Mathematical Modeling in the Environment

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Part 1

An Interdisciplinary Introduction to Selected Problems in Ground Water, Air Pollution, and Hazardous Materials

Part 2

Further Development of Modeling Concepts

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This book is dedicated to Joanne,
my wife, my best friend, and my greatest source
of encouragement and support.

Preface

I wrote this book because I felt it just had to be written.

Too few mathematics professors have an opportunity to leave the academic world to pursue a different career, in my case thirteen years in environmental consulting, and then return to academia to tell about it. I left academia in 1977 with a craving to learn more about "real world" applications. Even though I was certified by my graduate degree as an applied mathematician, I had never really delved deeply into a single, real applied problem, and that was not unusual. Within a short time, I found myself totally immersed in the issues of Love Canal, Bhopal, Three Mile Island, and many other major environmental cases of the day. I finally left the full-time consulting environment in 1990 because I had learned much, wanted to share it in the classroom, and wanted to think about it unfettered from the daily pressures of the "bottom line." I think that there is a great deal in this field that is likely to be of interest to mathematics teachers and students at all levels, as well as to other students and professionals.

This book is primarily intended for use in college-level mathematics or interdisciplinary courses. However, there are several quite distinct ways in which one might use it:

- as a text for a lower level, interdisciplinary course in an inherently interesting subject area, one with a strong problem solving component and with natural opportunities to support other important skills such as information research and computer use;
- as a text for a first or later course in applied mathematics, requiring minimal prerequisites but able to make use of more advanced background, if available, in selected sections and exercises;
- as a source of individual topics that could be incorporated into other courses, such as calculus;
- as fairly easy "bedtime reading" for faculty and graduate students, who may well find that the discussion here puts many traditionally more abstract concepts in a more concrete perspective, and who may also pick up more of an appreciation of how modeling is used in the real world;
- as a source of some well motivated derivations and exercises that could enrich many courses or independent study programs.

For many years, the mathematics community placed its highest value, at least implicitly, on "pure mathematics," and it continued to educate the bulk of both undergraduate and graduate students with scant attention to mathematical modeling of real world problems. Even courses in applied mathematics were more survey courses in mathematical techniques than courses intended to give the students a real taste of the very different world in which mathematical modeling for applications is usually carried out. Many of today's professors are the products of those years.

The grim job prospects for many recent PhDs in mathematics are now causing this to change; but, more generally, the need for math graduates at all levels to be able to compete for jobs with students of engineering, physics, economics, and other quantitative fields encourages a reexamination of our educational programs. To put it more positively, I think that many of the traditional aspects of mathematics education, such as the logical analysis of issues and the search for simple, basic principles of understanding, can prepare mathematics students with unique strengths that will let them make excellent contributions in real world applications and give them valuable advantages that others may not have — as long as they can learn to reach outside the limits of their own discipline.

But beyond serious and committed students of mathematics, there is also a larger group of students who have practically no appreciation for the degree to which mathematical modeling permeates the operations of our modern society. They range from beginning freshmen who may be trying to decide what field of study to major in to that somewhat disaffected group of college students who want to study the very least mathematics that their schools will permit. If one is to give these students an experience that will cause them to explore their options more fully or to reexamine their assumptions, one must get to them early, for their first college math course may be their last. There is no time to teach a plethora of techniques or abstract concepts. Nor should one want to. Let's take these students right out into the real world and show them the societal problems that modelers are grappling with day in and day out. You don't need a lot of mathematical machinery to get involved in some of these problems. Furthermore, most students are already well-disposed to studying environmental issues; they only have to get over the shock of seeing a mathematician in front of the classroom instead of a biologist or someone else!

The biggest challenge in developing this book was in offering material to more than one audience. I wanted to provide something engaging to the quite different groups that are characterized above. The solution was the division of the book into two parts. The first part (Chapters 1–4) uses no calculus; the closest it comes is the use of the exponential function e^x, with a self-contained introduction even to that. The second part (Chapters 5–7) further develops each of the three environmental themes from earlier, making various uses of one-dimensional calculus and multidimensional calculus in well defined subsections for which the prerequisites are spelled out. This part also touches upon other fields, either to reinforce previous study (e.g., linear algebra) when the student has had that material, or to try to provide an enticing introduction to some new ideas (e.g., numerical analysis, probability).

