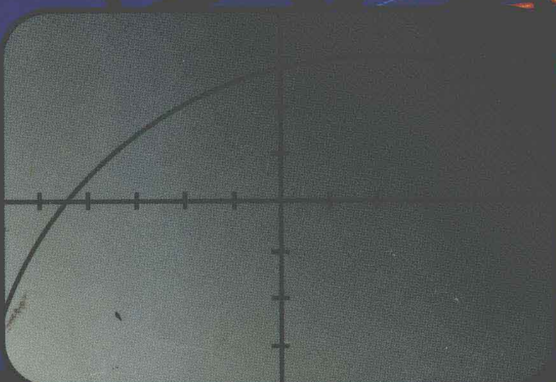
DEMANA WAITS CLEMENS GREENE

INTERMEDIATE ALGEBRA

A GRAPHING APPROACH



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Intermediate Algebra
A Graphing Approach

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PREFACE

In the past two decades we have seen powerful computational and graphical capabilities first available on mainframe computers, then on desktop computers, and recently on hand-held graphers. Technology is now convenient and affordable. The availability of technology causes mathematics teachers to rethink both the teaching and learning of mathematics.

Our goal in writing ***Intermediate Algebra: A Graphing Approach*** was to present a friendly, convenient, easy-to-use book which *fully* integrates graphing technology. **We assume that each student has access to a graphing calculator for both class work and for completing assignments.** Graphers are used throughout for concept development, for discovery learning, and for problem solving. At the same time, all of the essential problem-solving concepts and skills of the more traditional symbolic algebra course have been preserved.

Our own combined teaching experience of over one hundred years at The Ohio State University, Bluffton College, and Florida Community College at Jacksonville has given us many insights into the characteristics and needs of intermediate algebra students. There has always been a rich interplay between visualizing a concept and representing it symbolically. However, until recently, our ability to create visualizations has been limited to long, laborious, and often imprecise paper-and-pencil methods. Because of this past limitation, we generally have not fully utilized visualization as a teaching and learning vehicle. Many persons are visual thinkers and visual problem solvers. Modern hand-held graphers now make visualization methods accessible, precise, and time efficient.

There are a variety of pedagogical benefits to fully integrating a grapher into the classroom. We find that using a grapher helps students feel that mathematics is concrete and tangible. We have discovered that our classrooms are less "teacher centered" and more "student centered." As students interact with their grapher and with each other in the spirit of cooperative learning, they talk about mathematics with each other. And as students verbalize mathematics, they learn mathematics.

The content of this book will match a standard course syllabi for intermediate algebra as it has typically been taught. It includes all of the standing topics including paper-and-pencil problem-solving skills that intermediate algebra students are expected to learn. Students who successfully complete a course from this text should be successful in any subsequent college algebra and trigonometry course—even if that subsequent course does not integrate a grapher.

Features

A number of features have been included in this text to make it a more valuable learning and teaching tool.

ART

Throughout the book, pictures of grapher screens are provided that show true grapher output. Some of these screens show the home screen on which numerical calculations are completed. On occasion, grapher menus are shown to help students learn grapher technique. There are also instances in which grapher-produced tables of numerical data are shown. And of course, many figures show graphs. Adequate information is provided so that students are able to duplicate any of these graphs on their own grapher.

MARGIN BOXES

Three different types of margin boxes appear in the text. The purpose of these boxes is to provide information and helpful hints about both the mathematics and about the grapher and its use. The three types of boxes are:

- **Reminder.** The information in these boxes often reviews mathematical convention, language, or notation. In some cases, a prerequisite concept is reviewed briefly. In other cases, warnings about common pitfalls are given.
- **Grapher Note.** These notes clarify information about the grapher and its use. They sometimes refer students to the lab manual to learn some specific grapher skill. Others make suggestions about grapher use.

- **Try This.** These boxes ask students to engage in a brief activity. Sometimes it is an activity involving paper-and-pencil skills. More often it is a grapher activity.

