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MATHEMATICS

PREPARATORY TO

STATISTICS AND FINANCE

统计与财务数学

BY

GEORGE N. BAUER

UNIVERSITY OF NEW HAMPSHIRE

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MATHEMATICS PREPARATORY TO
STATISTICS AND FINANCE



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PREFACE

Many students in our universities are studying statistics and the mathematics of finance as part of a business course. They often meet with difficulties because of the mathematics involved. The presentation here offered is the result of a course designed to enable the students of the University of New Hampshire to carry successfully the work in statistics and the mathematics of finance. The material has been used with several groups of freshmen students for three years.

This is not a book dealing with the subject of statistics; neither does it attempt to present the subject of the mathematics of finance. It is a study of the simpler mathematical methods and principles that occur frequently in elementary courses in these subjects. But many of the principles and methods included are applicable to the study of phenomena in other fields. They have a wide range of usefulness.

The illustrative exercises have been drawn in part from business and statistical problems; but the aim has not been to present a large number of exercises of this character. The emphasis throughout has been placed on the underlying principles and methods. In order to have these appear the more clearly, many exceedingly simple exercises have been used.

It has been found desirable to treat logarithms rather more fully and more in detail than is customary, in order that the students may be at ease when logarithms arise in various combinations.

Throughout the entire work, an attempt was made to

secure a cumulative effect. As a rule, a principle once introduced recurs again and again so that a feeling of mastery may be attained.

The material here presented has been found ample as a basis for the first course in statistics. Additional preparation is needed for advanced, theoretical courses.

I wish to express my appreciation of the coöperation given so freely by all the members of the Department of Mathematics while using the material of this course with their several classes.

G. N. B.

DURHAM, N. H.

July, 1929.

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MATHEMATICS

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CHAPTER I

GRAPHS AND CHARTS

1. **A few quotations.** “Mathematics in general is fundamentally the science of self-evident things.” KLEIN, FELIX.

“The whole of mathematics consists in the organization of a series of aids to the imagination in the process of reasoning.” WHITEHEAD, A. N.

“Mathematics, the science of the ideal, becomes the means of investigating, understanding, and making known the world of the real. The complex is expressed in terms of the simple.” WHITE, WILLIAM F.

“Probably among all the pursuits of the university, mathematics preeminently demands self-denial, patience, and perseverance. . . .” TODHUNTER, ISAAC.

2. **The aim.** In the present course we shall aim to present some of the simpler mathematical methods and principles that are useful in dealing with problems of finance, business, and statistics.

But the principles and methods to be introduced are applicable to a much wider range of problems than those of finance, business, and statistics. It would require a long list to indicate even the general fields to which they apply, for such a list would include agriculture, architecture, chemistry, physics, biology, navigation, transportation, sociology, economics, finance, insurance, and a variety of

other subjects. No attempt is made to apply the principles to various fields; the main objective of our study is an understanding of the simpler mathematical methods and principles used in finance, business, and statistics.

3. Charts and graphs. Graphical methods are used extensively in modern business and in statistical studies. A fact, or a series of facts, can frequently be shown more effectively by graphical methods than by any other means. A graphical presentation is often more compelling than a verbal description or a table of figures.

The words graph and chart may be used interchangeably. Any diagram or drawing indicating any sort of relationship by means of dots, lines, or areas is a graph or a chart.

There are numerous charts in common use, and they serve a variety of purposes. Among the more common may be mentioned the plan chart; the circle chart, also called the disc chart, pie chart, 100% chart; the bar chart; the map chart; and the line chart. There is a great variety of others, but they do not enter the general scope of this course. Our chief concern centers about the so-called *line chart*.

4. Plan chart. The plan chart is used to show the layout of an office, a factory, or any other area; it is also used to show the successive steps in any operation, as in manufacturing an article or in shipping goods; and it is frequently used to show the plan of organization of a corporation, or any other group, the duties of each department of the organization, and to whom each member is responsible. Plan charts may assume a variety of forms, but the questions naturally studied by means of them are beyond the scope of the task we have set for ourselves, and so will not be pursued any further.

