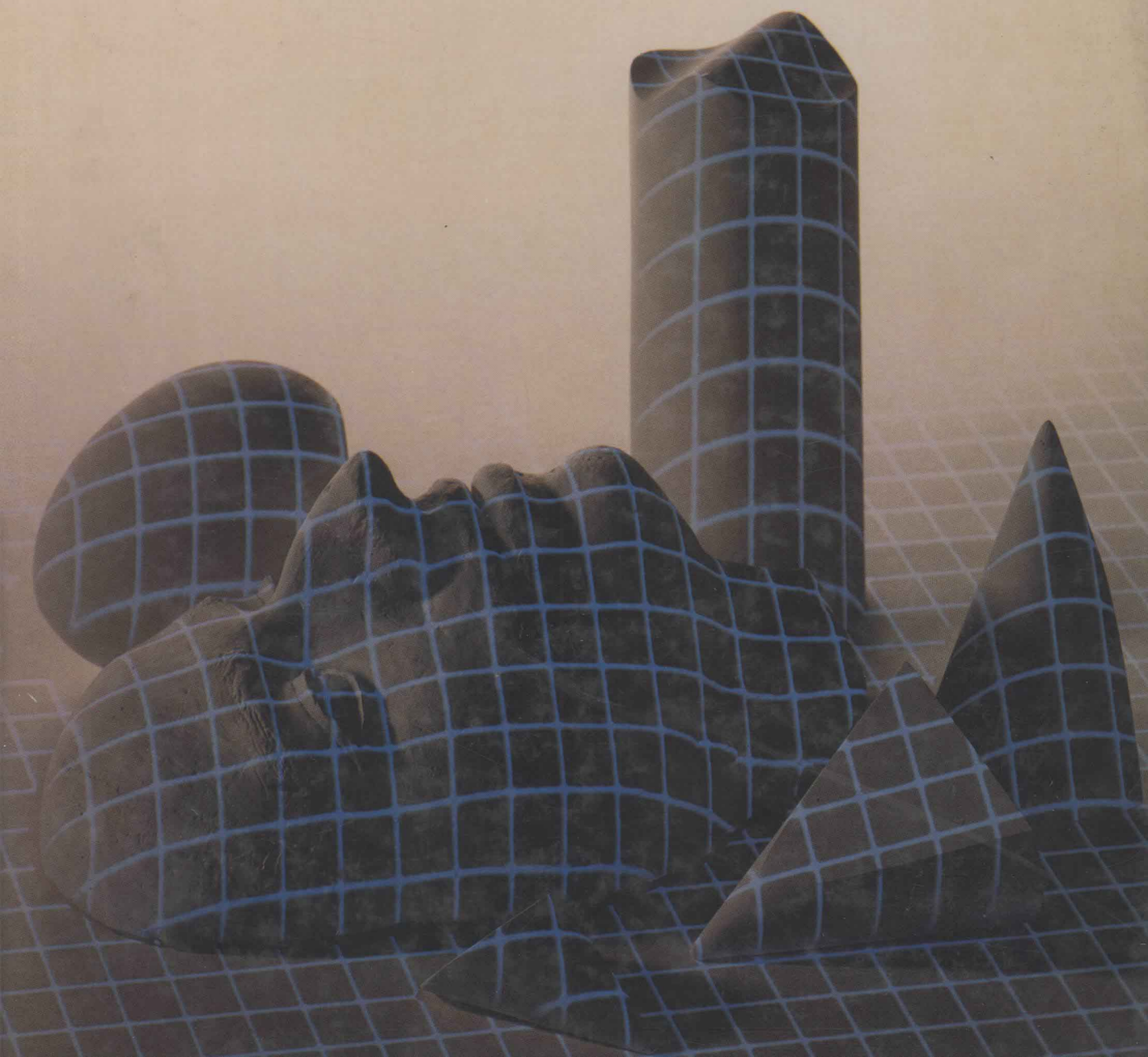


CALCULUS

with Analytic Geometry

Arthur B. Simon



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with Analytic Geometry

ARTHUR B. SIMON

California State University, Hayward

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ELEMENTARY FORMULAS FROM ALGEBRA AND GEOMETRY

Quadratic Formula

The equation $ax^2 + bx + c = 0$ with $a \neq 0$, has solutions

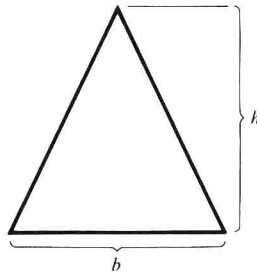
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Distance

