# ELECTRICAL MEASUREMENTS AND MEASURING INSTRUMENTS


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# PREFACE TO FIFTH EDITION

IT is now thirty years since the first edition of this book was published and I would like to express my deep and sincere appreciation of its acceptance by so many readers in all parts of the English-speaking world. During recent years I have visited many countries in which the book has been used and have been most gratified by the kind remarks made about its value to students and practising engineers. These comments have come from engineers met, sometimes in quite remote places, in East and Central Africa, India, Pakistan, Malaya, Hong Kong and Australia, and they have given me great pleasure.

With the passing of the years, and with the changes which have taken place in my own professional interests, it has become increasingly difficult to keep pace with the rapid advances made in the subject of Electrical Measurements. I have felt it imperative, therefore, to seek assistance in the task of revising the book in a way which, I hope, will prove satisfactory to the present-day reader. In this I have been extremely fortunate in receiving the co-operation of Dr. F. C. Widdis. He has had great experience in both the practical and academic sides of the subject, and he has been mainly responsible for the many changes made in preparing this fifth edition.

We hope that, so far as possible within the confines of a single volume, the latest practice has now been covered adequately and that the book will continue to be as useful as in the past.

Perhaps the most significant single change has been the adoption of the rationalized M.K.S. system of units, which is now increasingly used in teaching the subject. For the benefit of readers who find themselves in the transition stage of conversion from the C.G.S. to the M.K.S. system, a comprehensive comparison between the formulae in the two systems has been made, and we hope that this will be helpful to those who are not yet entirely conversant with the M.K.S. system.

E. W. GOLDING

LONDON,
DECEMBER, 1962

# PREFACE TO FIRST EDITION

WHEN this book was first contemplated, it was the author's intention to produce a short text-book covering the syllabuses of the B.Sc. (Eng.), City and Guilds (Final) and I.E.E. examinations. As the work progressed, however, it was realized that the groundwork for these examinations could not be adequately covered in a book of the size originally planned. Extension of the text being thus necessitated, the author felt that the scope and method of treatment could with advantage be modified to suit, also, the requirements of electrical engineers in general, for whom, owing to the rapid advances taking place in the electrical industry and in the interconnection of power systems, the subject of electrical measurements is assuming an ever-increasing importance. In particular, it was thought that engineers engaged in the standardizing and metering side of the industry could be catered for.

Some of the subjects dealt with—e.g. Transient Phenomena—are essentially mathematical, and cannot well be treated otherwise, but the mathematics throughout the book has been kept as simple as possible and should be followed easily by most readers. The theory of most of the methods of measurement has been given in full, but in many cases smaller type has been used for such theoretical discussions, so that the reader may omit them, if desired, and consider only the resulting expressions.

References and bibliographies are given at the ends of the chapters, their inclusion being both an acknowledgment of the sources of the author's information—in certain cases—and an augmentation of the text, which can, at the best, give only a general outline of so large a subject in the space of a single volume.

A number of worked examples have been given as illustrations, and a selection of examination questions (with answers) is included at the end of the book. In this connection the author's thanks are due to the Senate of the University of London, the City and Guilds of London Institute, and the Institution of Electrical Engineers for permission to use questions from their examination papers.

Most of the illustrations have been specially made for this book, but the author is glad to acknowledge his indebtedness to the many firms and authorities who have very kindly supplied him with information, drawings, and photographs relating to their apparatus. Individual acknowledgments are made in the text. Messrs. Elliott Bros., Ltd., had drawings specially made from photographs of their instruments. The author is indebted also to Messrs. A. T. Dover, B. Hague, and P. Dunsheath for permission to reproduce five illustrations from their books; to Messrs. B. G. Churcher and C. Dannatt,

Prof. W. M. Thornton, and Prof. S. P. Smith for their co-operation, and to Mr. N. A. Allen for information regarding cable tests with high-voltage direct current.

