LIAL • HORNSBY • SCHNEIDER

COLLEGE ALGEBRA

SEVENTH EDITION

Seventh Edition

College Algebra

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College Algebra
SEVENTH EDITION ▼▼▼

Preface

College Algebra, **Seventh Edition,** is written for students in a traditional college algebra course. We assume students have had at least one earlier course in algebra, but we include a thorough review in Chapter 1. For most students, the topics in Chapter 2 will also be review.

Although this book is intended for a traditional course, we have acknowledged the growing interest in utilizing graphing calculators to augment and deepen the concepts typically presented in College Algebra.

Changes in Content ▼▼▼

- The binomial theorem is introduced, using Pascal's Triangle, in the section that reviews polynomial operations. A complete presentation of the binomial theorem is given in the final chapter.
- The content in Chapters 3 and 4 has been rearranged to include relations and general function concepts, including symmetry, translation, and reflection, in one chapter. The conic sections are in a new chapter on analytic geometry, which includes a discussion of eccentricity.
- The chapter on polynomial and rational functions now precedes the chapter on exponential and logarithmic functions. We give more emphasis to general graphing techniques for polynomial functions, considering end behavior, turning points, and the expected number of zeros.
- The section on inverse functions introduces Chapter 5, on exponential and logarithmic functions, because this is the first real need for inverses.
- We have also combined systems of equations with matrices, because matrix solutions of linear systems have become more and more important with the increasing use of technology.

5

Exponential and Logarithmic Functions

In 1896 Swedish scientist Svante Arrhenius first predicted the greenhouse effect resulting from emissions of carbon dioxide by industrialized countries. In his classic calculation, he was able to estimate that a doubling of the carbon dioxide level in the atmosphere would raise the average global temperature by 7°F to 11°F. Since global warming would not be uniform, changes as small as 4.5°F in the average temperature could have drastic climatic effects, particularly on the central plains of North America. Sea levels could rise dramatically as a result of both thermal expansion and the melting of ice caps. The annual cost to the United States economy could reach $60 billion.

The burning of fossil fuels, deforestation, and changes in land use from 1850 to 1986 put approximately 312 billion tons of carbon into the atmosphere, mostly in the form of carbon dioxide. Burning of fossil fuels produces 5.4 billion tons of carbon each year which is absorbed by both the atmosphere and the oceans. A critical aspect of the accumulation of carbon dioxide in the atmosphere is that it is irreversible and its effect requires hundreds of years to disappear. In 1990 the International Panel of Climate Change (IPCC) reported that if current trends of burning of fossil fuel and deforestation

Sources: Clime, W., *The Economics of Global Warming*, Institute for International Economics, Washington, D.C., 1992.

Kraljic, M. (Editor), *The Greenhouse Effect*, The H. W. Wilson Company, New York, 1992.

International Panel on Climate Change (IPCC), 1990.

Wuebbles, D. and J. Edmonds, *Primer of Greenhouse Gases*, Lewis Publishers, Inc., Chelsea, Michigan, 1991.

Chapter Openers present a genuine application of the material to be presented.

deer population in using the equation),000 is the initial of growth. T is the e passed.

fter 4 years s 30,000 and the imately how many years?

can we expect in n is 45,000 and the

vorld population in al function defined by

$$A(t) = 2600e^{.018t},$$

where t is the number of years since 1950.

(a) The world population was about 3700 million in 1970. How closely does the function approximate this value?

(b) Use the function to approximate the population in 1990. (The actual 1990 population was about 5320 million.)

(c) Estimate the population in the year 2000.

57. A sample of 500 g of lead 210 decays to polonium 210 according to the function given by

$$A(t) = 500e^{-.032t},$$

where t is time in years. Find the amount of the sample after each of the following times.

(a) 4 years (b) 8 years

(c) 20 years (d) Graph $y = A(t)$.

58. Vehicle theft in the United States has been rising exponentially since 1972. The number of stolen vehicles, in millions, is given by

$$f(x) = .88(1.03)^x,$$

where $x = 0$ represents the year 1972. Find the number of vehicles stolen in the following years.

