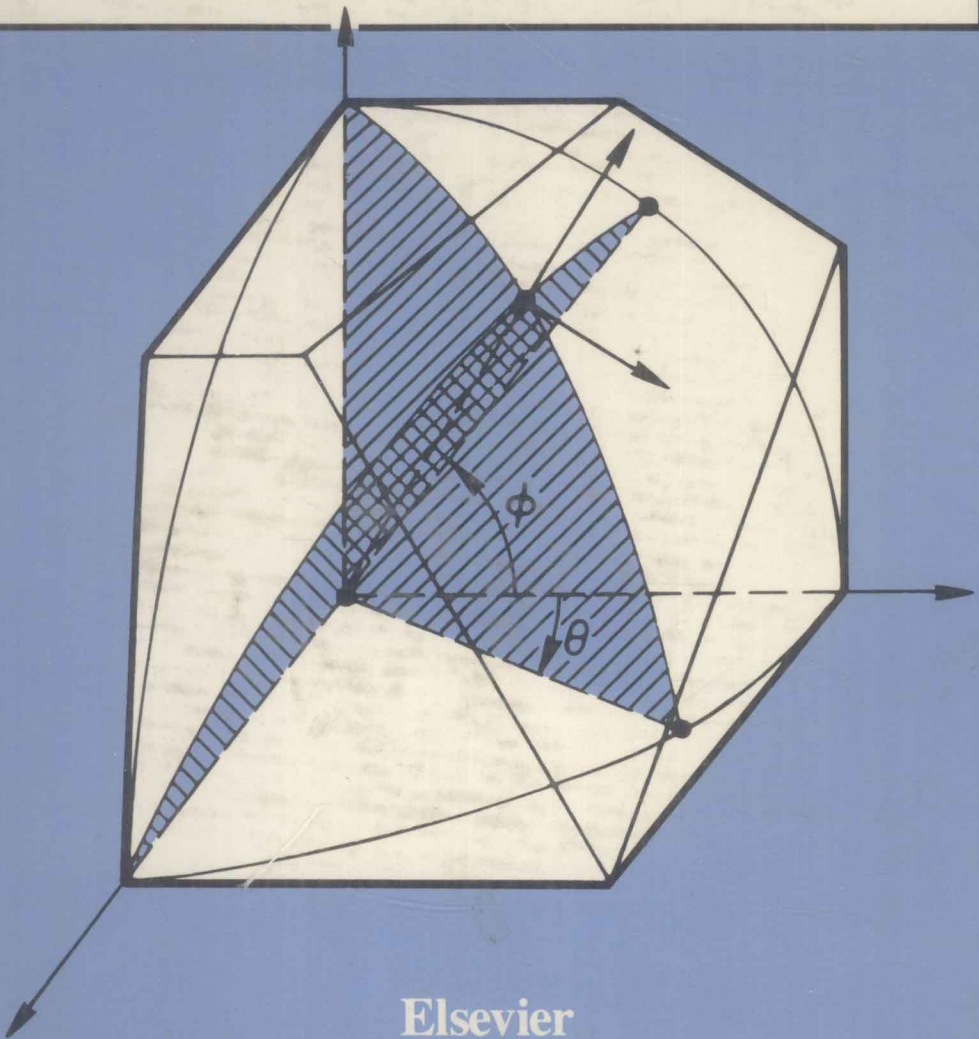
Advanced Strength and Applied Elasticity

Second SI Edition

A. C. Ugural S. K. Fenster



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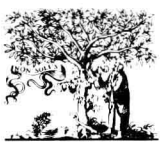

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Advanced Strength and Applied Elasticity

Second SI Edition

Preface to the Second SI Edition

The present edition seeks to preserve the objectives and emphases of the original and the SI version of the first edition to which the reader is referred. An effort has been made to provide a more complete and current text through the inclusion of new material dealing with the transverse normal stress in beams, composite beams, and the properties of composite areas. Digital computer programs for principal stresses and moments of inertia have been added in Appendixes B and C.

The entire text has been reexamined and many minor improvements have been made throughout by elimination and rearrangement. Some sections have been expanded to improve upon previous expositions. The footnote references, provided as an aid to the student who wishes to pursue further certain aspects of the subject, have been updated.

As before we have resisted the temptation to increase greatly the material covered. However, it was considered desirable to add a number of illustrative examples and a large number of problems that are important in engineering practice and design. In this context, as throughout, emphasis is placed upon the use of fundamentals in order to build students' understanding and ability to solve more complex problems. Answers to selected problems are given at the end of the book.

The authors are deeply indebted to the many readers who have contributed many valuable suggestions and comments. The authors also thank their colleagues who have found the book useful through the years, and who have encouraged the preparation of this edition.

A. C. Ugural
S. K. Fenster

Teaneck, New Jersey
January, 1987

Preface to the SI Version

The new edition of this text seeks to preserve the general character of the original version. While the change to the System of International Units provided the immediate impetus for this work, a major effort has been made to provide a more complete and current text through the inclusion of new material dealing with the following topics: polynomial solutions, metal fatigue, failure criteria for combined fluctuating loads, torsion of curved bars, thermal stresses in cylinders, elastic-plastic stresses in rotating disks, and the inelastic behavior of structural members. An appendix concerned with the practical solution of the stress cubic equation and direction cosines has been added.

The authors have resisted the temptation to increase greatly the material covered. However, it was considered desirable to add a number of new illustrative examples and problems of practical importance. It is hoped that clarity of presentation has been maintained, as well as simplicity as permitted by the nature of the subject, unpretentious depth, an effort to encourage intuitive understanding, and a shunning of the irrelevant.

