Chester Piascik

College Mathematics

with Applications to Management, Economics, and the Social and Natural Sciences

COLLEGE MATHEMATICS

WITH APPLICATIONS TO MANAGEMENT, ECONOMICS, AND THE SOCIAL AND NATURAL SCIENCES

CHESTER PIASCIK
Bryant College

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To my wife, Francine, and sister, Joan, who gave countless hours towards the preparation of early versions of this manuscript.

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PREFACE

This text is designed to provide the mathematical concepts required in today's undergraduate business administration curriculum. Typically the algebra backgrounds of the students taking these courses are varied. Thus, one of my goals has been to make the text as readable as possible without sacrificing its mathematical content.

Another goal of this text is to train the student to think graphically. Accordingly, I have included sections in Chapter 2 on graphing polynomial and rational functions and in Chapter 3 on graphing exponential functions. These sections are also designed to prepare the student for calculus. Since a calculus student is often sketching and working with higher degree polynomial functions, rational functions, and exponentials and logarithmic functions, a prior study of these topics will help to reduce their "shock effect" when encountered in calculus.

This text contains a more thorough coverage of mathematics of finance in response to managements' express needs. Feedback from management professionals indicates that this is the mathematical topic requiring more extensive coverage in undergraduate studies. Many business decisions involve an analysis of cash flows under various conditions. This text includes, in addition to the usual mathematics of finance topics, equations of value, deferred annuities, and complex annuities.

Since we live in an age of inexpensive computing, greater emphasis is placed upon problem formulation. In this regard, I have included one section (Section 6–8) at the end of Chapter 6 (Linear Programming) on formulating linear programming problems from various fields of application.

Chapter 7 (Probability) provides the instructor with many options. The section on Bayes' formula may be omitted if desired. Similarly, Section 7–7 (Counting, Permutations, Combinations, and Probability) may be omitted, as well as the sections on Markov chains, if time does not permit their treatment.

The calculus chapters are designed to provide the student with essential concepts and viable applications. Chapter 8 presents an intuitive approach to the concepts of average rate of change and instantaneous rate of change. These serve as the structural foundation for the "rate of change" and "slope of tangent line" concepts of the derivative. The "Limits" and "Differentiability and Continuity" topics have been placed in separate sections so that the instructor has the flexibility to choose the level of treatment of these topics.

The chapter on optimization stresses both graphics and problem-solving. Applications to real-world problems accompany the calculus concepts.

Applications appear at numerous places in the calculus chapters as they do throughout the text. Section 10–8 (Continuous Cash Flows) attempts to integrate these financial models with those presented in the Mathematics of Finance chapter. In this regard, the Improper Integrals section contains an application to the present value of a perpetual cash flow. Finally, continuous probability distributions are presented as an application of integral calculus. The intent is to introduce the concept of a continuous probability distribution for germination in future courses.

We have six business cases in this text. Each is intended to show how math techniques are actually applied to business problems to solve such practical problems as forecasting the price per share for a stock, calculating the present value, or determining the annual inventory cost for a company. By working through these problems, it is hoped the student will see how mathematical techniques are applied to solve real-world business problems.

Acknowledgments

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I thank Bryant College for its support with the secretarial and typing aspects of this project. A special thanks goes to Jackie David who coordinated the typing of this project. Her efficiency and willingness to help were invaluable. I am indebted to the typists who worked faithfully and diligently on this project. These include Mary Afeltra, Sherry Maddison, Pamela Eddleston, and Susan LaRosa.

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A very special thanks goes to Ho Key Min for valuable suggestions and comments regarding the applications in this text. Another very special thanks goes to Patricia A. Fox for invaluable suggestions and perceptive criticisms for both the text and Instructor's Manual.

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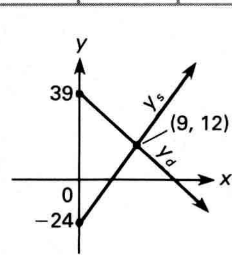
Chester Piascik

