PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS

Volume XXX

Part 1

SEVERAL COMPLEX VARIABLES

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SEVERAL COMPLEX VARIABLES

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These proceedings are dedicated to

Lillian Casey

who was the guiding force

behind six years of AMS meetings

PREFACE

The American Mathematical Society held its twenty-third Summer Research Institute at Williams College, Williamstown, Massachusetts, from July 28 to August 15, 1975. Several Complex Variables was selected as the topic for the institute. Members of the Committee on Summer Institutes at the time were Louis Auslander (chairman), Richard E. Bellman, S. S. Chern, Richard K. Lashof, Walter Rudin, and John T. Tate. The institute was supported by a grant from the National Science Foundation.

ORGANIZATION. The Organizing Committee for the institute consisted of Ian Craw, Hans Grauert, Robert C. Gunning (cochairman), David Lieberman, James Morrow, R. Narasimhan, Hugo Rossi (cochairman), Yum-Tong Siu, and R. O. Wells, Jr. (Editor of these PROCEEDINGS).

PROGRAM. The topic of the 1975 summer institute was the theory of functions of several complex variables. The emphasis in arranging the program was on the more analytical aspects of that subject, with particular attention to the relations between complex analysis and partial differential equations, to the properties of pseudoconvexity and of Stein manifolds, and the relations between currents and analytic varieties. However, there were also lectures and seminars on other aspects of that broad and active field of investigation, such as deformation theory, singularities of analytic spaces, value distribution theory, compact complex manifolds, and approximation theory.

There were six series of invited expository lectures, as well as twenty-two hour lectures of a general or survey nature; there were also eight series of seminars on current developments in the subject, six of which were planned and partially arranged in advance.

PRINCIPAL LECTURE SERIES.
Chern-Moser invariants by Daniel M. Burns, Jr. and Steven Shnider (2 lectures);
Power series methods in deformation theorems by Otto Forster (5 lectures);
Holomorphic chains and their boundaries by Reese Harvey (4 lectures);

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Methods of PDE in complex analysis by Joseph J. Kohn (5 lectures);

Tangential Cauchy-Riemann equations by Masatake Kuranishi (3 lectures);

Analysis on noncompact Kähler manifolds by Hung-Hsi Wu and Robert E. Greene (4 lectures).

HOUR LECTURES.

Theta functions with characteristic and distinguished subspaces of the Heisenberg manifold by Louis Auslander;

Hardy spaces and local estimates on boundaries of strongly pseudo-convex domains. I by Ronald Coifman;

Some recent developments in the theory of the Bergman kernel function. A survey by Klas Diederich;

Fibering of residual currents by Miguel Herrera;

The Monge-Ampere equation and complex analysis by Norbert Kerzman;

Stable homology and positive currents on abelian varieties by H. Blaine Lawson, Jr.;

A method of inverse functions for plurisubharmonic functions; applications to positive and closed currents by Pierre Lelong;

Holomorphic vector fields on projective varieties by David Lieberman;

Diffusion estimates in complex analysis by Paul Malliavin;

Rational singularities, Moishezon spaces, and compactifications of C^3 by James A. Morrow;

Rankin-Selberg method in the theory of automorphic forms by Mark Novodvorsky;

Hardy spaces and local estimates on boundaries of strongly pseudo-convex domains. II by Richard Rochberg;

Analysis of the boundary Laplacian and related hypoelliptic differential operators by Linda Rothschild;

Classification of quasi-symmetric domains by Ichiro Satake;

The Levi problem by Yum-Tong Siu;

Boundary values of the solutions of the $\bar{\partial}$ equation and characterization of zeros of functions in the Nevanlinna class by Henri Skoda;

Value distribution on parabolic spaces by Wilhelm Stoll;

Extending functions from submanifolds of the boundary by Edgar L. Stout;

Equisingularity and local analytic geometry by J. Stutz;

Constructability by Jean-Louis Verdier;

Boundary values of holomorphic functions on a Siegel domain and Cauchy-Riemann tangential equations by Michelle Vergne;

Deformations of strongly pseudoconvex domains in C^2 by R. O. Wells, Jr.

SEMINAR SERIES.