The subject matter of this text is usually covered in one-semester senior and one-semester graduate level courses in *Strength of Solids* and *Applied Elasticity*. However, inasmuch as sufficient material is presented for a full year of study, the book may simulate the development of courses in *Structural Analysis* and *Stress Analysis*. Topics selected and methods of presentation are also directed to the needs of practicing design and research engineers.

Because of extensive subdivision into a variety of topics and employment of more than one method of analysis, the text should provide flexibility in choice of assignments to cover courses of different length and content. Chapters 1 and 2 address the analysis of stress and strain and should be studied first. The treatment of two-dimensional problems in elasticity (Chapter 3) is illustrated repeatedly throughout and compared with the mechanics of materials approach. The remaining chapters may be studied in any order.

We are indebted to the many readers who have contributed numerous constructive suggestions. Our thanks also to our colleagues who have found the book useful through the years, and who have given encouragement to the preparation of the SI version.

A. C. Ugural
S. K. Fenster

Teaneck, New Jersey
January, 1981

Preface to the First Edition

This text is a development of classroom notes prepared in connection with advanced undergraduate and first year graduate courses in elasticity and the mechanics of solids. It is designed to satisfy the requirements of courses subsequent to an elementary treatment of the strength of materials. In addition to its applicability to aeronautical, civil, and mechanical engineering and to engineering mechanics curricula, the authors have endeavored to make the text useful to practicing engineers. Emphasis is given to *numerical techniques* (which lend themselves to computerization) in the solution of problems resisting *analytical treatment*. The stress placed upon numerical solutions is not intended to deny the value of classical analysis, which is given a rather full treatment. It instead attempts to fill what the authors believe to be a void in the world of textbooks.

An effort has been made to present a balance between the theory necessary to gain insight into the mechanics, but which can offer no more than crude approximations to real problems because of simplifications related to geometry and conditions of loading, and numerical solutions, which are so useful in presenting stress analysis in a more realistic setting. The authors have thus attempted to emphasize those aspects of theory and application which prepare a student for more advanced study or for professional practice in design and analysis.

The theory of elasticity plays three important roles in the text: it provides exact solutions where the configurations of loading and boundary are relatively simple; it provides a check upon the limitations of the strength of materials approach; it serves as the basis of approximate solutions employing numerical analysis.

To make the text as clear as possible, attention is given to the presentation of the fundamentals of the strength of materials. The physical significance of the solutions and practical applications are given emphasis. The authors have made a special effort to illustrate important principles and applications with numerical examples. Consistent with announced national policy, included in the text are problems in which the

physical quantities are expressed in the International System of Units (SI).

It is a particular pleasure, upon the completion of a project of this nature, to acknowledge the contributions of those who assisted the authors in the evolution of the text. Thanks are, of course, due to the many students who have made constructive suggestions throughout the several years when drafts of this work were used as a text. To Professor F. Freudenstein of Columbia University and Professor R. A. Scott of the University of Michigan, we express our appreciation for their helpful recommendations and valuable perspectives in connection with their review of the manuscript. And, as has always been the case, Mrs. Helen Stanek has provided intelligent editorial and typing assistance throughout the several drafts of this work; to her, the authors express their special thanks and appreciation.

A. C. Ugural
S. K. Fenster

Teaneck, New Jersey
May, 1975

List of Symbols

A	area
b	width
C	carry-over factor, torsional rigidity
c	distance from neutral axis to outer fiber
D	distribution factor, flexural rigidity of plate
$[D]$	elasticity matrix
d	diameter, distance
E	modulus of elasticity in tension or compression
E_s	modulus of plasticity or secant-modulus
E_t	tangent modulus
e	dilatation, displacement, distance, eccentricity
F	body force per unit volume, concentrated force
f_s	form factor for shear
G	modulus of elasticity in shear
g	acceleration of gravity (≈ 9.81 m/s^2)
h	depth of beam, height, membrane deflection, mesh width
I	moment of inertia of area, stress invariant
J	polar moment of inertia of area
K	bulk modulus, spring constant of an elastic support, stiffness factor, thermal conductivity, fatigue factor
$[K]$	stiffness matrix of whole structure
k	constant, modulus of elastic foundation, spring constant, strength coefficient, stress concentration factor
$[k]$	stiffness matrix of finite element
L	length, span
M	moment

M_t, M_{xy}	twisting moment or torque
m	area property, moment caused by unit load
N	factor of safety, fatigue life (cycles), force, newton
n	number, strain hardening index
l, m, n	direction cosines
P	concentrated force
p	distributed load per unit length or area, pressure
Q	first moment of area, heat flow per unit length, shearing force
$\{Q\}$	nodal force matrix of finite element
R	radius, reaction
S	elastic section modulus, shear center
r	radius, radius of gyration
r, θ	polar coordinates
s	distance along a line or a curve, second
T	surface load per unit area or stress resultant, temperature
t	thickness
U	strain energy
U_0	strain energy per unit volume
U^*	complementary energy
V	shearing force, velocity, volume
W	weight, work
u, v, w	components of displacement
Z	plastic section modulus
x, y, z	rectangular coordinates
α	angle, coefficient of thermal expansion
β	numerical factor
γ	shear strain, weight per unit volume or specific weight
δ	deflection, finite difference operator, variational symbol
$\{\delta\}$	nodal displacement matrix of finite element
Δ	change of a function
ε	normal strain
θ	angle, angle of twist per unit length
ν	Poisson's ratio
λ	axial load factor, Lamé constant
Π	potential energy

ρ	density (mass per unit volume), radius
σ	normal stress
τ	shear stress
ϕ	angle, stress function
φ	total angle of twist
ω	angular velocity

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