The author gratefully acknowledges the help which he has received from advanced students in the Electrical Engineering Department of University College, Nottingham, in the shape of proof reading. He would like, also, to tender his sincere thanks to Dr. H. Cotton for his continued and lively interest in the book during its preparation; to Mr. T. M. E. Ward for kindly reading the proofs of Chapters XX and XXI; and to his wife for her unfailing help and consideration while the book was being prepared.

Acknowledgment should also be made to the British Standards Institution for permission to make extracts from several of their instrument specifications. Copies of these specifications can be obtained from the Institution at British Standards House, 2 Park Street, London, W.1.

E. W. GOLDING

UNIVERSITY COLLEGE,
NOTTINGHAM, 1933

## AUTHORS' NOTE ON THE RATIONALIZED M.K.S. SYSTEM OF UNITS

In previous editions of the book the C.G.S. system of units was used throughout, but the rationalized M.K.S. (metre-kilogramme-second) system is now being used increasingly, especially in the teaching of electrical engineering, and it has been adopted throughout this edition. The derivation of the system is discussed in Chapter II. It has important advantages in that practical units—ampere, volt, ohm, etc.—are used throughout and the conversion factors necessary in the C.G.S. system are eliminated. The only complication in this system is the introduction of the primary magnetic constant $\mu_0 = 4\pi \times 10^{-7}$ and the primary electric constant $\varepsilon_0 = \dfrac{1}{36\pi \times 10^9}$, which must be memorized. Each of these constants has a value of unity in the C.G.S. system.

In the M.K.S. system, force is measured in "newtons" and lengths in metres. A newton is the force necessary to give a mass of 1 kilogramme an acceleration of 1 metre per sec per sec and it is equivalent to $10^5$ dynes in the C.G.S. system.

With the introduction of the M.K.S. system, the view has been expressed, in some quarters, that the classical methods of deriving electromagnetic and electrostatic formulae should be revised. This is because the unit magnetic pole used in the classical derivations has no physical existence, and it is possible, instead of using this concept, to explain the origin and properties of the magnetic field in terms of the effects of electric currents. The classical methods, however, provide a simple means of studying the properties of fields and are therefore adhered to in Chapter I. A very full discussion of alternative methods of developing field theory will be found in Ref. (24) at the end of Chapter 1.

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# INSETS

# CHAPTER I

## ELECTROSTATIC AND ELECTROMAGNETIC THEORY

### ELECTROSTATICS

**Coulomb's Law.** The earliest recorded facts in connection with the subject of electricity were obtained as a result of experiments carried out by the ancient Greek philosopher Thales of Miletus, about 600 B.C., and related to the forces of repulsion and attraction between bodies charged with static electricity. Those facts were qualitative only, and it was left for Coulomb, many centuries later, to state them in a quantitative form by his Inverse Square Law, which is the most fundamental law of electrostatics—

$$F \propto \frac{Q_1 Q_2}{\varepsilon r^2} \qquad . \quad . \quad . \quad . \quad (1.1)$$

where $F$ is the force between two small bodies charged respectively with $Q_1$ and $Q_2$ units of electricity, their centres being a distance $r$ apart, and $\varepsilon$ is a constant depending upon the medium in which the bodies are situated, and is called the "permittivity" of the medium.

In the rationalized M.K.S. system of units this expression is written as

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} \text{ newtons} \qquad . \quad . \quad . \quad (1.2)$$

where $Q_1$ and $Q_2$ are the charges in coulombs, $r$ is in metres, and $\varepsilon = \varepsilon_0 \varepsilon_r$ where $\varepsilon_0$ is the primary electric constant, having a value $\frac{1}{36\pi \times 10^9}$, and $\varepsilon_r$ is the permittivity of the medium relative to that of a vacuum. $\varepsilon_r = 1$ for a vacuum, and air may be considered to have the same value. In this system two infinitely small bodies each having unit charge and being 1 metre apart in air experience a force of $9 \times 10^9$ newtons.