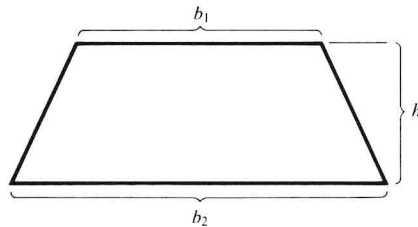
On the line $|x - y|$ = distance between x and y

In the plane $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ = distance between (x_1, y_1) and (x_2, y_2)

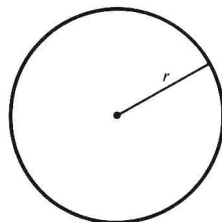
Area, Volume, and Surface Area:



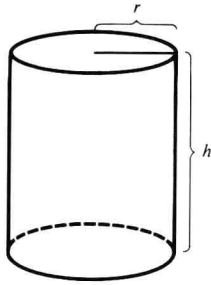
Triangle Area = $\frac{1}{2}bh$



Trapezoid Area = $\frac{1}{2}(b_1 + b_2)h$



Circle Area = πr^2
Circumference = $2\pi r$

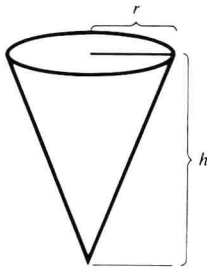


Right circular cylinder

$$\text{Volume} = \pi r^2 h$$

$$\text{Lateral surface area} = 2\pi r h$$

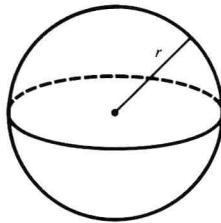
$$\text{Total surface area} = 2\pi r(r + h)$$



Right circular cone

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Lateral surface area} = \pi r \sqrt{r^2 + h^2}$$



Sphere Volume = $\frac{4}{3}\pi r^3$

$$\text{Surface area} = 4\pi r^2$$

Equations of Lines

General form $ax + by + c = 0$, a and b not both zero

Slope-intercept form $y = mx + b$

Point-slope form $y - y_1 = m(x - x_1)$

Two-point form $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Equations of Conic Sections (centers or vertex at the origin)

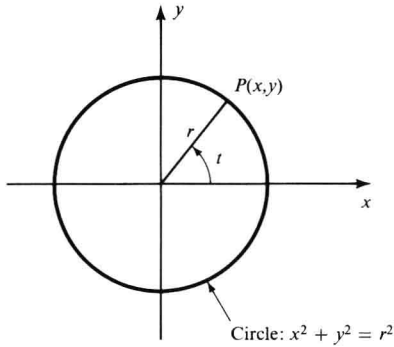
Circle of radius r $x^2 + y^2 = r^2$

Parabola $x^2 = 4py$

Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

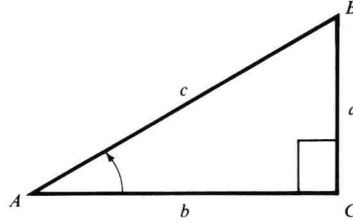
ELEMENTARY FORMULAS FROM TRIGONOMETRY



$$\sin t = \frac{y}{r}$$

$$\cos t = \frac{x}{r}$$

$$\tan t = \frac{y}{x}$$



$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b}$$

$$\cos^2 t + \sin^2 t = 1, \quad 1 + \tan^2 t = \sec^2 t, \quad \cot^2 t + 1 = \csc^2 t$$

$$\sin(-t) = -\sin t, \quad \cos(-t) = \cos t, \quad \tan(-t) = -\tan t$$

$$\cos t = \sin\left(\frac{\pi}{2} - t\right) \quad \sin t = \cos\left(\frac{\pi}{2} - t\right)$$

