Lecture Notes in Control and Information Sciences

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Proceedings of a Conference
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Editors
Alfred Auslender
Département de Mathématiques Appliquées
Université de Clermont-Ferrand II
B.P. 45
F-63170 Aubière (France)

Werner Oettli
Fakultät für Mathematik und Informatik
Universität Mannheim
Schloß
D-6800 Mannheim

Josef Stoer
Institut für Angewandte Mathematik und Statistik
Universität Würzburg
Am Hubland
D-8700 Würzburg

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PREFACE

This volume constitutes the proceedings of a conference held March 16-22, 1980, at Mathematisches Forschungsinstitut Oberwolfach. The purpose of this conference was to treat recent advances in general optimization theory as well as in problems of optimal control, and also to establish closer contacts between scientists, mainly from France and Germany, working in these fields. Included among the topics were new results in nonlinear analysis, used to obtain general optimality conditions. This more abstract approach was complemented by techniques using the specific structure of particular function spaces arising in optimal control problems. Also included were the development and evaluation of numerical methods for solving problems of this kind.

The organizers gratefully acknowledge the generous support received by Mathematisches Forschungsinstitut Oberwolfach.

A. Auslender W. Oettli J. Stoer

LIST OF PARTICIPANTS

M. ATTEIA, U.E.R. Sciences, Département de Mathématiques, 118, rue de Narbonne, 31077 Toulouse, France

A. AUSLENDER, Département de Mathématiques Appliquées, Université de Clermont, B.P. 45, 63170 Aubière, France

H.G. BOCK, Institut für Angewandte Mathematik der Universität Bonn, Wegelerstraße 6, D-5300 Bonn

J. BRÄUNINGER, Mathematisches Institut der Universität Stuttgart, Pfaffenwaldring 57, D-7000 Stuttgart

J. BRINK-SPALINK, Institut für Numerische und Instrumentelle Mathematik, Universität Münster, Roxelerstraße 64, D-4400 Münster

M. BROKATE, Institut für Mathematik, FU Berlin, Arnimallee 2-6, D-1000 Berlin 33

L. COLLATZ, Institut für Angewandte Mathematik, Universität Hamburg, Bundesstraße 55, D-2000 Hamburg 13

F. COLONIUS, Universität Bremen, Bibliothekstraße, Postfach 330 440, D-2800 Bremen 33

J.P. CROUZEIX, Département de Mathématiques Appliquées, Université de Clermont, B.P. 45, 63170 Aubière, France

U. ECKHARDT, Institut für Angewandte Mathematik, Universität Hamburg, Bundesstraße 55, D-2000 Hamburg 13

I. EKELAND, U.E.R. Mathématiques de la Décision, Université de Paris IX, 75775 Paris Cédex 16, France

J. FREHSE, Institut für Angewandte Mathematik, Universität Bonn, Beringstraße 4-6, D-5300 Bonn

K. GLASHOFF, Institut für Angewandte Mathematik, Universität Hamburg, Bundesstraße 55, D-2000 Hamburg 13

B. GOLLAN, Mathematisches Institut, Universität Würzburg, Am Hubland, D-8700 Würzburg

M. GRÖTSCHEL, Universität Bonn, Institut für Operations Research, Nassestraße 2, D-5300 Bonn 1

J. GWINNER, Fakultät für Mathematik und Informatik, Universität Mannheim, Seminargebäude A 5, D-6800 Mannheim

W. HACKBUSCH, Mathematisches Institut der Universität Köln, Weyertal 86-90, D-5000 Köln 41

J.B. HIRIART-URRUTY, Département de Mathématiques Appliquées, Université de Clermont, B.P. 45, 63170 Aubière, France

K.H. HOFFMANN, FU Berlin, Institut für Mathematik, Arnimallee 2-6, D-1000 Berlin 33

W. VAN HONSTEDE, Institut für Angewandte Mathematik, Universität Bonn, Wegelerstraße 6, D-5300 Bonn

P. HUARD, Electricité de France, 1, avenue du Général de Gaulle, 92141 Clamart, France

J.L. DE JONG, Onderafdeling der Wiskunde, Technische Hogeschool, P.O. Box 513, Eindhoven, The Netherlands

P. KALL, Universität Zürich, Weinbergstraße 59, CH-8006 Zürich, Switzerland

F. KAPPEL, II. Mathematisches Institut, Universität Graz, Elisabethstraße 11, 8010 Graz, Austria

H.W. KNOBLOCH, Mathematisches Institut der Universität Würzburg, Am Hubland, D-8700 Würzburg