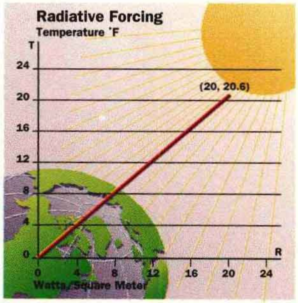
(a) 1975 (b) 1980 (c) 1985 (d) 1990

59.  (Refer to Example 8.) Carbon dioxide in the atmosphere traps heat from the sun. Presently, the net incoming solar radiation reaching the earth's surface is 240 watts per square meter (w/m²). The relationship between additional watts per square meter of heat trapped by the increased carbon dioxide R and the average rise in global temperature T (in °F) is shown in the graph. This additional solar radiation trapped by carbon dioxide is called **radiative forcing.** It is measured in watts per square meter.

(a) Is T a linear or exponential function of R?

(b) Let T represent the temperature increase resulting from an additional radiative forcing of R w/m². Use the graph to write T as a function of R.

(c) Find the global temperature increase when $R = 5$ w/m².



Source: Clime, W. *The Economics of Global Warming*, Institute for International Economics, Washington, D.C., 1992.

Exercises corresponding to the Chapter Openers are marked with a special symbol.

Titled Examples include detailed, step-by-step solutions and descriptive side comments. Examples relating to the Chapter Openers are also marked with a symbol.

Boxes highlight words, definitions, rules, and procedures.

Notes and Cautions highlight common student errors and address concepts that students often find difficult or confusing.

384 CHAPTER 5 EXPONENTIAL AND LOGARITHMIC FUNCTIONS

(b) The ratio A/K for a sample of granite from New Hampshire is .212. How old is the sample?
Since A/K is .212, we have

$$t = (1.26 \times 10^9)\frac{\ln[1 + 8.33(.212)]}{\ln 2} \approx 1.85 \times 10^9.$$

The granite is about 1.85 billion years old.

EXAMPLE 6 Analyzing global temperature increase

Carbon dioxide in the atmosphere traps heat from the sun. The additional solar radiation trapped by carbon dioxide is called *radiative forcing*. It is measured in watts per square meter. In 1896 the Swedish scientist Svante Arrhenius estimated the radiative forcing R caused by additional atmospheric carbon dioxide using the logarithmic equation $R = k\ln(C/C_0)$, where C_0 is the preindustrial amount of carbon dioxide, C is the current carbon dioxide level, and k is a constant. Arrhenius determined that $10 \le k \le 16$ when $C = 2C_0$.*

(a) Let $C = 2C_0$. Is the relationship between R and k linear or logarithmic?
If $C = 2C_0$, $C/C_0 = 2$, so $R = k \ln 2$ is a linear relation, because $\ln 2$ is a constant.

(b) The average global temperature increase T (in °F) is given by $T(R) = 1.03R$. (See Section 5.2, Exercise 59.) Write T as a function of k.
Use the expression for R given in the introduction above.

$$T(R) = 1.03R$$
$$T(k) = 1.03k\ln(C/C_0)$$

LOGARITHMS TO OTHER BASES A calculator can be used to find the values of either natural logarithms (base e) or common logarithms (base 10). However, sometimes it is convenient to use logarithms to other bases. For example, base 2 logarithms are important in computer science. The following theorem can be used to convert logarithms from one base to another.

Change-of-Base Theorem

For any positive real numbers x, a, and b, where $a \neq 1$ and $b \neq 1$:

$$\log_a x = \frac{\log_b x}{\log_b a}.$$

NOTE As an aid in remembering the change-of-base theorem, notice that x is above a on both sides of the equation.

*Source: Clime, W., *The Economics of Global Warming*, Institute for International Economics, Washington, D.C., 1992.

