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Wavelets, Approximation, and Statistical Applications

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Preface

The mathematical theory of *ondelettes* (wavelets) was developed by Yves Meyer and many collaborators about 10 years ago. It was designed for approximation of possibly irregular functions and surfaces and was successfully applied in data compression, turbulence analysis, image and signal processing. Five years ago wavelet theory progressively appeared to be a powerful framework for nonparametric statistical problems. Efficient computational implementations are beginning to surface in this second lustrum of the nineties. This book brings together these three main streams of wavelet theory. It presents the theory, discusses approximations and gives a variety of statistical applications. It is the aim of this text to introduce the novice in this field into the various aspects of wavelets. Wavelets require a highly interactive computing interface. We present therefore all applications with software code from an interactive statistical computing environment.

Readers interested in theory and construction of wavelets will find here in a condensed form results that are somewhat scattered around in the research literature. A practioner will be able to use wavelets via the available software code. We hope therefore to address both theory and practice with this book and thus help to construct bridges between the different groups of scientists.

This text grew out of a French-German cooperation (*Séminaire Paris-Berlin, Seminar Berlin-Paris*). This seminar brings together theoretical and applied statisticians from Berlin and Paris. This work originates in the first of these seminars organized in Garchy, Burgundy in 1994. We are confident that there will be future research work originating from this yearly seminar.

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Symbols and Notation

φ	father wavelet
ψ	mother wavelet
$S1, S2, \ldots$	symmlets
$D1, D2, \ldots$	Daubechies wavelets
$C1, C2, \ldots$	Coiflets
ISE	integrated squared error
$MISE$	mean integrated squared error
$\mathbb{R}$	the real line
$\mathbb{Z}$	set of all integers in $\mathbb{R}$
l_p	space of p-summable sequences
$L_p(\mathbb{R})$	space of p-integrable functions
$W_p^m(\mathbb{R})$	Sobolev space
$B_p^{sq}(\mathbb{R})$	Besov space
$\mathcal{D}(\mathbb{R})$	space of infinitely many times differentiable compactly supported functions
$S'(\mathbb{R})$	Schwartz space
H^λ	Hölder smoothness class with parameter λ
(f, g)	scalar product in $L_2(\mathbb{R})$
$\|\|f\|\|_p$	norm in $L_p(\mathbb{R})$
$\|\|a\|\|_{l_p}$	norm in l_p
$\|\|f\|\|_{spq}$	norm in $B_p^{sq}(\mathbb{R})$
ONS	orthonormal system
ONB	orthonormal basis
MRA	multiresolution analysis
RHS	right hand side
LHS	left hand side
DWT	discrete wavelet transform

$f * g$	convolution of f and g
$I\{A\}$	indicator function of a set A
a.e.	almost everywhere
$supp\ f$	support of function f
ess sup	essential supremum
$f^{(m)}$	m-th derivative
$\tau_h f(x) = f(x-h)$	shift operator
$\omega_p^1(f,t)$	modulus of continuity in the L_p norm
$K(x,y)$	kernel
δ_{jk}	Kronecker's delta
$\simeq$	asymptotic identical rate
$\sum_k$	sum over all $k \in \mathbb{Z}$
card Ω	cardinality of a set Ω

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Chapter 1

Wavelets

1.1 What can wavelets offer?

A wavelet is, as the name suggests, a small wave. Many statistical phenomena have wavelet structure. Often small bursts of high frequency wavelets are followed by lower frequency waves or vice versa. The theory of wavelet reconstruction helps to localize and identify such accumulations of small waves and helps thus to better understand reasons for these phenomena. Wavelet theory is different from Fourier analysis and spectral theory since it is based on a local frequency representation.

Let us start with some illustrative examples of wavelet analysis for financial time series data. Figure 1.1 shows the time series of 25434 log(ask) – log(bid) spreads of the DeutschMark (DEM) - USDollar (USD) exchange rates during the time period of October 1, 1992 to September 30, 1993. The series consists of offers (bids) and demands (asks) that appeared on the FXFX page of the Reuters network over the entire year, see Bossaerts, Hafner & Härdle (1996), Ghysels, Gourieroux & Jasiak (1995). The graph shows the bid - ask spreads for each quarter of the year on the vertical axis. The horizontal axis denotes time for each quarter.

The quarterly time series show local bursts of different size and frequency. Figure 1.2 is a zoom of the first quarter. One sees that the bid-ask spread varies dominantly between 2 - 3 levels, has asymmetric behavior with thin but high rare peaks to the top and more oscillations downwards. Wavelets provide a way to quantify this phenomenon and thereby help to detect mechanisms