5. Circle chart. The circle or disc chart is often used to show the ratios of various items that enter to make a

certain total. Let us apply it to the receipts of the United States government. The accompanying table is based on figures taken from the Statistical Abstract of the United States, 1924.

RECEIPTS OF THE U. S. GOVERNMENT FOR FISCAL YEAR
ENDED JUNE 30, 1924

	AMOUNT	%
Customs	\$ 545,012,000	8.0
Income Tax	1,841,759,000	27.1
Miscellaneous Tax	952,531,000	14.0
Public Debt Receipts	2,207,129,000	32.5
Postal Revenues	572,949,000	8.5
All Other Sources	668,598,000	9.9
Total	\$6,787,978,000	100.0

The circumference of a circle is divided into 360 parts called degrees, and each part measures an angle of one degree at the center of the circle. The numerical measure of any central angle is equal to the measure of the intercepted arc. The total to be represented by the circle chart is thought of as representing 100%. One percent of the circumference corresponds to an arc of 3.6 degrees and measures a central angle of 3.6 degrees. The various items to be represented by the circle chart are expressed in terms of percentages of the whole, and the different items are then reduced to degrees by the simple relation 1% equals 3.6° . Applying this to the figures above we have

RECEIPTS OF THE U. S. GOVERNMENT FOR FISCAL YEAR, 1924

	AMOUNT	%	DEGREES
Customs	\$ 545,012,000	8.0	28.8
Income Tax	1,841,759,000	27.1	97.6
Miscellaneous Tax	952,531,000	14.0	50.4
Public Debt Receipts	2,207,129,000	32.5	117.0
Postal Revenues	572,949,000	8.5	30.6
All Other Sources	668,598,000	9.9	35.6
Total	\$6,787,978,000	100.0	360.0

In comparing any two items of the circle chart, we may compare the angles at the center of the circle, or the lengths of their intercepted arcs, or the magnitude of their respective areas. This is due to two theorems in plane geometry.

In the same circle, or in equal circles, two central angles have the same ratio as their intercepted arcs.

In the same circle, or in equal circles, two central angles have the same ratio as the areas of the corresponding sectors.

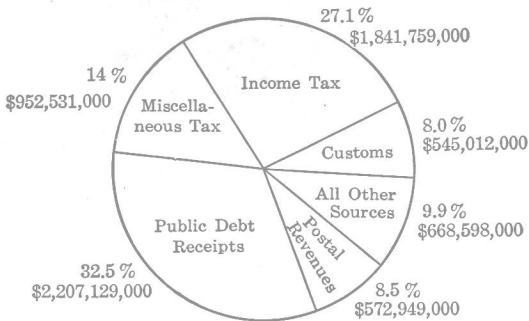


FIG. 1. Circle Chart Showing Receipts of the U. S. Government for Fiscal Year, 1924.

The illustration used above indicates the type of graphical representation made by means of the circle chart. Whenever any quantity under consideration is divided into its component parts or elements, and the total represented by means of the circle chart, the elements may be given in two ways, either as percentages of the whole, or in absolute amounts, as in the above figure.

EXERCISES

1. In analyzing the cost of a suit of clothes, it was found that 44% was given for labor, 34% for overhead, 11% for raw material, and 11% for profits. Make a pie chart representing the four items; also represent the amount of each item, the suit costing \$35.

2. In making up a family budget, with an income of \$2400, the following items appear. Make a circle chart.

Food.....	\$ 700
Clothing.....	380
Shelter.....	480
Operations.....	480
Advancement.....	240
Sundries.....	48
Savings.....	72
	\$2400

3. Represent by means of circle charts, the occupational distribution of children from 10 to 15 years of age gainfully employed in the United States in 1920. The following figures are taken from the fourteenth census of the United States.