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Inverse Function

Let f be a one-to-one function. Then g i

$(f \circ g)(x) = x$ for every

and $(g \circ f)(x) = x$ for every

A special notation is often used for inv
a function f, then g is written as f^{-1} (read "
$f(x) = 8x + 5$, and $g(x) = f^{-1}(x) = (x -$

EXAMPLE 3 Deciding whether two functions are inverses

Let functions f and g be defined by $f(x) = x$
tively. Is g the inverse function of f?
A graph indicates that f is one-to-one,
is one-to-one, now find $(f \circ g)(x)$ and $(g \circ$

$$(f \circ g)(x) = f(g(x)) = (\sqrt[3]{x+1})^3 - 1$$
$$= x + 1 - 1$$
$$= x$$
$$(g \circ f)(x) = g(f(x)) = \sqrt[3]{(x^3 - 1) + 1}$$
$$= \sqrt[3]{x^3}$$
$$= x$$

Since both $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, function g is indeed the inverse of function f, so that f^{-1} is given by

$$f^{-1}(x) = \sqrt[3]{x+1}.$$

CAUTION Do not confuse the -1 in f^{-1} with a negative exponent. The symbol $f^{-1}(x)$ does not represent $1/f(x)$; it represents the inverse function of f. Keep in mind that a function f can have an inverse function f^{-1} if and only if f is one-to-one.

The definition of inverse function can be used to show that the domain of f equals the range of f^{-1}, and the range of f equals the domain of f^{-1}. See Figure 4.

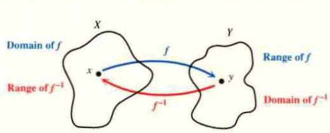


FIGURE 4

5.1 INVERSE FUNCTIONS 349

CONNECTIONS Inverse functions are used by government agencies and other businesses to send and receive coded information. The functions they use are usually very complicated. A simplified example involves the function $f(x) = 2x + 5$. If each letter of the alphabet is assigned a numerical value according to its position ($a = 1, \ldots, z = 26$), the word ALGEBRA would be encoded as 7 29 19 15 9 41 7. The "message" can be decoded using the inverse function $f^{-1}(x) = \frac{x - 5}{2}$.

FOR DISCUSSION OR WRITING

Use the alphabet assignment given above.

1. The function $f(x) = 3x - 2$ was used to encode the following message:

   37 25 19 61 13 34 22 1 55 1 52 52 25 64 13 10.

   Find the inverse function and decode the message.
2. Encode the message SEND HELP using the one-to-one function $f(x) = x^3 - 1$. Give the inverse function that the decoder would need when the message is received.

Connections Boxes point out the many connections between mathematics and the "real world" or other mathematical concepts.

For the inverse functions f and g discussed earlier $f(10) = 85$ and $g(85) = 10$; that is, (10, 85) belongs to f and (85, 10) belongs to g. The ordered pairs of the inverse of any one-to-one function f can be found by exchanging the components of the ordered pairs of f. The equation of the inverse of a function defined by $y = f(x)$ also is found by exchanging x and y. For example, if $f(x) = 7x - 2$, then $y = 7x - 2$. The function f is one-to-one, so that f^{-1} exists. The ordered pairs in f^{-1} have the form (y, x), so y can be used to produce x, since $x = f^{-1}(y)$. Therefore, the equation for f^{-1} can be found by solving $y = f(x)$ for x. Finally, x and y can be interchanged to conform to our convention of using x for the independent variable and y for the dependent variable.

$$y = 7x - 2$$
$$7x = y + 2 \qquad \text{Add 2.}$$
$$x = \frac{y + 2}{7} = f^{-1}(y) \qquad \text{Divide by 7.}$$
$$y = \frac{x + 2}{7} = f^{-1}(x) \qquad \text{Exchange } x \text{ and } y.$$
$$f^{-1}(x) = \frac{x + 2}{7}.$$

As a check, verify that $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$.

