OPTIONS PRICING

AN INTERNATIONAL PERSPECTIVE

GORDON GEMMILL

Options pricing

An international perspective

Gordon Gemmill

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To Gerda

Preface

This book is aimed at the middle ground: between those excellent, but mathematically difficult, treatments that are available and those books with many words (but few ideas) on how to make a fortune with options. It should not only provide an accessible review of the subject, starting at the simplest possible level, but should also have sufficient detail on applications to be relevant for professionals. Examples are drawn from European and American markets, providing an 'international perspective'.

The book is divided into three parts. Part One is concerned with 'basics'. If you know something about options, then I suggest that you skim the first chapter. However, the second chapter on strategies should be of interest. Part Two is concerned with 'valuation of stock options'. It is the theoretical core of the book. Individual chapters deal with arbitrage bounds, the binomial and Black/Scholes methods, necessary adjustments to the methods in order to value stock options realistically and whether the underlying assumptions of the models are likely to invalidate them. A good comprehension of Part Two should enable the reader to solve new valuation problems as they rise. Part Three is concerned with the 'valuation of use of options on various other kinds of asset'. There are separate chapters on currencies, stock indices, portfolio insurance, interest rates, warrants/convertibles and commodities. This part concludes with chapters on 'exotic' options such as those on average rates, and whether option markets operate efficiently and have any impact on the markets for the underlying assets.

I have not been able to find a unified, up-to-date and accessible treatment of these topics, even though the subjects are well developed in the literature. Some chapters have sections that are more difficult to follow, and these are indicated with a star. Their omission should not affect continuity. There are also appendices to several of the chapters, containing results which, although not central to the argument of the book, will be of interest to some readers.

A suite of programs that implements most of the models in this book can be obtained from the author. A set of exercises is also available on request.

Options are fun, but they are often misunderstood, and I hope that some of their intellectual excitement may be gained by the reader.

Acknowledgements

I would like to thank the following people for offering comments on chapters and providing information: Kevin Connolly, Paul Dawson, Alfred Kenyon, Jenny Tanner, Stephen Taylor, Patrick Thomas, Xavier Trabia and two anonymous referees. No doubt some errors remain, but there would have been many more without their help. I would also like to thank my colleagues at the City University for giving me the time to complete this work.

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PART ONE

Basics

1
Introduction

A whole book on options may seem rather 'excessive'. Are they not just contracts which allow for the purchase or sale of an asset at a fixed price at some future date? Not quite. The agreements just described would be forward contracts. Options are different because they give the holder the choice of whether to go through with the fixed-price deal or not. The holder of an option can simply abandon the deal if that is desirable, whereas the holder of a forward contract is obliged to complete the deal.

One person's right is another person's constraint. 'Freedom for the pike is death for the minnow.' If the holder of an option benefits from the right to abandon it, the issuer (or writer) of an option must suffer the consequences. There are two sides to every deal and risks in options are quite different, depending on which side you are on. The buyer can only *lose* what has been paid for the option, the premium, because the option can be abandoned. On the other hand, the writer of an option can only *gain* the premium and no more, but can lose a very much larger sum of money. Pay-offs are asymmetric.

DEFINITION *An option is a contract which confers the right but not the obligation to buy or sell an asset on a given date at a predetermined (exercise) price.*

UNIQUE FEATURE *An option may be abandoned and the buyer just loses what was paid for it.*

There are options to buy assets, known as *call* options, and options to sell assets, known as *put* options. The puts are less common than the calls and are more difficult to comprehend. Investors usually want the right to buy something. On the other hand, producers of commodities, such as farmers, are very interested in guaranteeing their selling prices and so are naturally oriented towards the purchase of put options. One way to remember the terms call and put is to think that the first 'calls' for the asset, while the second 'puts' it on the market.

The following gives an example of a call option on a share. It is 2 January and Mr Optimist is still suffering from the New Year celebrations. He has ulcer problems. This makes him think of buying some shares in a pharmaceutical company, Glaxo, which is well known for its ulcer drugs. The current share price is 850 pence. For £850 he could therefore buy 100 shares. Alternatively, in order to have a more exciting new year, he could buy a larger number of call options on Glaxo. He believes that a share price of more than 900 pence is likely by the end of March. His broker informs him that, for a cost of 50 pence per share, he can buy call options with an exercise price of 850 pence and a life of three months. For £850 he is able to buy options on 1700 shares.

What does Mr Optimist stand to gain or lose? If his view is correct and Glaxo's price rises, then he will eventually exercise his call options. For example, if the share price were to rise to 920 pence, then exercising his right to buy at 850 pence would give him a pay-out of 70 pence per share. Since he paid 50 pence per option, his new profit would be 20 pence per share. If Glaxo shares were to rise only to 900 pence, then exercise of the options would pay 50 pence

