Modern Mathematics for the Engineer

Second Series

Edited by

EDWIN F. BECKENBACH

SECOND SERIES

MODERN MATHEMATICS FOR THE ENGINEER

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Preface

For several years the University of California, through University Extension, has been conducting a highly successful series of lecture courses devoted to Modern Science for the Engineer. The lectures are available in book form in the *University of California Engineering Extension Series,* of which six volumes (see page ii) have preceded the present contribution.

The timeliness of the series—or perhaps the scientific alertness of the Eastern European countries—is attested to by the fact that, during its first three years of existence, the initial volume of "Modern Mathematics for the Engineer" was translated into Hungarian, Polish, and Russian.

Turning a second time to mathematics, University Extension appointed an Advisory Committee composed of individuals representing various universities and industrial organizations in California to plan the 1958–59 lecture series.

The Committee adopted as its objective the presentation of some exciting aspects of modern mathematics that either are presently applicable or promise soon to be applicable to science and engineering.

To achieve this objective, the Committee sought to obtain the services of a group of outstanding speakers who were experts on advanced applicable mathematics. The topics chosen were, for the most part, those that have had recent spectacular applications in mathematics, that have been applied or are likely soon to be applied in physical, sociological, and biological sciences, and that involve a degree of mathematical subtlety.

The high success of the course was ensured by the enthusiastic response of the distinguished group of lecturers who participated in the series. It was presented at five locations throughout the state: at Berkeley, Corona, Los Angeles, Palo Alto, and San Diego.

The volume that has resulted is intended for engineers, scientists, mathematicians, students, high-school and college teachers, and others who desire to become or remain informed concerning current applicable mathematical developments.

The first volume of "Modern Mathematics for the Engineer" was somewhat arbitrarily divided into three parts: Mathematical Models, Probabilistic Problems, and Computational Considerations. The present volume is similarly divided into three parts: Mathematical Methods, Statistical and Scheduling Studies, and Physical Phenomena.

Certainly the foregoing partitioning is not a sharp one; rather, it is one of emphasis. Thus the broad and deep mathematical methods developed in Part 1 are clearly motivated by physical problems; the solutions of the physical problems treated in Part 3 call for the use of the most modern mathematical methods; and the powerful probabilistic processes described in Part 2 are strongly evident in Chaps. 11 and 12 of Part 3.

Chapter 12, for example, might equally well have been combined with Chaps. 1 to 4 to constitute a comprehensive part entitled Operational Observations, or it might have been put in Part 2 because of its probabilistic content; but its greatest emphasis seemed to be on physical propagation phenomena, and it could well furnish the basis of a one-semester course in this branch of mathematical physics. Again, Professor Pólya's delightful concluding Chap. 16 is concerned with physical observations, but these observations lead to general conjectures—and the conjectures demand proof, which in turn involves ingenious and penetrating mathematical methods.

There are numerous examples and exercises throughout the book. Some of the exercises are at the ends of chapters, others are interspersed with the text, and still others are incorporated in the text. It is hoped that they will aid the reader in his over-all assimilation of the material and that they will add to the usefulness of the book.

The editor is most grateful to the authors for their excellent and prompt contributions to this volume; to the other Advisory Committee members, John L. Barnes, Clifford Bell, L. M. K. Boelter, George W. Brown, John C. Dillon, Gerald Estrin, James C. Fletcher, Bernard Friedman, Magnus R. Hestenes, John W. Miles, Russell R. O'Neill, Louis A. Pipes, C. T. Singleton, Ivan S. Sokolnikoff, Thomas H. Southard, Angus E. Taylor, Charles B. Tompkins, and John D. Williams for their efforts and excellent ideas; to the Course Coordinators, John C. Bowman, Bernard Friedman, Stanley B. Schock, and Victor Twersky for their smooth handling of lecture arrangements; once again to Clifford Bell, the Statewide Coordinator of the course, for his unobtrusive but highly valued leadership; and especially the editor thanks his secretaries, Mildred Webb and Patti Hansen, for their careful and efficient work.

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Foreword

The physical and economic world in which the modern engineer operates continues to grow more complex, putting ever greater demands on the mathematical models representing that world. During the five years since University Extension last offered a lecture series in "Modern Mathematics for the Engineer," reliance on a variety of these models has grown in an amazing fashion, due in no small measure to the adaptation of advanced mathematical techniques for use in connection with high-speed computing machines. Hilbert-space methods, always meaningful to the mathematicians who developed them, have now become useful also to the engineer and applied scientist, and the acceptance of probabilistic as well as deterministic analyses has become commonplace.

The present series was conceived with the objective of presenting some exciting aspects of modern mathematics. The course was designed for nonspecialists with training in engineering or science, high-school and college teachers of mathematics, and others desiring to remain *au courant* concerning mathematical developments. The material is intended to be quite understandable in the large, although not necessarily in complete detail, on the basis of the single lectures, and all of it should be applicable either now or in the reasonably near future to science and engineering.

We are pleased to share the stimulating experience of the second "Modern Mathematics for the Engineer" lecture series with you, the reader, through the pages of this book.

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During the last decade, there has been a remarkable expansion in the demand for advanced mathematics by the engineer. This demand has arisen in part because of the increasing complexities created by technological progress.

In advanced design it is frequently necessary to study a carefully constructed mathematical model before creating the physical model. The modern rockets and satellites, for example, could not have been built and successfully launched without careful mathematical analysis of the physical problem at hand.

Not only does mathematics enter into initial planning and design; it enters into testing programs as well. Data are collected and interpreted in accordance with a statistical theory. Once a product has been designed and tested, a mathematical theory of quality control is frequently used in the manufacture of the product.

More recently, mathematics has been found to be a useful tool in the field of production planning.

Thus mathematics enters into all phases of engineering and production.

The modern high-speed computing machine is playing an ever-increasing role in physical, biological, and social sciences, in engineering, and in business. The effective use of computers requires the aid of persons with a high degree of mathematical training and proficiency. Almost every branch of mathematics has been used on problems that have been successfully attacked with the help of computing machines.

These machines can be used for experimentation as well as for solving intricate mathematical problems. For example, a traffic problem has been simulated on a computing machine, and experiments have suggested means of traffic control that have significantly increased the flow of traffic in a congested area.