REFERENCES TO THE LAB MANUAL

A Lab Manual is available free to users of this text. Whenever a new grapher skill is introduced, that skill is referenced and taught in the lab manual. All of the popular brands of grapher models are included in this lab manual. This manual has a convenient organization that makes it easy for students to find the particular skill needed for identifying the necessary key strokes on the model of grapher they are using.

EXPLORE WITH A GRAPHER

Many sections of the book begin with a feature by this name. These feature activities use the grapher to introduce a new concept. Usually the pedagogical approach is "discovery learning" and often they conclude by asking the student to state the generalization that they have discovered. These activities are consistent with the pedagogical philosophy

I hear and I forget,

I see and I remember,

I do and I understand.

PROBLEM SITUATIONS

Throughout the text real-life problem situations are discussed. These problem situations allow both student and teacher to refer to various problem types by a name so that they can more easily be classified and revisited later in the course. We find that a graphing-calculator-based approach to problem solving dramatically changes results in the classroom. Instead of being bored and discouraged by conventional, contrived problems, students suddenly grow excited by their ability to explore problems that arise from real-world situations.

We have a three-strand philosophy regarding the interplay between algebraic and graphical methods for solving problems. These three strands can be stated as follows.

1. **Solve Algebraically, and Support Graphically or Numerically.**
   This means that the primary solution method is paper-and-pencil algebra. After an algebraic solution is found, the grapher is used to provide numerical or visual understanding of the problem. A numerical table or a graph can add credibility to the solution.
   We follow this strand when an algebraic solution is reasonably easy and manageable.

2. **Solve Graphically and/or Numerically, and Confirm Algebraically.**
   This means that the primary solution method is graphical or numerical. After a graphical or numerical solution has been found, which is probably an approximate solution, algebraic methods are used to confirm an exact solution.
   We follow this strand when the graphical visualization of the problem closely matches the physical description of the problem. In some cases, this graphical solution may even be a simulation of the problem.
3. **Solve Graphically** (because algebraic methods are not available, are too difficult, or are beyond the scope of this course.)
   This means that, for all practical purposes, a graphical solution is the only one available.

Throughout the text you will find the *Solve Algebraically, Support Graphically or Numerically,* and *Solve Graphically, and Confirm Algebraically* labels used in solutions to examples. These solutions model these various approaches to problem solving.

EXERCISE SETS

There are nearly 5,000 exercises in this text. Special care and attention has been given to crafting exercise sets that give students a balance between drill exercises, which help gain mastery of key skills, and extension exercises, which encourage creative problem solving.

Each exercise set begins with paired exercises that reflect the types of problems that have been solved in the development of the section. These exercises, which mirror the examples in the development, also include application problems similar to those discussed in the Problem Situations.

Besides the basic list of exercises there are several additional categories of exercises—each with its own objectives:

- **Translating Words to Symbols.** These exercises focus on the important task of translating word phrases to mathematical symbols. These exercises are designed to help students with the process of translating a problem situation to its mathematical formulation.
- **Extending the Ideas.** Each exercise set includes several exercises in this category. Their purpose is to encourage students to move beyond the specific skills taught in the lesson. These exercises require a higher level of creativity as students synthesize combinations of skills in solving problems.
- **Looking Back—Making Connections.** These exercises review previously learned concepts and skills. They also require that students apply mathematical skills to solve problems which come from other disciplines or from other concentrations within mathematics.

END-OF-CHAPTER MATERIAL

Chapter Summary. Each chapter ends with a Chapter Summary. This summary classifies the concepts, skills, and solution techniques that have been learned in the chapter using a three-column format. The first column names the concept or skill, the second column defines or describes the named item, and the third column gives an example of it. The material in this summary is organized according to natural classification schemes and is not listed in the order of the chapter development. This organization helps students comprehend a more global view of the chapter as they study for examinations.

Supplements

Instructor's Solutions Manual. The Instructor's Solutions Manual contains worked-out solutions to every problem in the text.

Student's Solutions Manual. The Student's Solutions Manual contains worked-out solutions to every odd-numbered problem in the text.