**Electric Field Round Charged Conductors.** If unit positive charge of electricity be placed in the neighbourhood of a charged body it will experience a force of attraction or repulsion according as the charged body is negatively or positively charged. If this unit charge be allowed to move freely, it will trace out a "line of electric force." For all points on this line the resultant force on the unit charge will be in a direction tangential to the line at the given point. The electric field in the neighbourhood of any charged conductor or system of charged conductors can be represented by such lines of force, arrow heads placed upon them giving the direction in which unit positive charge would move along the line.

The magnitude of the force upon unit positive charge, placed at any point, is a measure of the "electric force" or "field strength" at that point, it being assumed that the introduction of the unit charge does not affect the distribution of charge upon the conductors to which the field is due.

**The Electric Field.** In the preceding paragraph lines of force are spoken of as giving the direction of the field at any point. If we have two adjacent charges of opposite polarity and there are no

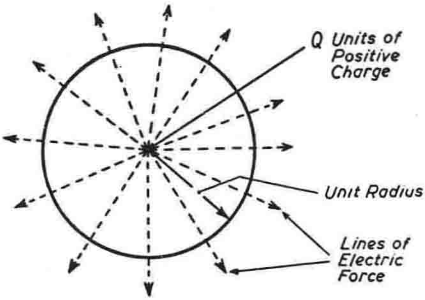


FIG. 1.1. LINES OF ELECTRIC FORCE ROUND A POSITIVE CHARGE

other charges in the vicinity, we can visualize lines of force emanating from one charge and terminating on the other. These lines of force can be looked upon as representing an electric flux between the charges. The unit of electric flux is now defined as being the total flux originating from a unit charge. It follows that

$$\psi = Q \text{ coulombs}$$

where $\psi$ is the electric flux radiating from a charge of $Q$ coulombs.

If a charge $Q$ is situated at the centre of a sphere of radius 1 metre then the electric flux density $D$ at the surface of the sphere is given by

$$D = \frac{Q}{4\pi} \text{ coulombs per sq. metre}$$

The electric field strength $E$ at any point is defined as the force on a unit charge situated at that point and it follows from Equation (1.2) that $E$, at the surface of a sphere of unit radius, is

$$E = \frac{Q}{4\pi\varepsilon} = \frac{D}{\varepsilon} \quad . \quad . \quad . \quad . \quad (1.3)^*$$

In air, $\varepsilon_r = 1$, so that $\varepsilon = \varepsilon_0$ and $E = \dfrac{D}{\varepsilon_0}$.

* Note that this law is similar to the magnetic law—
$H = B/\mu$ where $B$ = magnetic flux density,
$H$ = magnetic field strength,
$\mu$ = magnetic permeability of the medium.

$E$ is also termed the electric force and, since $E$ is the force in newtons on a charge of 1 coulomb, it will be shown subsequently that the unit of $E$ is 1 volt per metre.

**Tubes of Flux.** Fig. 1.2 represents a number of lines of electric

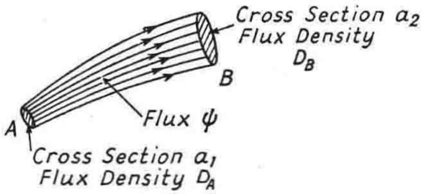


FIG. 1.2. TUBE OF FLUX

field strength forming a "tube of flux." If $A$ and $B$ are two points in an electric field such that the field strength at $A$ is greater than that at $B$, then, from Equation (1.3), the field strength at $A$ is

$$E_A = \frac{D_A}{\varepsilon_0}$$

where $D_A$ = lines per unit surface of cross-section of the tube at $A$.