5.4 EVALUATING LOGARITHMS: CHANGE OF BASE 385

This theorem is proved by using the definition of logarithm to write $= \log_a x$ in exponential form.

roof

t

$$y = \log_a x.$$
$$a^y = x \qquad \text{Change to exponential form.}$$
$$\log_b a^y = \log_b x \qquad \text{Take logarithms on both sides.}$$
$$y \log_b a = \log_b x \qquad \text{Property (c) of logarithms}$$
$$y = \frac{\log_b x}{\log_b a} \qquad \text{Divide both sides by } \log_b a.$$
$$\log_a x = \frac{\log_b x}{\log_b a} \qquad \text{Substitute } \log_a x \text{ for } y.$$

Any positive number other than 1 can be used for base b in the change of base theorem, but usually the only practical bases are e and 10, since calculators give logarithms only for these two bases. The change-of-base theorem is used to find logarithms for other bases.

Optional Graphing Calculator Boxes offer guidance for students using graphing calculators.

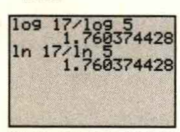
The change-of-base theorem is needed to graph logarithmic functions with bases other than 10 and e (and sometimes with one of those bases). For instance,

to graph $y = \log_3(x - 1)$, graph $y = \frac{\log(x - 1)}{\log 3}$ or $y = \frac{\ln(x - 1)}{\ln 3}$.

The next example shows how the change-of-base theorem is used to find logarithms to bases other than 10 or e with a calculator.

Many examples include optional graphing calculator coverage.

EXAMPLE 7
Using the change-of-base theorem



The result of Example 7(a) is valid for *either* natural or common logarithms.

Use natural logarithms to find each of the following. Round to the nearest hundredth.

(a) $\log_5 17$

Use natural logarithms and the change-of-base theorem.

$$\log_5 17 = \frac{\log_e 17}{\log_e 5} = \frac{\ln 17}{\ln 5} \approx \frac{2.8332}{1.6094} \approx 1.76$$

To check, use a calculator along with the definition of logarithm, to verify that $5^{1.76} \approx 17$.

Discovering Connections exercises tie together different topics and highlight the relationships among various concepts and skills.

DISCOVERING CONNECTIONS (Exercises 39–44)

The solution set of $f(x) = 0$ consists of all x-values for which the graph of $y = f(x)$ intersects the x-axis (i.e., the x-intercepts). The solution set of $f(x) < 0$ consists of all x-values for which the graph lies below the x-axis, while the solution set of $f(x) > 0$ consists of all x-values for which the graph lies above the x-axis.

In Chapter 2 we saw how a sign graph can be used to solve a quadratic inequality. Graphical analysis allows us to solve such inequalities as well. Work the following exercises in order. They demonstrate why we must reverse the direction of the inequality sign when multiplying or dividing an inequality by a negative number.

39. Graph $f(x) = x^2 + 2x - 8$. This function has a graph with two x-intercepts. What are they?

40. Based on the graph from Exercise 39, what is the solution set of $x^2 + 2x - 8 < 0$?

41. Now graph $g(x) = -f(x) = -x^2 - 2x + 8$. Using the terminology of Chapter 3, how is the graph of g obtained by a transformation of the graph of f?

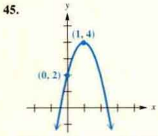
42. Based on the graph from Exercise 41, what is the solution set of $-x^2 - 2x + 8 > 0$?

43. How do the two solution sets of the inequalities in Exercises 40 and 42 compare?

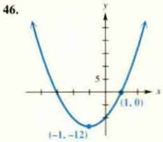
44. Write a short paragraph explaining how Exercises 39–43 illustrate the property involving multiplying an inequality by a negative number.

In Exercises 45 and 46, find a polynomial function f whose graph matches the one in the figure. Then use a graphing calculator to graph the function and verify your result.

45.



46.



t dollars from 1985
quadratic function
3x + 3954
5 and f(x) is in bil-
pply, what would be
Note: There are pit-
far into the future.)
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by the U.S. Court of
4 and 1990 can be
odel
9x + 31,676
984. Based on this
ber of cases com-

menced in 1996? (*Note:* There are pitfalls in using models to predict far into the future.)

49. Between 1985 and 1989, the number of female suicides by firearms in the United States each year can be modeled by

$$f(x) = -17x^2 + 44.6x + 2572$$

where $x = 0$ represents 1985. Based on this model, in what year did the number of such suicides reach its peak?

