeling process. The linear break-even model is developed and the assumptions underlying this frequently employed accounting model are discussed. Several of the assumptions are modified and the economist's short run break-even model is developed. This material is representative of the process that will be employed throughout the text in introducing accounting related mathematical and statistical models.

First, the symbols for the model are specified and any required submodels are developed. Then the model is developed and manipulated. Finally, the assumptions made in constructing the abstract model are specified. In some cases, the assumptions are modified and a related model is developed.

Model symbols for the linear break-even model are:

$$\begin{aligned}
\pi &= \text{profit} \\
p &= \text{unit selling price} \\
a &= \text{fixed cost} \\
b &= \text{unit variable cost} \\
x &= \text{quantity} \\
r &= \text{total revenue} \\
c &= \text{total cost}
\end{aligned}$$

Total revenue and total cost functions (submodels) can be specified as follows:

$$\begin{aligned}
r &= px \\
c &= a + bx. \qquad (1\text{-}1)
\end{aligned}$$

The profit function (a submodel) can now be specified:

$$\begin{aligned}
\pi &= r - c \\
\pi &= px - a - bx \\
\pi &= (p - b)x - a \qquad (1\text{-}2)
\end{aligned}$$

Since the objective of the break-even model is to determine that quantity where the profit is zero, a solution to the model can be obtained by setting the profit function equal to zero and then solving for quantity.

$$0 = (p-b)x - a$$
$$(p-b)x = a$$
$$x = \frac{a}{(p-b)} \qquad (1\text{-}3)$$

The same result is obtained by setting the total revenue equal to the total cost and solving once again for the quantity.

$$r = c$$
$$px = a + bx$$
$$px - bx = a$$
$$(p-b)x = a$$
$$x = \frac{a}{(p-b)}$$

The break-even quantity equals the fixed costs divided by the unit contribution margin.

Symbols in a model are classified as either variables or parameters. Variables can be subclassified as either (1) decision variables, (2) criterion variables, or (3) exogenous variables. Decision variables are those for which a solution is sought. In the break-even model the break-even volume is the solution variable. Criterion variables represent what is measured to determine when the solution has been reached. Profit is the criterion variable in the break-even model. When profit is zero, the break-even point has been reached. Exogenous variables have values that are determined outside of the system. In the break-even model, the selling price is assumed to be constant and is determined by market forces. Therefore the selling price is an exogenous variable.

Parameters are either constant values or random variables that specify relationships within the model. Parameters in the break-even model are the fixed and variable unit costs. These constants could be classified as exogenous variables if it is considered that the costs are determined outside the system. It is frequently difficult to classify a component of a model as either a parameter or an exogenous variable. When the value of a parameter varies according to a predefined probability distribution, the parameter is called a random variable. In the break-even model there are no random variables.

Most models are the result of integrating sub-models. In the break-even model, the total revenue and total cost functions are submodels. Linking these submodels together results in the profit model and permits determination of the break-even volume.

It is often informative to graphically represent a model. Relationships among the variables of the model can often be better understood when presented in this format. The break-even model is represented in Figure 1-2, page 8.