CONTENTS

LINEAR FUNCTIONS

1



1–1

THE REAL NUMBERS

All the numbers we will use in this text can be represented as points on a straight line. Such a representation can be constructed as follows. Begin with a straight line. Choose an arbitrary point, called the **origin**, on the line and label it 0. Then choose another point to the right of 0 and label it 1. Let the distance between 0 and 1 represent 1 unit of measure (see Figure 1–1). The point on the straight line 1 unit to the right of 1 is labeled 2, the point 1 unit to the right of 2 is labeled 3, etc. (see Figure 1–2). Also, the point on the straight line 1 unit to the left of 0 is labeled −1, the point 1 unit to the left of −1 is labeled −2, the point 1 unit to the left of −2 is labeled −3, etc. (see Figure 1–2). The straight line of Figure 1–2 is called the **real number line.** There is a one-to-one correspondence between the points on the real number line and the set of real numbers. In other words, any real number is associated with some point on this number line. Also, any point on this number line is associated with some real number. For example, the fraction 1/2 is associated with the point midway between 0 and 1; the number $1\frac{3}{4}$ is associated with the point three-quarters of the distance between 1 and 2; and the number −1/3 is associated with the point one-third the distance between 0 and −1 (see Figure 1–3).

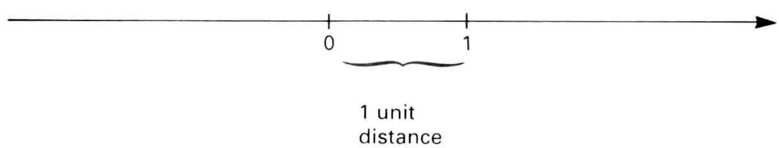


FIGURE 1–1

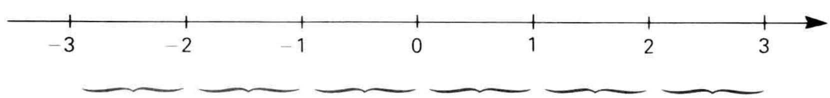


FIGURE 1–2

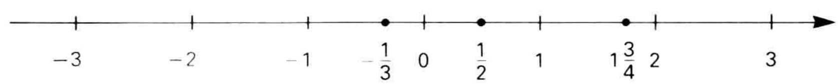


FIGURE 1–3

There are several types of real numbers:

1. **Counting Numbers** (also called **natural numbers**):

$$1, 2, 3, 4, 5, \ldots{}^*$$

2. **Whole Numbers:**

$$0, 1, 2, 3, 4, \ldots *$$

3. **Integers:**

$$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$$

4. **Rational Numbers**—All numbers that can be expressed as a quotient of two integers, where the denominator is not equal to 0. Examples are numbers such as 1/2, 3/4, 4/1, 3, 1.23, and nonterminating repeating decimals such as 5.838383 . . ., which can be written as $5.\overline{83}$.
5. **Irrational Numbers**—All real numbers that are not rational. Irrational numbers have decimal representations that are nonterminating and nonrepeating. Some examples are:

$$\begin{aligned} \sqrt{2} &= 1.4142135\ldots ** \\ \pi &= 3.1415926\ldots \\ e &= 2.718281\ldots \\ -\sqrt{5} &= -2.2360679\ldots \end{aligned}$$

Thus, a real number is either rational or irrational. The rational numbers include the integers, the integers include the whole numbers, and the whole numbers include the counting or natural numbers.

Inequality

If a number a lies to the left of a number b on the real number line, then "a is less than b." This is written $a < b$ (see Figure 1–4). Also, if a number b lies to the right of a number a on the real number line, then "b is greater than a." This is written $b > a$ (see Figure 1–5). Thus, the statement "5 is less than 6" is written $5 < 6$.

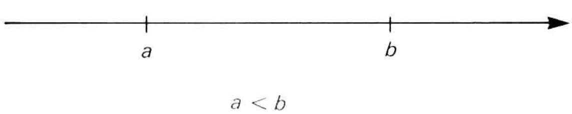


FIGURE 1–4

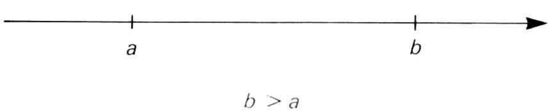


FIGURE 1–5

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*Here, the three dots indicate that the numbers continue indefinitely in the same manner.
**These dots indicate that the decimal representations are nonterminating.

The inequality phrases and their respective symbols are summarized as follows:

Inequality Phrase	Symbol
"Is less than"	$<$
"Is greater than"	$>$
"Is less than or equal to"	$\leq$
"Is greater than or equal to"	$\geq$
"Is not equal to"	$\neq$

Intervals

Sometimes, it is necessary to refer to all real numbers located between two numbers a and b on the real number line (see Figure 1–6). Such a set of numbers is called an **interval** and is expressed as all real numbers x such that

$$a < x < b$$

Observe that the endpoints, a and b, are not included in the above interval. This situation is graphically expressed by using an open circle at each endpoint (see Figure 1–6). If the endpoints are to be included, then the set must be written as

$$a \leq x \leq b$$

and graphically expressed by using a solid circle at each endpoint (see Figure 1–7).