50. The number of infant deaths during the past decade has been decreasing. Between 1980 and 1989, the number of infant deaths per 1000 live births each year can be approximated by the function

$$f(x) = .0234x^2 - .5029x + 12.5$$

where $x = 0$ corresponds to 1980.

Solve the problem involving a polynomial function model. See Example 8.

65. From 1930 to 1990 the rate of breast cancer was nearly constant at 30 cases per 100,000 females whereas the rate of lung cancer in females over the same period increased. The number of lung cancer cases per 100,000 females in the year t (where $t = 0$ corresponds to 1930) can be modeled using the function defined by $f(t) = 2.8 \times 10^{-4}t^3 - .011t^2 + .23t + .93$. (*Source:* Valanis, B., *Epidemiology in Nursing and Health Care*, Appleton & Lange, Norwalk, Connecticut, 1992.)

(a) Use a graphing calculator to graph the rates of breast and lung cancer for $0 \le t \le 60$. Use the window [0, 60] by [0, 40].

(b) Determine the year when rates for lung cancer first exceeded those for breast cancer.

(c) Discuss reasons for the rapid increase of lung cancer in females.

66. The number of military personnel on active duty in the United States during the period 1985 to 1990 can be determined by the cubic model $f(x) = -7.66x^3 + 52.71x^2 - 93.43x + 2151$, where $x = 0$ corresponds to 1985, and $f(x)$ is in thousands. Based on this model, how many military personnel were on active duty in 1990? (*Source:* U.S. Department of Defense.)

67. A survey team measures the concentration (in parts per million) of a particular toxin in a local river. On a normal day, the concentration of the toxin at time x (in hours) after the factory upstream dumps its waste is given by $g(x) = -.006x^4 + .14x^3 - .05x^2 + .02x$, where $0 \le x \le 24$.

(a) Graph $y = g(x)$ in the window [0, 24] by [0, 200].

(b) Estimate the time at which the concentration is greatest.

(c) A concentration greater than 100 parts per million is considered pollution. Using the graph from part (a), estimate the period during which the river is polluted.

68. During the early part of the twentieth century, the deer population of the Kaibab Plateau in Arizona experienced a rapid increase because hunters had reduced the number of natural predators and because the deer were protected from hunters. The increase in population depleted the food resources and eventually caused the population to decline. For the period from 1905 to 1930, the deer population was approximated by $D(x) = -.125x^5 + 3.125x^4 + 4000$, where x is time in years from 1905.

(a) Graph $y = D(x)$ in the window [0, 50] by [0, 120,000].

(b) From the graph, over what period of time (from 1905 to 1930) was the deer population increasing? Relatively stable? Decreasing?

69. The table lists the total annual amount (in millions of dollars) of government-guaranteed student loans from 1986 to 1994. (*Source: USA TODAY.*)

Year	*Amount*
1986	8.6
1987	9.8
1988	11.8
1989	12.5
1990	12.3
1991	13.5
1992	14.7
1993	16.5
1994	18.2

(a) Graph the data with the following three function definitions, where x represents the year.

(i) $f(x) = .4(x - 1986)^2 + 8.6$

(ii) $f(x) = 1.088(x - 1986) + 8.6$

(iii) $f(x) = 1.455\sqrt{x - 1986} + 8.6$

(b) Discuss which function definition models the data best.

70. The table lists the number of Americans (in thousands) who are expected to be over 100 years old for selected years. (*Source:* U.S. Census Bureau.)

Year	*Number*
1994	50
1996	56
1998	65
2000	75
2002	94
2004	110

(a) Use graphing to determine which polynomial best models the number of Americans over 100 years old where $x = 0$ corresponds to 1994.

(i) $f(x) = 6.057x + 44.714$

Many exercises and examples are based on **Real Data,** and many require reading graphs and charts.

Writing and **Conceptual Exercises** are included to aid students in applying the concepts presented.

New Features

Several new features have been incorporated in this edition. The design has been developed to enhance the pedagogical features and increase their accessibility.

