# SOCIAL SCIENCE STATISTICS

## **Social Science Statistics**

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#### To Dean and Ehren



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## **Social Science Statistics**

## **Preface**

As statistics become a more widespread and important form of communication in business and government, statistical literacy is becoming a mandatory characteristic for job applicants in many fields. Among the skills in demand is the ability to bridge the communication gap between the computer operator/professional statistician and the traditional manager who has little or no computer/statistical literacy. It is assumed that many students who use this text will need the skills described here in their future research or occupations. They may be asked to determine the appropriate statistics for analysis of data, to interpret the findings of statistical analysis, and to transmit their understandings to others who may or may not be sophisticated in statistical and research terminology. In other words, the premise is not that students using this text will become professional statisticians but that they will apply statistics in their graduate research or their future occupations.

#### **PURPOSE**

Given the stated assumptions, this text seeks to achieve several goals. It presents the traditional basics of statistics, including the language, concepts, and skills needed for statistical literacy. In addition, it seeks to generate sufficient understanding and appreciation of the role of statistics in the research process to permit the student to make decisions about when statistical analysis is needed, to evaluate theoretical or programmatic problems and determine which statistics are most appropriate, to interpret the statistical results, and finally, to communicate the statistical results and interpretations in terms that are meaningful for further research or for applied decision making.

#### **AUDIENCE AND PEDAGOGY**

This text is designed for use in an introductory statistics course that emphasizes social science methods and problems. Students of sociology, political science, history, public administration, business management, social psychology, speech communication, and many other areas in which statistics are applied will find the text appropriate. It is assumed that the typical students in the course will have some math background but will not be mathematically sophisticated. It is also assumed that students will have no familiarity with the specialized language of statistics.

Although it is necessary for the professional statistician to understand formula derivation and other aspects of theoretical statistics, a mathematically sophisticated presentation of statistics often intimidates the typical social science student (particularly if math skills are rusty or anxiety is high) and makes attainment of a lasting, practical understanding of the use of statistics less likely. At the same time a simple how-to-calculate approach is inadequate. Students must be encouraged to understand the reasons behind the uses of the statistics and the concepts of research and statistical analysis that allow them to make informed decisions about the uses and interpretations of statistics.

Given these assumptions about the probable audience for the text, and the perception that a knowledge of statistical basics often is mistakenly taken for granted, causing needless frustration for the student, every effort has been made to explain concepts, terminology, and procedures thoroughly, using a vocabulary that gradually increases in sophistication. Throughout the book, basic information for each statistic is provided in a statistical summary box; then formulas and calculation examples are presented and the results of each statistical analysis are interpreted. For the first examples, the interpretations are simple verbalizations, but they become more complex as additional concepts and statistics are presented.

Because most students can grasp the complexities of a statistic more easily after they have some concrete calculative experience, additional discussion of characteristics, uses, and other important issues related to each statistic is delayed until after an initial calculation example has been worked through. Practice problems (with complete answers) are provided after every statistic or group of statistics has been presented to allow students to test and reinforce their skills and their understanding of meanings and procedures. At the end of each chapter, a summary discussion, a summary of the formulas, a self-test, and exercises are presented. The self-test problems are designed to help students determine whether they have grasped the concepts and procedures presented in the chapter; therefore, complete and detailed answers to the self-test problems are given at the end of the chapters. The exercises provide an opportunity for instructors to test the students' understanding of the concepts in each chapter; answers to the exercise problems are provided in the instructor's manual.

A unique feature of the text is that each section is summarized in a decision-making framework in which the characteristics of the statistics and criteria for their use are examined in discussion and illustrated through sample decision-making problems. Additional decision-making problems are offered in the form of practice and exercise problems for

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students to test their own understandings. The final chapter integrates decision-making techniques for all statistics presented in the text.

#### ORGANIZATION

The chapters in the text are organized according to several criteria, including the analytical goals of description versus inference, the level of measurement, and the number of variables to be analyzed. The introductory chapter concentrates on the basic concepts and procedures used in statistics. The remainder of the text is divided into several parts, each devoted to statistics that are designed for particular tasks.

Part I familiarizes the student with basic statistical language and procedures through examination of the first category of statistics—those designed for examination of single variables. Students are gradually introduced to such basic skills as formula reading and are encouraged to think about levels of measurement, shapes of distributions, and other factors that may affect the selection of a statistic for description. Chapter 2 examines techniques for organizing data, chapter 3 offers a very brief discussion of graphic presentations of data, and chapter 4 introduces basic statistical notation and procedures with which the student must be familiar before actually examining the first statistical formulas.