(1) Singularities of analytic spaces (Egbert Brieskorn, chairman);
(2) Function theory and real analysis (Stephen Greenfield, chairman);
(3) *q*-convexity and noncompact manifolds (Yum-Tong Siu, chairman);
(4) Value distribution theory in several variables (Wilhelm Stoll, chairman);
(5) Compact complex manifolds (A. Van de Ven, chairman);
(6) Problems in approximation (John Wermer, chairman);
(7) Group representations and harmonic analysis (Kenneth I. Gross, chairman);
(8) Differential geometry and complex variables (Peter Gilkey, chairman).

SUMMARY. The broad areas emphasized by this summer institute can be summarized as follows:

The $\bar{\partial}$-equation. The relation between complex analysis and partial differential equations centers about the Cauchy-Riemann or $\bar{\partial}$-equations. Kohn gave a series of lectures which provided a survey of the role of the $\bar{\partial}$-equations. Several lectures and seminar talks were devoted to the regularity up to the boundary of solutions of the $\bar{\partial}$-equation, with applications to complex analysis. The structure imposed by the $\bar{\partial}$-equation on the smooth boundary of a region in $\boldsymbol{C}^n$ was discussed in several seminars as well. Most lectures on these topics were in the seminars led by S. Greenfield and Y.-T. Siu.

Holomorphic chains. Reese Harvey gave a survey of the characterization of holomorphic chains and their boundaries among other currents, with a number of applications to classical problems in complex analysis; other lectures discussed recent research on holomorphic chains, which includes, for instance, value distribution theory.

Differential geometry. R. E. Greene and H. Wu gave a survey of the characterization of Stein manifolds by curvature conditions on an underlying Kähler metric. A seminar was organized by P. Gilkey on differential geometry and complex analysis in an extension of this. The survey lectures by D. M. Burns, and S. Shnider discussed recent developments in the theory of higher order invariants associated to real hypersurfaces in $\boldsymbol{C}^n$.

Singularities. A seminar led by E. Brieskorn was devoted to the singularities of analytic spaces, with an emphasis on the complex analytic properties rather than the algebraic properties treated in the preceding summer institute on algebraic geometry.

Value distribution theory. Wilhelm Stoll lectured on general value distribution theory and led a seminar in which a number of recent developments in this area were discussed.

Compact complex manifolds. A number of analytic results on compact manifolds were discussed in a seminar led by A. Van de Ven.

Approximation theory. While no attempt was made to cover recent developments in complex analysis and function algebras, there was some discussion of approximation theory in a seminar led by J. Wermer.

Harmonic analysis. Symmetric spaces and boundaries of homogeneous domains have been much investigated in complex analysis and lead quite naturally to recent work on group representation and harmonic analysis, some of which was discussed in a seminar led by K. Gross.

PROCEEDINGS. The proceedings of the 1975 summer institute are published here in two volumes. The hour lectures and seminar papers accepted for publication appear in the seminar series most appropriate to the subject matter of the given paper. These are principally research reports describing current research of the authors, while some are of a general expository nature in a given area. The principal lecture series are represented by six survey articles which have been interlaced in these volumes with the seminar series, with an attempt being made for some relationship between the seminar series and the survey articles they juxtapose.

Volume 1: *Seminar Series*: Singularities of Analytic Spaces
Principal Lecture 1: M. Kuranishi

Seminar Series: Function Theory and Real Analysis
Principal Lecture 2: J. J. Kohn
Seminar Series: Compact Complex Manifolds
Principal Lecture 3: Reese Harvey

Volume 2: *Seminar Series*: Noncompact Complex Manifolds
Principal Lecture 4: R. Greene and H. Wu
Seminar Series: Differential Geometry and Complex Analysis
Principal Lecture 5: D. Burns, Jr., and S. Shnider
Seminar Series: Problems in Approximation
Principal Lecture 6: O. Forster
Seminar Series: Value Distribution Theory
Seminar Series: Group Representation and Harmonic Analysis

The detailed list of authors and titles of papers is given in the table of contents fo each volume. At the conclusion of each volume is an author index for authors o articles, as well as authors of papers cited in the bibliographies for each particula part.