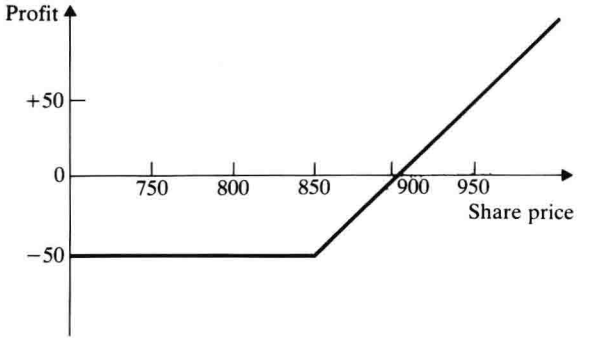

Figure 1.1 Profit at maturity on purchased Glaxo call

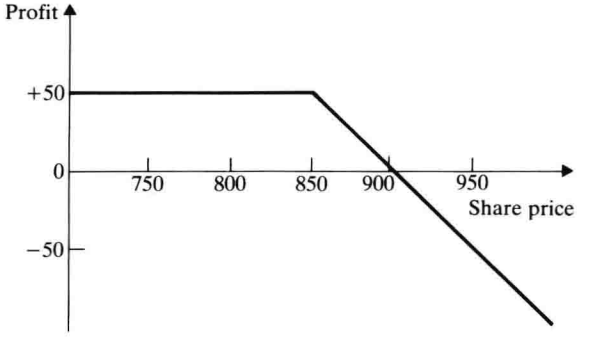

Figure 1.2 Profit at maturity on written Glaxo call

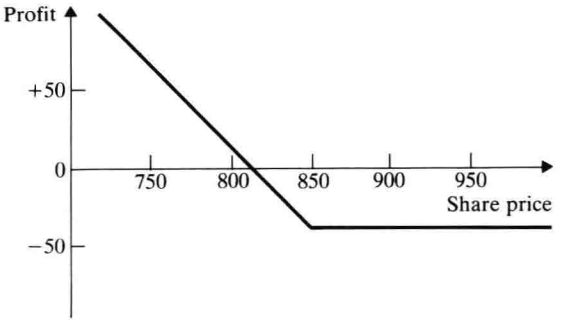

Figure 1.3 Profit at maturity on purchased Glaxo put

gross, which would be a profit of zero after deducting the cost. Nevertheless, even if Glaxo shares only rose to 851 pence, it would pay Mr Optimist to exercise. That way he could at least recoup 1 penny per share, to set against the 50 pence per share cost of the options. However, if Glaxo fell below 850 pence, he would not exercise: it would be irrational to go ahead and buy at 850 pence if the market price were less than that.

Mr Optimist's potential profit is illustrated in Fig. 1.1. At a share price of less than 850 pence he loses the whole outlay of 50 pence. Above 850 pence he claws back the loss until, above 900 pence, he makes a profit.

The seller (or 'writer') of the Glaxo option can only gain what the buyer loses and so faces the profit diagram of Fig. 1.2. He or she can only gain a maximum of 50 pence, but can lose much more. The seller's view must be that Glaxo is unlikely to exceed 900 pence in March.

Now suppose that there is another character, Mr Pessimist, who takes a rather negative view of Glaxo's prospects. He could have sold the call to Mr Optimist, but that would have entailed high risk. Instead he can buy a put

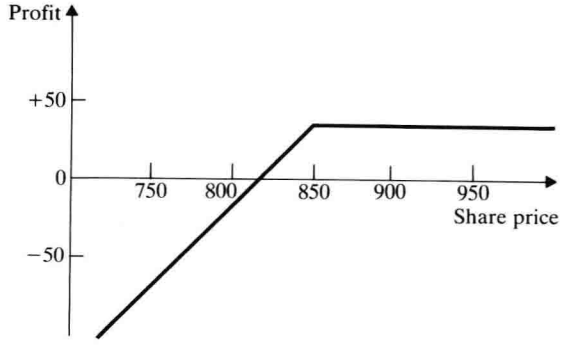

Figure 1.4 Profit at maturity on written Glaxo put

option, with an exercise price of 850 pence, maturing in March and costing 35 pence. Mr Pessimist rubs his hands with glee at the thought of the Glaxo price falling below 815 pence, the break-even level for this strategy (see Fig. 1.3).

Once again the writer has the mirror-image of the pay-out to the buyer. The latter can only lose 35 pence on the purchased put, while the former can only gain 35 pence at most on the sold put, as shown in Fig. 1.4.

1.1 A cautionary tale

Writers of options on shares are typically professionals who control their risks. Often they are 'covered' because they already own the shares on which call options are written. If the share price rises and the options are exercised, they deliver the shares. Small investors are mostly buyers of 'naked' options, i.e. options that are not covered in any way. They also prefer to buy calls rather than puts, finding it psychologically easier to have the right to buy than the right to sell.

In the Crash of October 1987, the buyers of put options made huge gains and the writers of these options lost. One disaster (out of many) at the time related to a schoolboy, aged 13, who had written naked puts on the FTSE 100 share index in London. He was unobtainable (at school) during the day and by the time he had been located his account was more than £1 million in deficit. Needless to say, he could not pay. This not only indicates the risk of being an option writer, but it also indicates the need for adequate back-office controls by a broker.

1.2 Early exercise

Some options give the buyer an additional right. The option may not only be exercised at the maturity date, but also on any date before then. These are known as *American* options. Options that do not allow such early exercise are known as *European* options. The source of this terminology is obscure. One cannot help thinking that it was invented by an American who found Europeans to be rather inflexible. Most options that are traded on exchanges the world over are American, but many of the off-exchange ('over-the-counter') deals are for European options.

Since an American option gives an extra right, it must be worth at least as much as a European option and could be worth more. Later in the book we shall examine the circumstances in which it is worth more.

1.3 History and markets

Option contracts on commodities, such as wheat, have been used by merchants since the middle ages. Sometimes it suited a merchant to make a simple