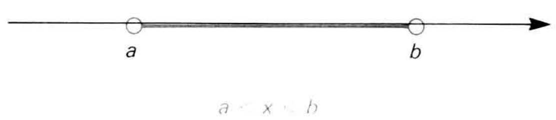

FIGURE 1–6

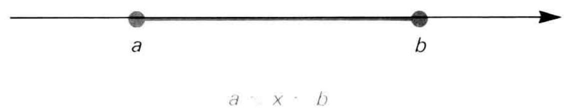

FIGURE 1–7

EXAMPLE 1–1 Graph all real numbers x such that $5 \leq x \leq 10$.

Solution This interval includes all real numbers between 5 and 10. The endpoints are included. The graph appears in Figure 1–8.

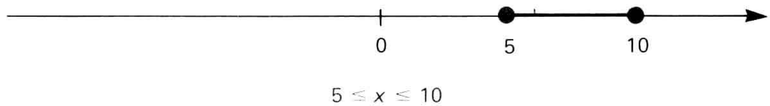

FIGURE 1–8

EXAMPLE 1–2 Express the interval of Figure 1–9 by using the variable x.

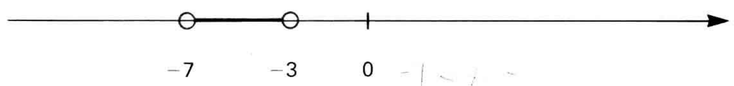

FIGURE 1–9

Solution This interval includes all real numbers between −7 and −3. The endpoints are not included. Hence, the interval is written as all real numbers x such that $-7 < x < -3$.

EXAMPLE 1–3 Graph all real numbers x such that $x \leq 9$.
Solution This interval includes all real numbers less than or equal to 9. The endpoint, 9, is included. The graph appears in Figure 1–10.

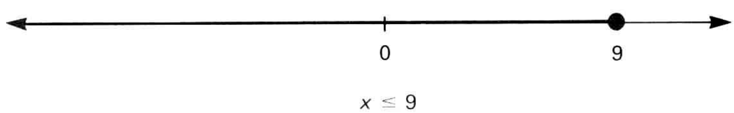

FIGURE 1–10

Absolute Value

The **absolute value** of a number x, written $|x|$, is the distance on the real number line from 0 to x. Thus, the absolute value of −3, written $|-3|$, is 3 since the distance from 0 to −3 is 3 units (see Figure 1–11). Also, the absolute value of 3, written $|3|$, is 3 since the distance from 0 to 3 is 3 units (see Figure 1–12). Note that the absolute value of a number is always nonnegative.

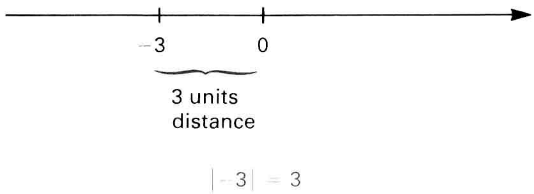

FIGURE 1–11

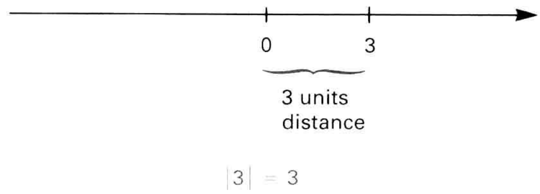

FIGURE 1–12

EXAMPLE 1–4 Solve for $|-7|$.
Solution $|-7| = 7$ since the distance between 0 and -7 on the real number line is 7 units.

EXAMPLE 1–5 Solve for $|8|$.
Solution $|8| = 8$ since the distance between 0 and 8 on the real number line is 8 units.

EXAMPLE 1–6 Graph all real numbers x such that $|x| \leq 5$ on the real number line.
Solution This interval includes all numbers x on the real number line such that the distance between 0 and x is less than or equal to 5. Thus, any real number x within 5 units distance of 0 has an absolute value less than or equal to 5. Hence, we write

$$-5 \leq x \leq 5$$

This interval is graphed in Figure 1–13.