- Each chapter opens with a genuine application of the material to be presented. Corresponding examples and exercises, identified with a special icon, are located throughout the chapter.
- We have made an effort to point out the many connections between mathematical topics in this course and those studied earlier, as well as connections between mathematics and the "real world." Optional Connections boxes presenting such topics are included in many sections throughout the book. Most of them include thought-provoking questions for writing or class discussion. In addition, we have included a feature in most exercise sets called Discovering Connections. These groups of exercises tie together different topics and highlight the relationships among various concepts and skills.
- Graphing calculator comments and screens are given throughout the book as appropriate. These are identified with an icon, so that an instructor may choose whether or not to use them. We know that many students own graphing calculators and may need guidance for using them, even if they are not a required part of the course.
- Many examples and exercises are based on real data, and many require reading charts and graphs.
- The exercise sets have been completely rewritten and contain many new exercises, including more conceptual and writing exercises (which are marked by symbols in the instructor's edition), as well as optional graphing calculator exercises. Those exercises that require applying the topics in a section to ideas beyond the examples are marked as challenging in the instructor's edition.
- Cautions and notes are included to highlight common student errors and misconceptions. Some of these address concepts that students often find difficult or confusing.

Supplements

For the Instructor

Annotated Instructor's Edition With this volume, instructors have immediate access to the answers to every exercise in the text, excluding proofs and writing exercises. In a special section at the end of the book, each answer is printed next to or below the corresponding text exercise. In addition, challenging exercises, which will require most students to stretch beyond the concepts discussed in the text, are marked with the symbol ▲. The conceptual (◎) and writing (✍) exercises are also marked in this edition so instructors may assign these problems at their discretion. (Graphing calculator exercises are marked by 🖩 in both the student's and instructor's editions.)

Instructor's Resource Manual Included here are two forms of a pretest; six versions of a chapter test for each chapter—four open-response and two multiple-choice—additional test items for each chapter; and two forms of a final examination. Answers to all tests and additional exercises also are provided. Answers to most of the textbook exercises are included as well.

Instructor's Solution Manual This manual includes complete, worked-out solutions to every even exercise in the text (excluding most writing exercises).

Test Generator/Editor for Mathematics with QuizMaster is a computerized test generator that lets instructors select test questions by objective or section or use a ready-made test for each chapter. The software is algorithm driven so that regenerated number values maintain problem types and provide a large number of test items in both multiple-choice and open-response formats for one or more test forms. The **Editor** lets instructors modify existing questions or create their own including graphics and accurate math symbols. Tests created with the **Test Generator** can be used with **QuizMaster,** which records student scores as they take tests on a single computer or network, and prints reports for students, classes, or courses. CLAST and TASP versions of this package are also available (IBM, DOS/Windows, and Macintosh).

For the Student

Student's Solution Manual Complete, worked-out solutions are given for odd-numbered exercises and chapter review exercises and all chapter test exercises in a volume available for purchase by students. In addition, all new cumulative review exercises with worked-out solutions are provided.

Student's Study Guide Written in a semiprogrammed format, the study guide includes a pretest and posttest for each chapter, plus exercises that give additional practice and reinforcement for students.

Videotapes A new videotape series has been developed to accompany *College Algebra,* Seventh Edition. In a separate lesson for each section of the book, the series covers all objectives, topics, and problem-solving techniques within the text.

Interactive Mathematics Tutorial Software with Management System is an innovative software package that is objective-based, self-paced, and algorithm driven to provide unlimited opportunity for review and practice. Tutorial lessons provide examples, progress-check questions, and access to an on-line glossary. Practice problems include hints for the first incorrect responses, solutions, textbook page references, and on-line tools to aid in computation and understanding. Quick Reviews for each section focus on major concepts. The optional **Management System** records student scores on disk and lets instructors print diagnostic reports for individual students or classes. Student versions, which include record-keeping and practice tests, may be purchased by students for home use.

Acknowledgments ▼▼▼

We are grateful to the many users of the sixth edition and to our reviewers for their insightful comments and suggestions. It is because they take the time to write thoughtful reviews that our textbooks continue to meet the needs of students and their instructors.