R. O. WELLS, JR.
Houston, Texas
May 1976

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Function Theory and Real Analysis

Principal Lecture 2

Compact Complex Manifolds

Principal Lecture 3

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SEMINAR SERIES

SINGULARITIES OF ANALYTIC SPACES

Proceedings of Symposia in Pure Mathematics
Volume 30, 1977

HIGHER DERIVATIONS AND THE ZARISKI-LIPMAN CONJECTURE

JOSEPH BECKER

1. A well-known conjecture in algebraic geometry is that if R is the affine ring of an algebraic variety V over a field k of characteristic zero, and $\mathrm{Der}_k^1(R)$ the module of derivations of R over k is free, then R is a regular ring, e.g., the embedding dimension equals the Krull dimension. In this note we show how this conjecture is related to some questions about higher derivations that have been studied recently by several authors.

First we review the literature on the above conjecture. Lipman **[14]** has proven that $\mathrm{Der}_k^1(R)$ free implies that R is normal, e.g., integrally closed in its quotient field. Scheja and Storch **[23]** have proven R is regular under the additional hypothesis that R has embedding codim $\leqq 1$, e.g., the variety is a hypersurface. Moen **[16]** and Hochster **[10]** have proven the same under the additional hypothesis that R is a homogenous complete intersection. Recently **[11]** Hochster has improved this to require only that R be a quasi-homogenous domain.

Assuming V is normal, so that everything extends across the supposed singular locus, the conjecture has a nice geometric statement for analytic varieties: If the tangent bundle to the regular points of V is holomorphically trivial, then V is actually nonsingular. It is not enough just to require that the tangent bundle be topologically trivial, because Brieskorn's example **[8]** of a locally contractible hypersurface singularity has topologically trivial tangent bundle, but its module of tangent vector fields must be nonfree by the above result of Scheja and Storch. It is interesting to note that the above discussion does not contradict the Grauert equivalence of holomorphic and continuous vector bundles because, for a singular variety, Reg V is not Stein. Unfortunately, all progress on this problem to date has been by algebraic methods.

It is elementary (in terms of well-established machinery in commutative algebra) to prove the following [3], **[20]**:

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(a) There is an R-module $\Omega_k^1(R)$ of differentials of R over k such that $\mathrm{Hom}_R(\Omega_R^1(R), R) = \mathrm{Der}_k^1(R)$; just let $\Omega_k^1(R)$ be the free R-module on the symbols dr, $r \in R$, modulo the submodule generated by the relations $d(r+s) = dr + ds$, $d(rs) = rds + sdr$, and $dl = 0$ for $l \in k$, $r, s \in R$. The embedding dim of R is the rank of $\Omega_k^1(R)$ as an R-module.

(b) For prime ideal p in R, $\Omega_k^1(R)_p = \Omega_k^1(R) \otimes_R R_p$ and R_p is flat over R since localization is an exact functor. By Noetherianness of R, $\Omega_k^1(R)$ has a finite presentation; so by tensoring and Homing in the opposite orders and comparing the resulting exact sequences we see that $\mathrm{Der}_k^1(R_p) = \mathrm{Der}_k^1(R)_p$. Hence it suffices to consider only local rings, to prove the conjecture.

(c) The algebraic closure $\bar{k}$ of k is faithfully flat over k, so $\mathrm{Der}_k(R) \otimes_k \bar{k} = \mathrm{Der}_{\bar{k}}(R \otimes_k \bar{k})$. So it suffices to assume k is algebraically closed.

(d) The analogous conjecture for commutative local rings R over an algebraically closed field of characteristic zero can easily be seen to be a first-order theorem in logic (this means the hypothesis and conclusion can be stated in finitely many symbols) so by Tarski's theorem **[26]** on the elimination of quantifiers, it is true in all algebraically closed fields if it is true in one algebraically closed field. Hence we may take the field to be the complex numbers $\mathbf{C}$.

(e) If m is the maximal ideal of R and $\hat{R}$ the completion of R in its m-adic topology then $\Omega_k^1(R)^\wedge = \Omega_k^1(\hat{R})/\bigcap_{i=1}^\infty m^i\Omega_k^1(R)$, $\mathrm{Der}_k^1(\hat{R}) = \mathrm{Der}_k^1(R)^\wedge$ and $\hat{R}$ is faithfully flat over R so it suffices to consider only complete local rings, that is formal power series rings modulo an ideal (by Cohen's theorem).

(f) If $n =$ Krull dim of $R \equiv$ geometric dim of V, then $\Omega_{\mathbf{C}}^1(R)$ is free if and only if it is generated over R by n elements.