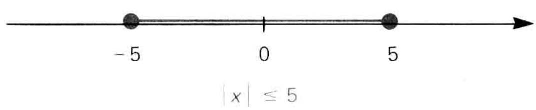

FIGURE 1–13

EXERCISES

1. State whether each of the following is true or false:

(A) $3 < 7$	(B) $-3 < -7$	(C) $-2 < -5$
(D) $2 < 5$	(E) $-6 < -2$	(F) $-3 > -7$
(G) $-2 > -5$	(H) $0 < 5$	(I) $0 > -3$
(J) $9 > 6$	(K) $8 > 10$	(L) $-6 < -1$

2. State whether each of the following is true or false:
   (A) Every counting number is a whole number.
   (B) Every whole number is an integer.
   (C) Every counting number is an integer.
   (D) Every integer is a rational number.
   (E) Every rational number is a real number.
   (F) Every integer is a whole number.
   (G) Every whole number is a counting number.
   (H) Every irrational number is a real number.
3. State whether each of the following is true or false:
   (A) 7 is a rational number.
   (B) $\frac{3}{5}$ is a rational number.
   (C) $-\frac{2}{3}$ is a rational number.
   (D) $\sqrt{11}$ is a rational number.
   (E) $\sqrt{11}$ is an irrational number.
   (F) 3.56345 . . . is an irrational number.
   (G) 4.7065 is an irrational number.
   (H) 2.767676 . . . is a rational number.
4. Sketch each of the following on the real number line:
   (A) $-5 \le x \le -1$ (B) $7 \le x \le 11$
   (C) $-4 < x < -2$ (D) $9 < x < 15$
   (E) $-3 < x \le 2$ (F) $2 \le x < 9$
   (G) $5 \le x$ (H) $x \ge 5$
   (I) $x \le -3$ (J) $x < 10$
   (K) $x > -2$ (L) $x > 4$
   (M) $2 < x$ (N) $x \ge -1$
   (O) $x \ne 2$ (P) $x = -3, x \ne 5$
5. Solve for each of the following:
   (A) $|0|$ (B) $|-1|$ (C) $|1|$
   (D) $|-21|$ (E) $|-2|$ (F) $|15|$
   (G) $|-15|$ (H) $|-20|$ (I) $|20|$
6. Sketch each of the following intervals on the real number line:
   (A) $|x| \le 4$ (B) $|x| < 8$
   (C) $|x| \le 10$ (D) $|x| < 10$
   (E) $|x| \ge 6$ (F) $|x| > 6$
   (G) $|x| \ne 5$ (H) $|x| \ne 3$

1–2

FUNCTIONS

A **function** is a rule that associates a unique **output value** with each element in the set of possible **input values.** Consider, for example, the conversion of temperature from degrees Fahrenheit to degrees Celsius. Given a temperature in degrees Fahrenheit (input value), we can find the corresponding value in degrees Celsius (output value) by the following rule:

$$\underbrace{\text{Celsius temperature}}_{\substack{\text{output}\\ \text{value}}} = \frac{5}{9}(\underbrace{\text{Fahrenheit temperature}}_{\substack{\text{input}\\ \text{value}}} - 32)$$

If C is temperature in degrees Celsius and F is temperature in degrees Fahrenheit, then this rule may be expressed by the equation

$$C = \frac{5}{9}(F - 32)$$

To determine the Celsius temperature (output value) associated with 50 degrees Fahrenheit, we substitute $F = 50$ (input value) into the equation and obtain

$$\begin{aligned} C &= \frac{5}{9}(50 - 32) \\ &= \frac{5}{9}(18) \\ &= 10 \end{aligned}$$

Thus, 10 degrees Celsius is associated with 50 degrees Fahrenheit. Since only one value of C is associated with a value of F, then this equation defines C as a function of F.

Observing the equation

$$\underset{\substack{\uparrow\\ \text{output}\\ \text{value}}}{C} = \frac{5}{9}(\underset{\substack{\uparrow\\ \text{input}\\ \text{value}}}{F} - 32)$$

note that the output value, C, is dependent upon the input value, F. Thus, C is called the **dependent variable**, and F is called the **independent variable.** This relationship is usually indicated by saying that C is a *function* of F.

Functional Notation

Often, a letter is used to represent a function. Specifically, if the letter f is used to name the function defined by the equation

$$y = 5x^2 + 2x + 7$$

then the dependent variable, y, is represented by the symbol $f(x)$, read "f of x." Thus, the preceding equation is written as

$$f(x) = 5x^2 + 2x + 7$$

To find the output value associated with $x = 3$, we replace x with 3 to obtain