M. KOHLMANN, Institut für Angewandte Mathematik, Universität Bonn, Wegelerstraße 6, D-5300 Bonn

B. KORTE, Institut für Operations Research, Universität Bonn, Nassestraße 2, D-5300 Bonn 1

P. KOSMOL, Mathematisches Seminar, Universität Kiel, Olshausenstraße 40-60, D-2300 Kiel 1

W. KRABS, Fachbereich Mathematik,TH Darmstadt, Schloßgartenstraße 7, D-6100 Darmstadt

D. KRAFT, Institut für Dynamik der Flugsysteme, D-8031 Oberpfaffenhofen

P.J. LAURENT, Institut National Polytechnique de Grenoble, Domaine Universitaire, B.P. 53, 38041 Grenoble-Cedex, France

C. LEMARECHAL, I.N.R.I.A., Rocquencourt, B.P. 105, 78150 Le Chesnay, France

F. LEMPIO, Lehrstuhl für Mathematik, Universität Bayreuth, Postfach 3008, D-8580 Bayreuth

U. MACKENROTH, Fachbereich Mathematik und Physik, Universität Bayreuth, Postfach 3008, D-8580 Bayreuth

K. MALANOWSKI, Systems Research Institute, Polska Akademia Nauk, ul. Newelska 6, 01-447 Warszawa, Poland

D. MANTHEY, Lehrstuhl für Mathematik VII, Universität Mannheim, D-6800 Mannheim

H. MAURER, Institut für Numerische und Instrumentelle Mathematik, Universität Münster, Roxelerstraße 64, D-4400 Münster

E. NURMINSKI, IIASA, Schloßplatz 1, Laxenburg, Austria

H.J. OBERLE, TU München, Fachbereich Mathematik, Arcisstraße 21, D-8000 München 2

W. OETTLI, Lehrstuhl für Mathematik VII, Universität Mannheim, D-6800 Mannheim

D. PALLASCHKE, GMD, Schloß Birlinghoven, D-5205 St. Augustin

J.P. PENOT, Faculté des Sciences, Avenue Philippon, 64000 Pau, France

H.J. PESCH, TU München, Fachbereich Mathematik, Arcisstraße 21, D-8000 München 2

G. PIERRA, E.N.S.M.A., 20, rue Guillaume le Troubadour, 86034 Poitiers, France

K. RITTER, Mathematisches Institut A, Universität Stuttgart, Pfaffenwaldring 57, D-7000 Stuttgart 80

S.M. ROBINSON, University of Wisconsin, 610 Walnut Street, Madison, WI 53706, USA

R.T. ROCKAFELLAR, University of Washington, Department of Mathematics, Seattle, WA 98195, USA

E. SACHS, TU Berlin, Fachbereich Mathematik, Straße des 17. Juni 135, D-1000 Berlin 12

S. SCHAIBLE, Department of Finance, University of Alberta, Edmonton, Alberta T6G 2G1, Canada

K. SCHITTKOWSKI, Institut für Angewandte Mathematik und Statistik der Universität Würzburg, Am Hubland, D-8700 Würzburg

E. SPEDICATO, Istituto Universitario di Bergamo, Via Salvecchio, 24100 Bergamo, Italy

P. SPELLUCCI, Fachbereich Mathematik, TH Darmstadt, Schloßgartenstraße 7, D-6100 Darmstadt

J. STOER, Institut für Angewandte Mathematik und Statistik der Universität Würzburg, Am Hubland, D-8700 Würzburg

K.H. WELL, Institut für Dynamik der Flugsysteme, D-8031 Oberpfaffenhofen

J. ZOWE, Lehrstuhl für Angewandte Mathematik, Universität Bayreuth, Postfach 3008, D-8580 Bayreuth

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Part 1:
Optimization

QUASI-CONVEX DUALITY

by M. ATTEIA and A. EL QORTOBI

Université Paul Sabatier

U.E.R. Mathématiques Informatique Gestion

118, route de Narbonne

31062 - TOULOUSE CEDEX

The study of quasi-convex duality developed in this paper is based on the notion of projective polarity studied by M. ATTEIA (Cf. [1]).

We shall give only the main elementary properties of quasi-convex duality which, formally, are quite analogous to those of convex duality.