Similarly, at $B$

$$E_B = \frac{D_B}{\varepsilon_0}$$

If $\psi$ is the electric flux in the tube, and $a_1$ and $a_2$ are the areas of cross-section of the tube at $A$ and $B$, these areas being measured perpendicular to the direction of the field at the points, then

$$E_A = \frac{D_A}{\varepsilon_0} = \frac{\psi}{a_1\varepsilon_0}$$

$$E_B = \frac{D_B}{\varepsilon_0} = \frac{\psi}{a_2\varepsilon_0}$$

**Electric Field Inside a Charged Spherical Conductor.** Imagine a hollow sphere of conducting material which has been given a charge of $Q$ positive units of electricity. If its area of surface be $S$ the density of charge on the surface (which will be uniform) is $\frac{Q}{S}$ per unit of surface. The electric field at the surface will be at all points normal to the surface, since the sphere is of conducting material. This follows from a consideration of the fact that, if it were not so, the field would have a tangential component which would produce a movement of charge until the direction of the field became normal. Consider a point $P$ inside the sphere (Fig. 1.3)

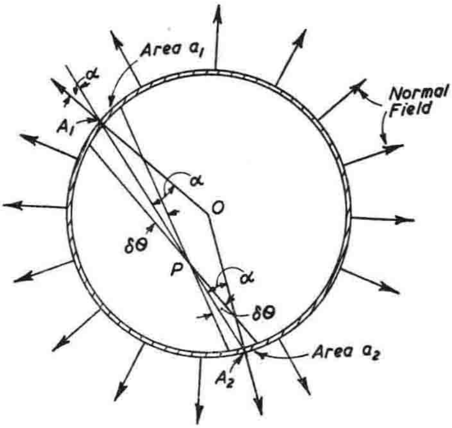


FIG. 1.3. FIELD INSIDE A SPHERICAL CONDUCTOR

at which small areas of surface $a_1$ and $a_2$ subtend a solid angle $\delta\theta$ as shown. Points $A_1$ and $A_2$ are mid-points of the areas $a_1$ and $a_2$. Angle $OA_1P$ = angle $OA_2P = a$.

Let $A_1P = d_1$, $A_2P = d_2$

$\varepsilon$ = permittivity of the medium inside the sphere.

Then $$\text{Charge on area } a_1 = \frac{Q}{S} \cdot a_1$$

$$\text{,, ,, ,, } a_2 = \frac{Q}{S} \cdot a_2$$

Since the field is everywhere normal to the surface, the field strength at $P$ due to charge on $a_1$ is

$$\frac{Qa_1}{S} \frac{\cos a}{4\pi\varepsilon\, d_1^{\,2}} \text{ in direction } A_1P$$

Similarly, the field strength at $P$ due to charge on $a_2$ is

$$\frac{Qa_2}{S} \cdot \frac{\cos a}{4\pi\varepsilon\, d_2^{\,2}} \text{ in direction } A_2P$$

directly opposite to direction $A_1P$.

Now, the solid angle subtended at the centre of a sphere of radius $R$ by any area $A$ on its surface is $\dfrac{A}{R^2}$.

Hence,

$$\text{Solid angle } \delta\theta = \frac{a_1 \cos a}{d_1^2} = \frac{a_2 \cos a}{d_2^2}$$

Thus, the field strengths at $P$ due to charges on $a_1$ and $a_2$ are opposite and are each equal to $\frac{Q}{4\pi\varepsilon S} \cdot \delta\theta$, giving a resultant field strength due to these two charges of zero.