For a more elementary class, the first part is enough for a semester's course, especially if the nonmathematical aspects of the material are explored as well. Even if such aspects are not treated as fully as I myself might like to do, there are additional topics in part 2, labeled as such, that do not use any calculus, as well as others that use only one-dimensional calculus. General education courses usually require minimal prerequisites and are sometimes offered even in the freshman year. I believe that this material is particularly well suited for such courses, as it uses no calculus, is interdisciplinary (math, science, societal issues, computers, information research, and field-trip opportunities), and deals with a subject about which students are motivated to learn. It also lends itself to group work in class and homework projects. A number of high schools have also incorporated material from the first part as enrichment units in their junior and senior level math courses.

A more advanced class could easily become quite involved in the mathematical issues raised in the second part of the book, and some of the exercises will be quite challenging even to the professional. Either a traditional course or a seminar structure could be used in such a class. The approach of this book is not to focus on mastering a wide range of techniques nor bringing students to the current state-of-the-art, however. Rather, it first immerses the students in an applied area with engaging nonmathematical assignments (e.g., reading news accounts of local environmental emergencies, web browsing), and then gives them experience with a number of important math topics (e.g., Laplace's equation, heat flow, diffusion, numerical methods, Gaussian distribution, physical modeling, fluid mechanics, and others) that structure the modeling approach to these issues.

Two key components found throughout are:

- constant interplay between physical reasoning and mathematical modeling;
- "learning by doing," as the material is presented via a combination of text and interspersed exercises, the latter suitable (if desired) for group work, even in class, or student presentation and discussion. (The questions are sometimes open-ended and invite disagreement and discussion.)

Computers are a natural tool for mathematical modeling, and I believe it would be difficult and less than optimal to try to teach about modeling without making use of them. There is a natural progression in the text. Roughly speaking, Chapter 2 is hand calculation, although some problem types could be simplified and organized with something like a spreadsheet program. Chapter 3 would benefit further from some programming device, such as a programmable calculator or computer spreadsheet. Chapter 4 is based on using one of several commercial packages, although one free one that has minimal computer hardware requirements is available on a single diskette. On the non-math side, on-line computer resources are encouraged for some of the information research questions.

A teaching challenge is that the teacher has to build up enough self-confidence to talk about various scientific concepts, such as soil, molecules, vapor pressure, and fire. Nothing is needed beyond what is in the book, but we in the mathematics profession simply are not used to teaching concepts that don't fit the nice structure of an equation or theorem. The students seem to handle it very well, even very weak students who always hated science in high school. Dealing with some of these concepts in response to computer menu requests seems to make it more motivating for the students to think about what they really mean. And a class field trip to a fire department (or a class speaker), where uniformed personnel throw these same concepts around constantly, makes them seem so much less academic that the students are no longer intimidated. The *Supplementary Material and Solutions Manual* is intended to help teachers over these obstacles, as well as to offer complete solutions to every single exercise, mathematical and nonmathematical, in the text.

Organizing this material into the above framework was quite a challenge. I started with many more topics in my initial teaching efforts, but gradually decided to provide a more thorough treatment of three themes that had a universal presence and many good local examples.

I sincerely hope that the reader will discover new interests in this material and that this work will make at least some small contribution to the improvement of our corps of citizens, leaders, and professionals as we approach the important environmental decisions of the future.

Charles R. Hadlock
Waltham, Massachusetts

Acknowledgments

I am deeply indebted to five institutions and to numerous individuals for their help in bringing this project to fruition.

Bentley College, in offering me a professorship in 1990, gave me the opportunity to realize my hopes of bringing into the classroom so many of the new viewpoints and interesting ideas I had encountered while working in the environmental industry since 1977. Dean Lee Schlorff and all my colleagues in the Mathematical Sciences Department have been strongly supportive of this curricular innovation. Barbara Nevils offered to be the first "guinea pig" in trying to learn and teach this material without previous environmental background, and her success at this convinced me of the viability of trying to share it with the greater mathematical community. David Carhart, Norman Josephy, Erl Sorenson and others have offered valuable comments and suggestions on the manuscript. Alex Schuh verified my solutions to almost every problem in the book and recommended valuable improvements. Alice Laye provided superb typing and production assistance.