1 - Projective polar of a set

Let X be a separated locally convex space and X' its topological dual. We suppose that X (resp. X') is equipped with the weak topology $\sigma(X, X')$ (resp. $\sigma(X', X)$).

If A is a subset of X, we denote by :

$$A^{o} = \{x' \in X' \; ; \; \forall x \in A, \; \langle x, x'\rangle \leq 1\},$$

$$A^{\nabla} = \{x' \in X' \; ; \; \forall x \in A, \; \langle x, x'\rangle \geq 1\},$$

and by $\overline{co}(A)$, the closed convex hull of A. If A is closed and convex we put:

$$A_{\infty} = \bigcap_{\lambda \in \mathbb{R}_{+}^{*}} \lambda(A-a), \text{ where a is an arbitrary element of A.}$$

We can easily verify that :

$$A^{o} = \Big(\bigcup_{\lambda \in [0,1]} \lambda \; \overline{co}(A)\Big)^{o}, \quad A^{\nabla} = \Big(\bigcup_{\lambda \geq 1} \lambda \; \overline{co}(A)\Big)^{\nabla}, \quad (A^{\nabla})_{\infty} = -\,(A^{o})_{\infty}.$$

Definition 1 : The set $A^{\#} = A^{o} \cup A^{\nabla}$ is called the *projective polar* of A.

Proposition 1 :

(i) If $(C^{+}(A)) = \{(x, \sigma) \in X \times \mathbb{R}_{+}^{*} \; ; \; x \in \sigma A\}$

and $(C^+(A))^o = \{(x',\sigma') \in X' \times \mathbb{R} \; ; \; \forall (x,\sigma) \in C^+(A), \; \langle x, x'\rangle + \sigma\sigma' \le 0\}$

then : $(C^+(A))^o \cap (X' \times \{-1\}) = A^o \times \{-1\}$,

$(C^+(A))^o \cap (X' \times \{1\}) = -(A^\nabla) \times \{1\}$.

As $(C^+(A))^o$ is a (closed) convex cone we say that $A^\#$ *is projectively convex*.

(ii) $A^\#$ is closed.
If $A \ni 0_X$, then $A^\nabla = \{\phi\}$ and $A^\# = A^o$ is convex.

(iii) If $(A_i)_{i \in I}$ is a family of subsets of X, then :

$$\Big(\bigcup_{i \in I} A_i\Big)^\# \subset \bigcap_{i \in I} (A_i^\#) \quad \text{and} \quad \Big(\bigcap_{i \in I} A_i\Big)^\# \supset \bigcup_{i \in I} (A_i^\#).$$

Generally, these inclusions are strict.

Definition 2 : The set $A^{oo} \cap A^{\nabla\nabla}$ is called the *projective bipolar* of A, and we denote it by $A^{\#\#}$.

Proposition 2 : $A^{\#\#} = \overline{co}(A)$

Remark 1 : Generally $(A^o \cup A^\nabla)^\#$ is different from $A^{\#\#}$.

2 - *Projective polar of a functional*

Suppose that $f \in \overline{\mathbb{R}}^X$.

For each $\lambda \in \mathbb{R}$, we denote :

$\tilde{S}_\lambda(f) = \{x \in X \; ; \; f(x) \le \lambda\}$ (resp. $S_\lambda(f) = \{x \in X \; ; \; f(x) < \lambda\}$).

Let k be a decreasing bijective map from $\mathbb{R}$ to $\mathbb{R}$ and :

$$\forall x' \in X', \; f^o(x') = \operatorname{Inf} \{k(\lambda) \; ; \; x' \in (S_\lambda(f))^o\}$$

$$f^\nabla(x') = \operatorname{Inf} \{k(\lambda) \; ; \; x' \in (S_\lambda(f))^\nabla\}.$$

We can easily prove that :

$$\forall x' \in X', \; (k^{-1} \circ f^o)(x') = \operatorname{Inf} \{f(x) \; ; \; \langle x, x'\rangle > 1\}$$

$$(k^{-1} \circ f^\nabla)(x') = \operatorname{Inf} \{f(x) \; ; \; \langle x, x'\rangle < 1\}.$$

Definition 3 : The functional : $f^{\#} = \mathrm{Min}(f^{o}, f^{\nabla})$ is called the *projective polar of* f.

Now, for simplicity, we suppose that : $\forall\lambda \in \mathbb{R}$, $k(\lambda) = -\lambda$.