Chapters 5 and 6 present several univariate statistics that summarize the characteristics of a distribution in terms of a single value. These chapters are designed to provide an understanding of measures of central tendency and dispersion, but they also provide the first formulas, the first statistical calculation experience, and the first interpretive efforts. For the first statistical formulas, instructions are presented in detail. Some students may find this information repetitive of previously covered concepts, whereas others may find it necessary to review mathematical procedures and concepts before mastering the formulas and calculations (see the math review in appendix A). When students have completed part I, they should have a complete grasp of basic formula reading and computation procedures. Chapter 7 is devoted to a more intensive examination of the meaning and application of one measure—the standard deviation—through analysis of the concepts of normal curve and z-scores. Finally, chapter 8 brings all of these statistics together in a decision-making framework.

Part II of the text presents statistics for describing two-variable relationships, including correlation and prediction measures appropriate for each measurement level. Chapter 9 presents basic concepts, discusses the appropriate data distributions for bivariate analysis, and introduces the interpretations that are used for bivariate statistical analysis. Chapters 10, 11, and 12 present bivariate statistics appropriate for nominal, ordinal, and interval level variables, respectively. For every

statistic presented, calculation and interpretation are discussed, as are the issues that affect the selection of each statistic as the appropriate tool for analysis. Chapter 13 summarizes the bivariate statistics and presents a series of problems in which the student must select the correct statistic for analysis.

Part III examines statistical inference procedures, including estimation and hypothesis testing. In the discussion of inference and the principles that legitimize it, every effort has been made to cut through the inherent complexity of such concepts as probability to offer a clear, yet thorough discussion of the process of statistical inference. Chapter 14 presents the vocabulary of statistical inference and discusses such concepts as probability and sampling distributions. Chapter 15 is devoted to point and interval estimation procedures, and chapter 16 introduces hypothesis testing, using the binomial z-test for illustration. A single hypothesis-testing format is then used in chapters 17 through 19 in the presentation of various difference-of-means tests, analysis of variance, and chi-square tests. Chapter 19 covers inferential tests for ordinal data and inferential application of some of the statistics covered in part II. As in part II, part III provides extensive discussion of the uses, interpretation, and related issues for each statistic. Practice problems, self-test problems, and exercises again permit students to check their understanding and skills.

Part IV (chapter 20) summarizes the entire text and presents two decision-making discussions. The first set of decision-making problems covers only the inferential statistics presented in part III. The second decision-making discussion integrates the univariate and bivariate descriptive statistics with the inferential statistics to help the student become aware of the full array of statistical options and the criteria that must be considered when selecting a statistic.

It is recognized that time may not permit coverage of all of these statistics in a single course. Thus, the text is designed to allow the instructor maximum flexibility in coverage. For instance, chapters 1 through 4 are written at a level that students should find relatively easy to grasp with a minimum of assistance; also, if preferred, only selected statistics from chapters 5 and 6 may be given class attention. Similarly, since the basic concepts on which bivariate description is based are discussed in chapter 9, the instructor may select among the statistics covered in chapters 10 through 13 for class discussion.

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# **Basic Concepts in Applied Statistics**

#### An Introduction

#### STATISTICS AS TOOLS—TRITE BUT TRUE

Statistics! The word alone is enough to strike fear in the hearts of many students who find themselves in a program that requires a course in the subject. Yet most of us deal with statistics daily: We hear reports that the *average* cost of food has risen. The nightly news reports on price *indices* of many types. Students want to know the *mean* when an exam is returned and often ask whether an instructor grades on a *curve*. Americans in general are concerned about the crime *rate*. People who attend horse races are well informed on the *odds*. Despite these common uses of statistics, however, most people remain convinced that they can neither understand nor use statistics.

In reality, statistics are simply tools. As is true of any tool, once we know its purpose, the appropriate techniques for using it, and any necessary safety precautions, we can use a statistic easily and effectively. This understanding of statistics as tools is becoming more vital because as statistics become more common in public communication, they will also become basic tools in business and government. The age of the computer has increased the capability of businesses to accumulate and store massive amounts of information. It also has made possible the quick and accurate analysis of such information through statistical techniques.

The onset of the age of communication, based on computer and statistical analysis, has created a major personnel gap for many organizations. People who are trained to operate computers are readily available, as are professional statisticians; and many people are well