Graphing Calculator Lab Manual. This manual combines instructions for using the latest Casio, Sharp, and Texas Instruments calculators with the text.

Instructor's Manual and Printed Test Bank. Included in this manual and printed test bank are three alternate test forms that correspond to every chapter, a sample syllabus, and an essay discussing the implementation of technology in the classroom.

OmniTest II. A unique algorithm-based test generator for Demana, Waits, Clemens, Greene, *Intermediate Algebra: A Graphing Approach* produces virtually unlimited versions of problems appropriate for the graphing calculator. These printed problems include graphic representation of the images which appear on the graphing calculator. This provides a permanent record of the calculator image with which instructors and students may annotate instructional notes and illustrative comments. These printed problems are also useful in a cooperative learning environment.

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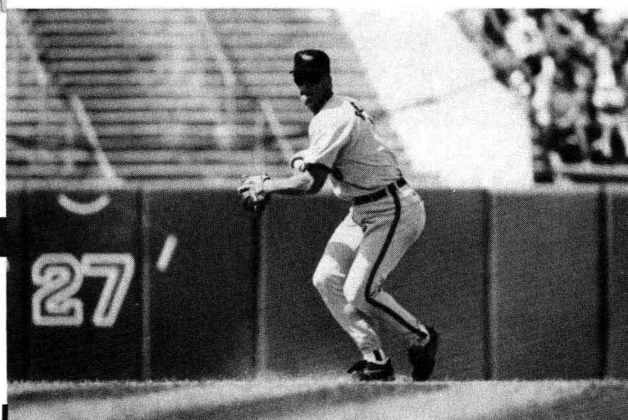

CHAPTER 1

Numerical Mathematics and the Graphing Calculator

AN APPLICATION

When a baseball is thrown straight up into the air with an initial velocity of 88 ft/sec, the equation

$$h = -16t^2 + 88t$$

describes the ball's height t seconds later. Many questions relating height and time can be answered using this equation.

1.1 Real Numbers and a Graphing Calculator

Repeating and Terminating Decimals ▪ Comparing Real Numbers and the Real Number Line ▪ Inequalities and Intervals on the Number Line ▪ Mathematical Notation and Exponents ▪ Order of Operations

Numbers are used to represent quantity. Number concepts as they are applied to real-world problems become the motivation for many topics in algebra.

The **natural numbers**, or **counting numbers**, are the numbers in the infinite set

$$N = \{1, 2, 3, \cdots\}.$$

Many applications require only the natural numbers. For example, the natural numbers are used to report data on the number of students enrolled in each class of a university. When the number 0 is included, the set then is called the set of **whole numbers.**

For some applications, we need to extend our number set to include the set called **integers**. Integers are the numbers in the set

$$I = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}.$$

For example, to communicate gain versus loss or profit versus debt, the numbers are selected from the integers.

In still other settings, we need to use numbers from the set of **rational numbers**. The rational numbers are the set of numbers of the form $\frac{m}{n}$, where m and n are integers, $n \neq 0$. For example, the numbers used to represent the three shaded regions of the chart in Figure 1.1 are rational numbers

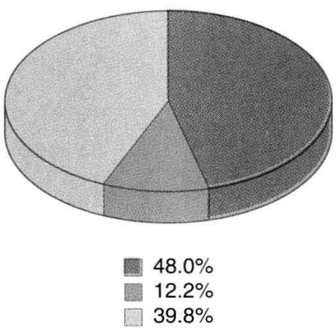

FIGURE 1.1 A chart like this is called a Pie Chart.

$$\frac{98}{802} = 0.1221\cdots, \qquad \frac{385}{802} = 0.4800\cdots, \qquad \frac{319}{802} = 0.3977\cdots.$$

Rational numbers can be reported in any one of three representations—as a decimal, as shown in the above three examples; as a fraction, and as a percent, as shown in the figure. For example,

$$\frac{2}{5} \qquad 0.4 \qquad 40\%$$

are three different representations of the same rational number.