Proposition 3 :

(i) $f^{\#}$ is l.s.c. (and projectively quasi-convex because its sections are projectively convex).

$$f^{o}(0_{X'}) = -\infty \text{ and } f^{\nabla}(0_{X'}) = -\mathrm{Inf}\ \{f(x)\ ;\ x \in X\}.$$

(ii) $f \leq g \Rightarrow f^{\#} \geq g^{\#}$.

(iii) $\forall\alpha \in \mathbb{R}$, $(f+\alpha)^{\#} = f^{\#} - \alpha$

$\forall\alpha \in \mathbb{R}_{+}^{*}$, $(\alpha f)^{\#} = \alpha . f^{\#}$

$\forall\alpha \in \mathbb{R}^{*}$, $(f.\alpha)^{\#} = f^{\#}.\frac{1}{\alpha}$.

(iv) If $(f_i)_{i \in I} \subset \overline{\mathbb{R}}^{X}$, then:

$$(\mathrm{Inf}_{i \in I}\ f_i)^{\#} \geq \mathrm{Sup}_{i \in I}\ (f_i^{\#}) \quad \text{and} \quad (\mathrm{Sup}_{i \in I}\ f_i)^{\#} \leq (\mathrm{Inf}_{i \in I}\ f_i)^{\#}.$$

Examples 1

(i) $X = \mathbb{R}$, $f(x) = \begin{cases} \mathrm{Log}|x| & \text{if } |x| > 1 \\ 0 & \text{otherwise} \end{cases}$

then : $\forall x' \in \mathbb{R}$, $f^{\nabla}(x') = 0$

and : $f^{\#}(x') = \begin{cases} \mathrm{Log}|x'| & \text{if } |x'| < 1 \text{ and } x' \neq 0 \\ 0 & \text{if } |x'| \geq 1 \\ -\infty & \text{if } x' = 0. \end{cases}$

(ii) $X = \mathbb{R}$, $p \in \mathbb{N}$, $f(x) = x^{2p+1}$,

then : $f^{\#}(x') = \begin{cases} -\infty & \text{if } x' = 0 \\ -\dfrac{1}{x'^{2p+1}} & \text{if } x' \neq 0. \end{cases}$

(iii) $X = \mathbb{R}^2$, $f(x, y) = \begin{cases} -xy & \text{if } x \geq 0 \text{ and } y \geq 0 \\ +\infty & \text{otherwise} \end{cases}$

then :

$$f^{\#}(x', y') = \begin{cases} \frac{1}{4x'y'} & \text{if } (x' > 0 \text{ and } y' > 0) \\ +\infty & \text{if } (x'>0 \text{ and } y'<0) \text{ or } (x' \leq 0 \text{ and } y'>0) \\ -\infty & \text{if } (x' \leq 0 \text{ and } y' < 0). \end{cases}$$

(iv) X is a separated locally convex space.
If A is a subset of X, we put :

$$\forall x \in A,\ \delta_A(x) = \begin{cases} 0 & \text{if } x \in A \\ +\infty & \text{otherwise} \end{cases}$$

$$\Delta_A(x) = \begin{cases} -\infty & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases}$$

$$p_A(x) = \text{Inf } \{\lambda \in \mathbb{R}_+^* \ ; \ x \in \lambda A\}.$$

Then : $(\delta_A)^{\#} = \Delta_{A^{\#}}$, $(\Delta_A)^{\#} = \delta_{A^{\#}}$

and $(p_A)^{\#} = -\dfrac{1}{\delta_A^*}$

where : $\forall x' \in X'$, $\delta_A^*(x') = \text{Sup}\{\langle x, x'\rangle \ ; \ x \in A\}$.

(v) X is a separated locally convex space.
For each $x' \in X'$ we denote by :

$A(x') = \{x \in X \ ; \ \langle x, x'\rangle > 1\}$ and $B(x') = \{x \in X \ ; \ \langle x, x'\rangle < 1\}$.

If $f \in \overline{\mathbb{R}}^X$, then:

$\forall x' \in X'$, $f^{\circ}(x') = (f + \delta_{A(x')})^*(0_{X'})$, $f^{\nabla}(x') = (f + \delta_{B(x')})^*(0_{X'})$,
(where for any $g \in \overline{\mathbb{R}}^X$ and $x' \in X'$, $g^*(x') = \text{Sup}\{\langle x,x'\rangle - g(x) \ ; \ x \in X\}$.