A SHORT TABLE OF INTEGRALS

$$1. \int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + C, n \neq -1 \\ \ln|x| + C, n = -1 \end{cases}$$

$$3. \int a^x dx = \frac{a^x}{\ln a} + C$$

$$5. \int \cos x dx = \sin x + C$$

$$7. \int \csc^2 x dx = -\cot x + C$$

$$9. \int \csc x \cot x dx = -\csc x + C$$

$$11. \int \cot x dx = \ln|\sin x| + C$$

$$13. \int \csc x dx = \ln|\csc x - \cot x| + C$$

$$15. \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$17. \int \frac{dx}{\sqrt{a^2 + x^2}} = \ln|x + \sqrt{a^2 + x^2}| + C$$

$$19. \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C$$

$$2. \int e^x dx = e^x + C$$

$$4. \int \sin x dx = -\cos x + C$$

$$6. \int \sec^2 x dx = \tan x + C$$

$$8. \int \sec x \tan x dx = \sec x + C$$

$$10. \int \tan x dx = -\ln|\cos x| + C$$

$$12. \int \sec x dx = \ln|\sec x + \tan x| + C$$

$$14. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$16. \int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + C$$

$$18. \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C$$

$$20. \int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

SOME CONVERSION FACTORS

$$1 \text{ mi (mile)} = 5,280 \text{ ft (feet)} = 63,360 \text{ in (inches)}$$

$$1 \text{ km (kilometer)} = 1,000 \text{ m (meters)} = 100,000 \text{ cm (centimeters)}$$

$$1 \text{ m} \approx 39.37 \text{ in} \approx 3.281 \text{ ft}$$

$$1 \text{ in} \approx 2.540 \text{ cm}$$

$$1 \text{ mi} \approx 1.609 \text{ km}$$

$$1 \text{ l (liter)} = 1,000 \text{ cm}^3 \approx 1.057 \text{ qt (quarts)}$$

$$1 \text{ hr (hour)} = 3,600 \text{ sec (seconds)}$$

$$1 \text{ ft/sec} \approx 0.3048 \text{ m/sec} \approx 0.6818 \text{ mph (miles per hour)}$$

$$1 \text{ mph} \approx 0.4471 \text{ m/sec} \approx 1.467 \text{ ft/sec}$$

$$1 \text{ revolution} = 2\pi \text{ rad (radians)} = 360^\circ \text{ (degrees)}$$

$$1 \text{ rad} \approx 57.30^\circ$$

$$1^\circ \approx 0.0175 \text{ rad}$$

$$1 \text{ revolution/min} \approx 0.1047 \text{ rad/sec}$$

$$1 \text{ kg (kilogram)} \approx 2.205 \text{ lbs (pounds)}$$

$$1 \text{ N (newton)} = 10^5 \text{ dynes} = 0.2248 \text{ lbs}$$

$$1 \text{ lb} \approx 4.448 \text{ N}$$

$$1 \text{ J (joule)} = 1 \text{ N}\cdot\text{m} = 10^7 \text{ ergs} \approx 0.7375 \text{ ft}\cdot\text{lbs}$$

$$1 \text{ hp (horsepower)} = 550 \text{ ft}\cdot\text{lbs/sec} \approx 746 \text{ W (watts)}$$

To my wife and partner
DOLORES

PREFACE

Now that calculus is a required course in such a wide range of disciplines, calculus classes are populated by students with diverse interests and levels of preparation. The modern text, therefore, must be accessible to a large majority of students and at the same time demonstrate the wide applicability of the subject. With this in mind, I have tried to write a calculus book that minimizes mathematical formalism and maximizes the variety of applications.

Each chapter is preceded by a short introduction that describes what new topics are introduced and explains why they are important. The introductions also contain advice to the student about studying the material. Most sections begin with an informal introduction followed by an intuitive discussion of the concepts. But elimination of all formality is neither possible nor desirable. Therefore, the final definitions and results are stated with as much precision as practical. In this way, correct statements can be learned from material that is readable and understandable. The results are usually accompanied by informal justifications; the formal proofs are presented as optional material either at the end of the section or at the end of the chapter. These proofs can be covered or not as time permits.

In addition to a complete coverage of the traditional topics of calculus, I would like to point out some unusual features of this text:

1. The function $f(x) = e^x$ and the trigonometric functions are introduced informally in Chapter 3 (Derivatives), after a thorough review of exponents and trigonometry in Chapter 1 (Introduction). Not only do these functions make the chain rule more meaningful, but they provide a wealth of interesting examples and motivating applications early in the course. The traditional (usual) development of these transcendental functions along with the natural logarithm is contained in Chapters 7 and 8.