And note that in the pie chart, the decimal form of each rational number is represented as a percentage, rounded to the nearest tenth.

Repeating and Terminating Decimals

When a rational number $\frac{m}{n}$ is written in its decimal representation by finding $m \div n$, the resulting decimal may have an infinite number of digits. Each rational number can be represented either as a repeating or terminating decimal.

The following are examples of both types of rational numbers:

Repeating Decimals	**Terminating Decimals**
$\frac{1}{3} = 0.333\cdots$	$\frac{11}{25} = 0.44$
$\frac{2}{7} = 0.285714285714\cdots$	$\frac{43}{20} = 2.15$

REMINDER

The authors assume that all students have access to a graphing calculator. Throughout the remainder of the book, we shall refer to this tool as a *grapher.*

EXPLORE WITH A GRAPHER

Use a grapher to find the decimal representation of each of the following rational numbers. Decide which you think are **terminating** and which are **repeating**.

1. $\frac{5}{8}$ 2. $\frac{1}{33}$ 3. $\frac{5}{11}$

4. $\frac{9}{20}$ 5. $\frac{11}{5}$ 6. $\frac{5}{24}$

Formulate a Definition Write several sentences that explain your definition of the words repeating and terminating.

GRAPHER NOTE

Refer to your grapher lab manual to review methods of performing computations on a grapher. Some graphers have a command FRAC that can be used to change a decimal representation to a fraction representation. Check whether the grapher you use has this command or an equivalent command.

A **repeating decimal** has to the right of the decimal point a block of digits that continues repeating. When the repeating digit is 0, however, the zeros are not written or displayed on the grapher screen and such a decimal is called a **terminating decimal**.

In a repeating decimal, the block of digits that repeat are often indicated by using a "bar" above the repeating block, as shown next.

$$\frac{1}{3} = 0.333\cdots \qquad \frac{2}{7} = 0.285714285714\cdots$$
$$= 0.\overline{3} \qquad = 0.\overline{285714}$$

For some repeating decimals, the block of digits that repeats is longer than the number of digits on your calculator display. For example, when you find $1 \div 19$, the repeating block, while it exists, is not evident on the calculator display.

EXAMPLE 1 Finding Repeating and Terminating Decimals

Determine with a calculator, if possible, whether the decimal representation of each of the following rational numbers is repeating or terminating:

a) $\frac{143}{999}$

b) $\frac{723}{250}$

c) $\frac{9}{13}$

SOLUTION

a) $\frac{143}{999} = 143 \div 999$

$= 0.143143\cdots$

$= 0.\overline{143}$

The decimal representation of $\frac{143}{999}$ is repeating.

b) $\frac{723}{250} = 2.892$

The decimal representation of $\frac{723}{250}$ is terminating.

c) $\frac{9}{13} = 0.6923076923\cdots$

$= 0.\overline{692307}$

The decimal representation of $\frac{9}{13}$ is repeating. ■

GRAPHER NOTE

Because a calculator can display (or remember internally) only a finite number of digits, calculator output is in effect a terminating decimal. However, it is a close approximation to the actual repeating decimal.

Some numbers are neither repeating nor terminating. Any number whose decimal representation is neither repeating nor terminating is called an **irrational** number. The **radical symbol**, $\sqrt{\ }$, is used to describe the positive square root of a number. For example, $\sqrt{25} = 5$ since $5 \cdot 5 = 25$. Although $\sqrt{25} = 5$ is a rational number, most radicals are irrational. Both of the following are irrational numbers. Confirm these two decimal values on your grapher.

$$\sqrt{2} = 1.414213562\cdots \qquad \pi = 3.141592654\cdots$$

We are now able to define the set of real numbers.

TRY THIS

Show that on your calculator 123456×0.33333 and $123456 \times \frac{1}{3}$ are not equal. Explain why.

DEFINITION 1.1 Real Numbers

The **real numbers** consist of all numbers that are either rational or irrational. The points on a number line are a model for these numbers.