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Margaret L. Lial
E. John Hornsby, Jr.
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An Introduction to Scientific and Graphing Calculators

In the past, some of the most brilliant minds in mathematics and science spent long, laborious hours calculating values for logarithmic and trigonometric tables. These tables were essential to solve equations in real applications. Because they could not predict what values would be needed, the tables were sometimes incomplete. During the second half of the twentieth century, computers and sophisticated calculators appeared. These computing devices are able to evaluate mathematical expressions and generate tables in a fraction of a second. As a result, the study of mathematics is changing dramatically.

Although computers and calculators have made a profound difference, they have *not* replaced mathematical thought. Calculators cannot decide whether to add or subtract two numbers in order to solve a problem—only you can do that. Once you have made this decision, calculators can efficiently determine the solution to the problem. In addition, graphing calculators also provide important graphical and numerical support to the validity of a mathematical solution. They are capable of exposing errors in logic and pointing to patterns. These patterns can lead to conjectures and theorems about mathematics. The human mind is capable of mathematical insight and decision making, but is not particularly proficient at performing long arithmetic calculations. On the other hand, calculators are incapable of possessing mathematical insight, but are excellent at performing arithmetic and other routine computations. In this way, calculators complement the human mind.

If this is your first experience with a scientific or graphing calculator, the numerous keys and strange symbols that appear on the keyboard may be intimidating. Like any learning experience, take it a step at a time. You do not have to understand every key before you begin using your calculator. Some keys may not even be needed in this course. The following explanations and suggestions are intended to give you a brief overview of scientific and graphing calculators.

It is not intended to be complete or specific toward any particular type of calculator. You may find that some things are different on your calculator. *Remember, a calculator comes with an owner's manual.* This manual is essential in learning how to use your calculator.

Scientific Calculators ▼▼▼

Two basic parts of any calculator are the keyboard and the display. The keyboard is used to input data—the display is used to output data. Without correct input, the displayed output is meaningless. Most scientific calculators do not display the entire arithmetic expression that is entered, but only display the most recent number inputted or outputted. If an arithmetic expression is entered incorrectly, it is not possible to edit it. The entire expression must be entered again.

Order of Operation

The order in which expressions are entered into a calculator is essential to obtaining correct answers. Operations on a calculator can usually be divided into two basic types: unary and binary. Unary operations require that only one number be entered. Examples of unary operations are $\sqrt{x}$, x^2, $x!$, $\sqrt[3]{x}$, and $\log x$. When entering a unary operation on a scientific calculator, the number is usually entered first, followed by the unary operation. For example, to find the square root of 4, press the key [4], followed by the square root key. Binary operations require that two numbers are entered. Examples of binary operations are $+$, $-$, $\times$, $\div$, and x^y. When evaluating a binary operation on a scientific calculator, the operation symbol is usually entered between the numbers. Thus, to add the two numbers 4 and 5, enter [4] [+] [5] [=]. However, on some calculators, such as those made by Hewlett Packard, it is necessary to use *Reverse Polish Notation* (RPN). In RPN the operation is entered last, after the operands. One advantage of RPN is that parentheses are usually not necessary.

Every calculator has a set of built-in precedence rules that can be found in the owner's manual. For example, suppose that the expression $3 + 4 \times 2 =$ is entered, from left to right, into a scientific calculator. The output will usually be 11 and not 14. This is because multiplication is performed before addition in the absence of parentheses. Parentheses can always be used to override existing precedence rules. *When in doubt, use parentheses.* Try evaluating $\frac{24}{4-2}$. It should be entered as $24 \div (4 - 2) =$ in order to obtain the correct answer of 12. This is because division has precedence over subtraction.

Scientific Notation

Numbers that are either large or small in absolute value are often displayed using scientific notation. The numeric expression 2.46 E12 refers to the large number 2.46×10^{12}, while the expression 2.46 E−12 refers to the small positive number 2.46×10^{-12}. Try multiplying one billion times ten million. Observe the output on your calculator.