2. Differential equations are introduced in Section 4–7 and are woven into the material throughout the rest of the book. They are used in Section 4–8 to enrich the discussion of velocity and acceleration, in Section 5–7 to compute the escape velocity of a rocket, in Chapter 7 to motivate the definition of the natural logarithm, in Section 8–3 to discuss simple harmonic motion, and so on. Finally, Chapter 18 is devoted entirely to differential equations and their applications.

3. There are six case studies marked *Optional Reading*; they can be read and discussed as time and desire permit. Each one presents a complete discussion and solution of a practical problem using the material just studied.

Section 8–5 Mercator and $\int \sec x \, dx$ (an application of the integral of the secant to map making)

Section 9–7 The Logistic Model of Population Growth (an application of partial fraction integration)

Section 10–4 The Predator-Prey Model (an application of L'Hôpital's rule and improper integrals)

Section 11–9 Infinite Series and Music Synthesizers (an application of series to producing musical sounds)

Section 13–6 Brachistochrones and Tautochrones (an application of parametric equations)

Section 15–10 An Application (of Lagrange Multipliers) to Economics

4. A section on probability is included in Chapter 6 (Applications of the Definite Integral). Only the simplest concepts of probability are necessary to provide many interesting examples and applications. In later chapters, these same concepts give rise to practical applications of improper integrals and infinite series.

5. Calculator use is encouraged. After repeated warnings and an explicit example demonstrating how round-off procedures can produce gross errors, calculators are used to advantage in the following ways: (a) to visualize the function concept (function machine), (b) to illustrate limits such as $(1 + h)^{1/h}$ as h approaches 0, and (c) to ease the burden of tedious computations, for example, in numerical integration (Simpson's rule) and in estimating the sum of an infinite series.

6. Chapter 1 contains a thorough review of precalculus material. Though the trend in recent years is to present only a brief review in order to introduce the derivative as soon as possible, experience shows that students have no difficulty at all in learning how to differentiate and integrate. The real difficulties in calculus are more basic: (a) translating a problem into functional notation, (b) simplifying algebraic expressions, and (c) solving equations and inequalities. Therefore, Chapter 1 addresses these trouble spots in some detail. The topics reviewed, along with examples of each, are prominently displayed at the beginning of each section. Thus, at a glance, the instructor or individual student can decide to cover as much or as little of the material as seems appropriate.

7. Wherever it is appropriate, the exercises are grouped into four clearly marked categories. The first group contains the drill exercises, the second contains applications in the form of word problems, and exercises in the third group require the student to verify or prove something. Each category is graded in difficulty. The fourth group contains optional exercises that are apt to be more of a challenge than the others. Answers to underlined exercises appear in the back of the book.

There is ample material for the usual three-semester or four-quarter calculus course. The instructor will find plenty of room for any convenient approach. The formal definition of a limit and the formal proofs of most statements are presented as optional material, which can be covered at any desirable time or omitted altogether. There is also plenty of opportunity to rearrange the material. For example, Section 1–9, on inverse functions, can be delayed until Chapter 7 where its use is essential. The material on the exponential e^x and trigonometry need not be taken up until Chapters 7 and 8. Chapter 11, on infinite series,

can be studied any time after Chapter 10. Another point of flexibility involves the conic sections. Some students have already studied conic sections and others have not. To accommodate both groups, Chapter 12 contains a detailed account of the conics whereas Section 13–1 contains only a brief review of their important properties. Either presentation is sufficient preparation for later work.

SUPPLEMENTAL MATERIALS

A student manual, containing worked-out solutions to many of the underlined exercises, is available through your book store. This manual was prepared by Professor Edward L. Keller, *California State University, Hayward*.

A teacher's manual, containing the answers not provided in the book, is available on request.

ACKNOWLEDGEMENTS

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Typing I would like to thank my wife, Dolores, who not only did all of the typing, but also made many suggestions for improvements in the visual illustrations.

Editing I am indebted to everyone at Scott, Foresman who helped put this book together, but especially to my editor, Jack Pritchard. I have never seen an editor work harder or be of more assistance to a struggling author. I am extremely grateful and hope to have the opportunity to work with him again.

Arthur B. Simon
Moss Beach, California

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