OCCUPATIONS OF CHILDREN 10 TO 15 YEARS OF AGE
GAINFULLY EMPLOYED IN 1920

	TOTAL	MALES	FEMALES
POPULATION 10 TO 15 YEARS OF AGE.....	12,502,582	6,294,985	6,207,597
ALL OCCUPATIONS.....	1,060,858	714,248	346,610
Agriculture, forestry, and animal husbandry.....	647,309	459,283	188,071
Extraction of minerals.....	7,191	7,045	146
Manufacturing and mechanical industries.....	185,337	104,335	81,002
Transportation.....	18,912	15,617	3,295
Trade.....	63,368	49,234	14,134
Public service.....	1,130	1,085	45
Professional service.....	3,465	1,979	1,486
Domestic and personal service.....	54,006	16,082	37,924
Clerical occupations.....	80,140	59,633	20,507

(a) Make a circle chart for the males showing the distribution of those gainfully employed, according to occupation. As a preliminary step, compute the percentage of each group and reduce to degrees.

(b) Make a circle chart for the females showing the distribution of those gainfully employed, according to occupation.

(c) Make a circle chart for the males and females combined showing the distribution of children gainfully employed, according to occupation.

6. Bar charts. Bar charts offer a simple means for representing items graphically and for comparing them. Let us represent the receipts of the United States government as given in Art. 5, by means of the bar chart. In constructing a chart it is necessary to select a certain unit of measurement. The unit selected will necessarily depend upon the range of values to be represented graphically. We have given the following information, with the items arranged in order of magnitude.

RECEIPTS OF THE U. S. GOVERNMENT FOR FISCAL YEAR, 1924

	AMOUNT	%
(a) Public Debt Receipts.....	\$2,207,129,000	32.5
(b) Income Tax.....	1,841,759,000	27.1
(c) Miscellaneous Tax.....	952,531,000	14.0
(d) All Other Sources.....	668,598,000	9.9
(e) Postal Revenues.....	572,949,000	8.5
(f) Customs.....	545,012,000	8.0
Total.....	\$6,787,978,000	100.0

Let us select as a unit representing \$500,000,000 the bar given below. Arranging the items in order of magnitude, we then have the following representation:

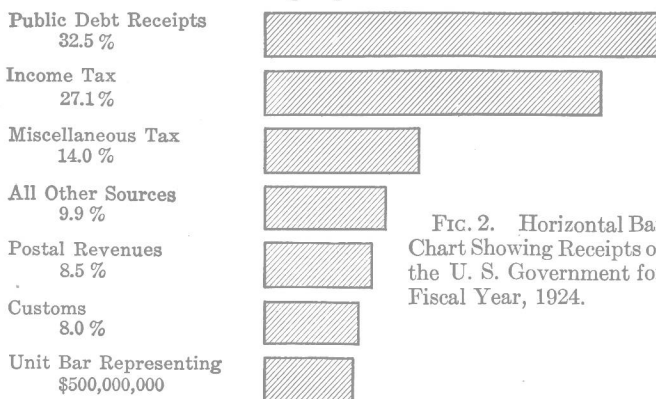


FIG. 2. Horizontal Bar Chart Showing Receipts of the U. S. Government for Fiscal Year, 1924.

For some purposes it is better to construct the bars vertically, rather than horizontally.

It is not essential that the items be arranged in order of magnitude; sometimes the items are arranged according to the time sequence, and sometimes it is desirable to bring a small item in sharp contrast with a large item. The nature of the problem will often indicate the order in which the items are to be presented.

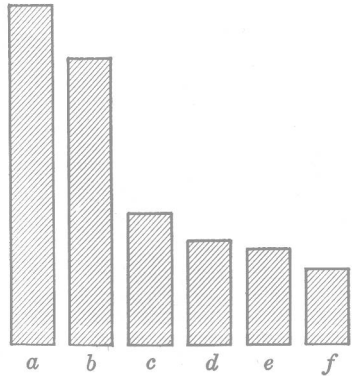


FIG. 3. Vertical Bar Chart Showing Receipts of the U. S. Government for Fiscal Year, 1924.