As the same is true for all similar pairs of areas such as $a_1$ and $a_2$, the *total field strength at any point inside a charged spherical conductor is zero.*

**Field in the Neighbourhood of a Charged Straight Conductor.** Fig. 1.4 ($a$) represents a long, thin, straight conductor which carries a

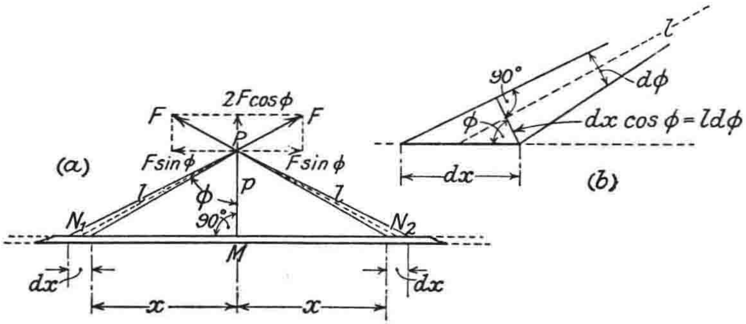


FIG. 1.4. ELECTROSTATIC FIELD NEAR A STRAIGHT CONDUCTOR

uniform charge of $Q$ per unit length. $P$ is a point whose perpendicular distance from the conductor is $p$, and $p$ is small compared with the length of the wire. Consider two elements of the conductor each of length $dx$, as shown at $N_1$ and $N_2$, the elements being equidistant from $P$.

Let $$N_1P = N_2P = l$$

Then, if the elements $dx$ are so small that the charges on them can be considered as concentrated at $N_1$ and $N_2$, the forces ($F$) upon unit positive charge placed at $P$ will be each equal to $\frac{Qdx}{4\pi\varepsilon l^2}$, from Equation (1.2), where $\varepsilon$ is the permittivity of the medium.

The directions of these forces will, as shown, each make an angle of $(90 - \phi)$ with the direction of the conductor, and will together be equivalent to one force of $2F \cos \phi$ in direction $MP$, the horizontal components neutralizing one another.

The same applies to all such pairs of elements as those shown, so that the total force upon unit charge at $P$—i.e. the field strength at

$P$—due to the whole length of the wire, will be in the direction $MP$, and is given by

$$E_p = \int_{x=0}^{x=\infty} \frac{2Q \cos \phi}{4\pi\varepsilon l^2} dx$$

where $E_p$ = total field strength at $P$, if the distance $p$ is small compared with the length of the wire.

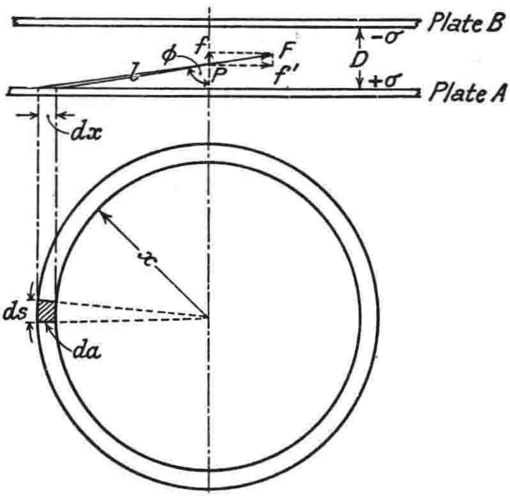


FIG. 1.5. ELECTROSTATIC FIELD BETWEEN TWO CHARGED PLATES

From Fig. 1.4 ($b$), it can be seen that, if $dx$ is very small, then

$$l d\phi = dx \cos \phi = dx \cdot \frac{p}{l}$$

$$\therefore \frac{dx}{l^2} = \frac{d\phi}{p}$$ where $d\phi$ is the angle subtended at $P$ by $dx$

$$\therefore E_p = \int_{\phi=0}^{\phi=+\frac{\pi}{2}} \frac{2Q \cos \phi}{4\pi\varepsilon p} d\phi = \frac{2Q}{4\pi\varepsilon p} \text{ in the direction } MP \qquad (1.4)$$

**Field in the Space Between Two Charged Parallel Conducting Plates.** Fig. 1.5 represents the two conducting plates, which are close together. Their extent is supposed to be so great as compared with their distance apart that the electrostatic field on or near their common axis is unaffected by the fringing field at the edges of the