In the following we will work in the category of complex analytic varieties although, as indicated above, we might just as well work over complete local rings. We let $\mathcal{O}$ denote the ring of germs at the origin of convergent power series in n variables. If V is a complex analytic variety with local ring at a point $R = \mathcal{O}/I$, then a derivation of R is a holomorphic tangent vector field

$$D = \sum_{i=1}^n a_i(z)\frac{\partial}{\partial z_i}, \qquad a_i \in \mathcal{O}/I,$$

such that $D(I) \subset I$. By analogy we introduce the higher order differential operators

$$\mathrm{Der}^k(V) = \left\{D = \sum_{|\alpha| \le k} a_\alpha(z)\frac{\partial}{\partial z^\alpha} : D(I) \subset I,\ a_\alpha \in \mathcal{O}/I\right\},$$
$$\mathrm{Der}\,(V) = \bigcup_{k=1}^\infty \mathrm{Der}^k(V).$$

If V is a manifold, it is easily seen that this is just the usual definition of differential operator. Also we introduce the constant coefficient differential operators

$$C(V) = \left\{D = \sum^{\text{finite}} c_\alpha \frac{\partial}{\partial z^\alpha} : D(f)(0) = 0 \text{ for all } f \in I\right\}.$$

Clearly we have a map $\rho : \mathrm{Der}\,(V) \to C(V)$ given by $\rho(\sum a_\alpha(z)\partial^\alpha) = \sum a_\alpha(0)\partial^\alpha$. It is trivial to check all of the following:

(i) $\mathrm{Der}\,(V)$ is a noncommutative ring under composition and a module over $\mathcal{O}(V) = \mathcal{O}/I$ (note $\partial_t(a(t)\partial_t) - (a(t)\partial_t)\,\partial_t = a'(t)\partial_t$), and the associated graded ring

$\operatorname{Gr} \operatorname{Der}(V) = \bigoplus_{k=0}^{\infty} \operatorname{Der}^k(V)/\operatorname{Der}^{k-1}(V)$ is a commutative ring and a module over $\mathcal{O}(V)$.

(ii) Not every constant coefficient differential operator (ccdo) is a variable coefficient differential operator (vcdo): Consider the example $\partial/\partial x$ on $x^2 - y^3$ in C^2.

(iii) $D = \sum_{|\alpha| \leqq k} a_\alpha \partial^\alpha \in \operatorname{Der}^k(V)$ if and only if $D(z^\beta f) \in I$ for all $f \in I$ and all $|\beta| \leqq k - 1$. As a consequence the differential operators of degree $\leqq k$ form a coherent sheaf of $\mathcal{O}$-modules.

(iv) If V is irreducible, then $\operatorname{Der}(V)$ is a domain: This is not at all clear from the ring point of view, but is clearly true at a regular point by looking at the symbol. So this claim just follows from (iii) above, and the obvious fact that the above sheafs have no embedded components in their primary decomposition.

(v) If $V = \bigcup_{i=1}^{n} V_i$ is the decomposition of V into irreducible components and $D \in \operatorname{Der}(V)$, then D restricts to be an element [22] of each $\operatorname{Der}(V_i)$.

(vi) If $\phi : V \to W$ is a holomorphic mapping, it induces a pull-back of differential operators; $\phi^*(\operatorname{Der}(W))$ is just the meromorphic differential operators (means the coefficients are meromorphic) on V which preserve the subring $\phi^*(\mathcal{O}(W))$. If $\operatorname{codim} \operatorname{Sg} V \geqq 2$, these operators have holomorphic coefficients. For additional details of this construction, see [24].

We now put [5] a topological structure on $\mathcal{O}(V)$ which shows why the ccdo are important. Let $\hat{\mathcal{O}}(V)$ be the completion of $\mathcal{O}$ in its m-adic topology. The simple topology on $\hat{\mathcal{O}}$ as a Frèchet space given by the seminorms $\rho_\beta(\sum a_\alpha z^\alpha) = a_\beta$ is not as fine as the Krull topology determined by the metric $\|f - g\| = e^{-\operatorname{ord}(f-g)}$ and so has fewer continuous functions (but does have the same linear continuous functions). It is well known [27] that the vector space continuous dual to the formal power series is the polynomials: Every polynomial $\sum_{|\alpha| \leqq k} c_\alpha y^\alpha$ induces a map: $\hat{\mathcal{O}} \to C$ by $f \to \sum_{|\alpha| \leqq k} c_\alpha (\partial f/\partial x^\alpha)(0)$. On the other hand for any continuous linear $L: \hat{\mathcal{O}} \to C$, $L = \sum (L(x^\alpha)/\alpha!)\, \partial/\partial x^\alpha$, and the sum is finite or else $L(\sum x^\alpha/L(x^\alpha)) = 1 + 1 + \cdots = \infty$. ($L$ commutes with infinite sums because it is continuous.) We will denote $C(V) = \hat{\mathcal{O}}(V)^*$.