Precision and Accuracy

Precision refers to the number of digits a calculator will display. When $\frac{1}{3}$ is evaluated, a calculator may display 0.333333333. This answer is approximate. The displayed precision of most calculators is between 8 and 12 digits. Accuracy is different from precision. It refers to the number of correct digits that an answer contains, compared to the true value. If a scale is misread as 129.6 pounds, when the actual answer is 145.8 pounds, then the number 129.6 has four digits of precision, but only one digit of accuracy. Many times when using a calculator to solve a real application, it will display ten digits of precision, but only a few digits will be accurate or meaningful. For example, suppose you drive 100 miles on 3 gallons of gas. A calculator would say that your mileage is $100 \div 3 \approx 33.33333333$. The precision of this answer is ten digits. The accuracy is probably not ten digits unless both the mileage and amount of gasoline were measured in an exceedingly accurate manner. It would be more reasonable or accurate to say that the mileage is about 33 miles per gallon, rather than 33.33333333 miles per gallon.

Second and Inverse Keys

Because the size of the keyboard is limited, there is often a 2nd or INV key. This key can be used to access additional features. These additional features are usually labeled above the key in a different color.

Graphing Calculators ▼▼▼

Graphing calculators provide several features beyond those found on scientific calculators. The bottom rows of keys on a graphing calculator are often similar to those found on scientific calculators. Graphing calculators have additional keys that can be used to create graphs, make tables, analyze data, and change settings. One of the major differences between graphing and scientific calculators is that a graphing calculator has a larger viewing screen with graphing capabilities.

Editing Input

The screen of a graphing calculator can display several lines of text at a time. This feature allows the user to view both previous and current expressions. If an incorrect expression is entered, a brief error message is displayed. It can be viewed and corrected by using various editing keys—much like a word-processing program. You do not need to enter the entire expression again. Many graphing calculators can also recall past expressions for editing or updating.

Order of Operation

Arithmetic expressions on graphing calculators are usually entered as they are written in mathematical equations. As a result, unary operations like $\sqrt{x}$, $\sqrt[3]{x}$, and $\log x$ are entered first, followed by the number. Unary operations like x^2 and $x!$ are entered after the number. Binary operations are entered in a manner similar to most scientific calculators. The order of operation on graphing calculators is also important. For example, try evaluating the expression $\sqrt{2 \times 8}$. If this expression is entered as it is written, without any parentheses, a graphing calculator may display 11.3137085 and not 4. This is because a square root is performed before multiplication. To prevent this error, use parentheses around 2×8.

Calculator Screen

If you look closely at the screen of a graphing calculator, you will notice that the screen is composed of many tiny rectangles or points called pixels. The calculator can darken these rectangles so that output can be displayed. Many graphing calculator screens are approximately 96 pixels across and 64 pixels high. Computer screens are usually 640 by 480 pixels or more. For this reason, you will notice that the resolution on a graphing calculator screen is not as clear as on most computer terminals. With a graphing calculator, a straight line will not always appear to be exactly straight and a circle will not be precisely circular. Because of the screen's low resolution, graphs generated by graphing calculators may require mathematical understanding to interpret them correctly.

Viewing Window

The viewing window for a graphing calculator is similar to the viewfinder in a camera. A camera cannot take a picture of an entire view in a single picture. The camera must be centered on some object and can only photograph a subset of the available scenery. A person may want to photograph a close-up of a face or a person standing in front of a mountain. A camera with a zoom lens can capture different views of the same scene by zooming in and out. Graphing calculators have similar capabilities. The xy-coordinate plane is infinite. The calculator screen can show only a finite, rectangular region in the xy-coordinate plane. This rectangular region must be specified before a graph can be drawn. This is done by setting minimum and maximum values for both the x- and y-axes. Determining an appropriate viewing window is often one of the most difficult things to do. Many times it will take a few attempts before a satisfactory window size is found. Like many cameras, the graphing calculator can also zoom in and out. Zooming in shows more detail in a small region of a graph, whereas zooming out gives a better overall picture of the graph.