7. Modified form of bar chart. It is at times convenient to represent various items by one bar, using shadings or cross hatchings to distinguish the several items. This may be illustrated by a simple example. Let us take a family budget with the items,—

1. Food.....	\$ 700
2. Clothing.....	380
3. Shelter.....	480
4. Operation.....	480
5. Advancement.....	240
6. Sundries.....	48
7. Savings.....	72
Total.....	\$2400

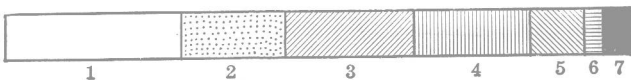


FIG. 4. Modified Bar Chart Showing the Items of a Family Budget.

GRAPHS AND CHARTS

The chart given as Figure 4 is self-explanatory.

A second modification of the bar chart is illustrated by the accompanying figure. Here the total length represents one item, namely the gross earnings. By inference, the upper part, with the cross-hatchings, represents the total cost of operation. The legends on the figure are an essential part of the chart and are self-explanatory.

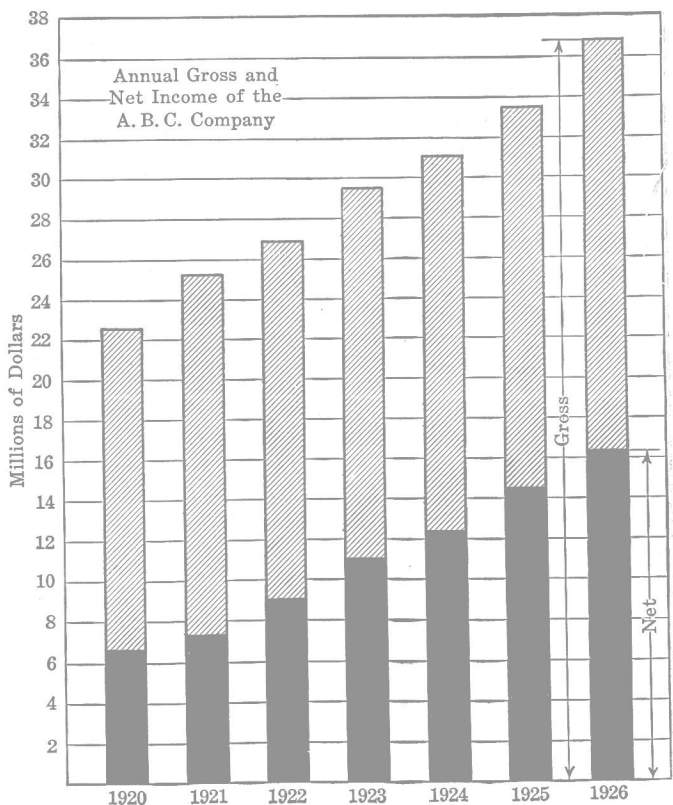


FIG. 5.

8. Balance sheet chart. It is sometimes useful to compare one group of items with another group, as in the balance sheet. This may be done by means of two bars placed side by side. As an illustration, let us use a simple, fictitious balance sheet. Suppose that a business firm has

Fixed Assets	\$25,000
Current Assets	72,000
	\$97,000

and

Fixed Liabilities	\$34,000
Current Liabilities	32,000
Net Worth	31,000
	\$97,000

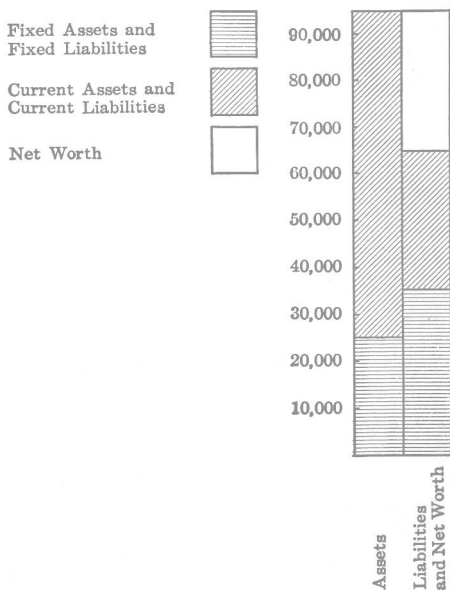


FIG. 6. Simple Balance Sheet Chart Illustrating a Fundamental Principle of Accounting.