The National Science Foundation provided generous support over the course of three years, and it was their acceptance of my proposal that represented the "alea iacta est" of the project. I must single out my project manager, Myra Smith, from Cornell University but on a temporary assignment at NSF, whose enthusiasm and confidence in this project were an inspiration. My NSF project advisory board, consisting of Ira Gessel (Brandeis), William Keane (Boston College), and James Ward (Bowdoin), were a constant source of good advice and encouragement.

The Massachusetts Institute of Technology provided the perfect working environment for much of the writing by offering me a visiting professorship for the academic year 1996–97. Not only did I have within the Department of Earth, Atmospheric, and Planetary Sciences a stimulating group of colleagues and excellent resources, but I also had an opportunity to teach from the manuscript to a group of lively and encouraging students from a wide range of disciplines. Daniel Burns, Timothy Grove, and Thomas Jordan made all this possible.

The Mathematical Association of America, in the personages of Andrew Sterrett and Don Albers, expressed strong interest in this project right from the start. During production, it has been a pleasure to work with Elaine Pedreira Sullivan and Beverly Ruedi.

Last but not least among the institutional groups, the many professional opportunities I received by virtue of my position at Arthur D. Little, Inc., really provided the underlying substance for the whole project. I value greatly my many stimulating associations with colleagues and clients, especially the late Dan Egan (EPA) who was most supportive in my early years of working in this field.

During the preparation of this work, I have also had the benefit of many valuable discussions with professionals in the field and with fellow mathematicians and educators, not listed above, some of whom provided detailed reviews of portions of the manuscript. I would particularly like to acknowledge contributions from Ben Fusaro (Florida State University), Anita Goldner (Framingham State College), Jack Guswa (HSI/GeoTrans), John Hagopian (Hazmat America), Don Miller (St. Mary's College), Steven Pennell (UMass Lowell), and Lou Rossi (UMass Lowell).

I would also like to recognize the key contribution made by a number of my former students at Bentley and at MIT, whose sharp intellect, attention to detail, and interest in the subject were both an encouragement to follow through on the project and a constant check against ambiguity and error. These include Matthew Berman, Wendy Brown, Ellen Clark, Diane Curry, Jennifer Forman, Hau Hwang, Caroline Martorano, Walter VanBuskirk, and many others.

In listing all the above contributors, I must myself take responsibility for any residual errors, and I would appreciate having them called to my attention for correction in later printings.

Contents

Part 1

An Interdisciplinary Introduction to Selected Problems in Ground Water, Air Pollution, and Hazardous Materials

1

Introduction

This book has two principal objectives:

1. To provide an in-depth introduction to several very important environmental issues.
2. To show how mathematical models are used routinely to help analyze these issues.

These objectives are closely interrelated. On the one hand, essentially every environmental problem has key quantitative aspects. For example, it is not enough simply to know that there is pollution somewhere, for, unfortunately, there is pollution almost everywhere. One needs to know how serious the pollution is, if it is getting better or worse, and how much it might be improved by implementing various possible remedial actions. These are all quantitative questions for which one would expect quantitative methods to be useful. On the other hand, by trying to make precise quantitative statements about environmental problems, one will actually develop a more precise and refined understanding of the underlying problems themselves.

The level of environmental awareness of society (both in the US and in many other countries) has grown immensely in the last decade or two. For example, the following are all practices that were routinely accepted a relatively short time ago, and, while still widely practiced, are now under much closer scrutiny and control:

- The disposal of garbage and other wastes in municipal dumps or "landfills," often located in swampy areas less desirable for construction and habitation. Now we recognize the migration potential of contaminants contained in waste materials, and we further understand that wetland areas are key entry points to surface and groundwater pathways through which such contaminants can easily move. Furthermore, the filling in of wetlands by waste disposal and other human activities eliminates a vital source of stability in the hydrologic cycle and can have far-reaching ecological implications.
- The use of buried underground storage tanks, often made of ordinary steel, for fuel oil and other chemicals. We now better recognize the "obvious" fact that such tanks will eventually corrode and leak, and that the resulting chemical contamination may persist for long periods of time and travel long distances before it is detected. By then it can be very hard and prohibitively expensive to clean up the problem.
- The disposal of sewage in such a way that it can enter lakes, rivers, and harbors in relatively untreated form. The scale of this problem ranges from large cities, where untreated sewage has often been allowed to flow into a harbor, to small bungalows and cottages on the