Two questions frequently occurring in the literature are:

(I) Is the ring of constant coefficient operators representable? That is, is ρ onto? For example, is every constant coefficient operator at p the specialization at p of a section of the sheaf of variable coefficient operators?

(II) Is the ring of differential operators at a point finitely generated? That is, does there exist $k > 0$ so that the set of kth order operators $\operatorname{Der}^k(V)$ generates $\operatorname{Der}(V)$ as an algebra over $\mathcal{O}(V)$? Is the associated graded ring finitely generated?

Neither is true in general, but both have been shown to hold for irreducible curves. Clearly $\operatorname{Der}(V)$ finitely generated implies $\operatorname{Gr} \operatorname{Der}(V)$ finitely generated but not conversely.

Remark. The set of C linear continuous (Krull topology) functions from $\hat{\mathcal{O}}$ to $\hat{\mathcal{O}}$ is precisely the set of infinite length differential operators $\sum a_\alpha(z)\, \partial/\partial z^\alpha$ such that $\operatorname{ord} a_\alpha \to \infty$ as $|\alpha| \to \infty$. However there exists [9] an $\mathcal{O}(V)$-module M such that $\operatorname{Hom}_{\mathcal{O}}(M, \mathcal{O}) = \operatorname{Der}(V)$ and $\operatorname{Hom}_C(M, C) = C(V)$. There is a commutative diagram

$$\begin{array}{ccc} & \nearrow & \mathcal{O} \\ M & & \downarrow \\ & \searrow & C \end{array}$$

Hence the name representable stems from the fact that the functor $C(V)$ is represented as the specialization of the functor $\mathrm{Der}(V)$.

EXAMPLE. The curve $x^2 = y^3$ has normalization $t \to (t^3, t^2)$ so the local ring is isomorphic to $\boldsymbol{C}\{t^3, t^2\}$. Then ∂_t is a ccdo but not a vcdo. It is represented by the vcdo $\partial_t - t\partial_t^2$.

The history and present state of the art on these questions is:

(A) If V is reducible at p, then V is not representable at p.

PROOF. Let $V = V_1 \cup V_2$, $F \in I(V_1) - I(V_2)$. Since ideals in a Noetherian ring are closed in the Krull topology, by the Hahn-Banach theorem for Frèchet spaces there exists $D \in C(V)_p$ so that $D(F)_p = 1$, $DI(V_2) = 0$. If D has a representative $\tilde{D} \in \mathrm{Der}\,(V)$, $1 = D(F)_p = \tilde{D}(F)_p = \lim_{q \in V_2} \tilde{D}(F)_q = \lim_{q \in V_2} 0 = 0$.

(B) Bloom **[5]** has proven that if V is an irreducible curve, then V is representable. In fact every ccdo of degree d can be represented by a vcdo of degree $= \max(d, e)$, where e is the exponent of the conductor. ($\mathcal{O}(V)$ is a subring of $\boldsymbol{C}\{t\}$ and e is the smallest integer such that $t^n \in \mathcal{O}(V)$ for all $n \geqq e$.) He also gives an example ($x^2 = yz^2$) of a variety which is not representable at an irreducible point which is the limit of reducible points. It seems likely that this should always be the case.

(C) Stutz **[24]** extended Bloom's result to varieties having a Puiseux series normalization, a condition **[25]** which holds on an arbitrary variety in the complement of a subvariety of codim 2.

(D) M. Jaffee **[12]** has completely determined all the differential operators on the curve $x^p = y^q$ by studying the double grading on Der given by degree and strength, where

$$\text{strength}\left(\sum_{\alpha,\beta} a_{\alpha\beta} x^\alpha \frac{\partial}{\partial y \beta}\right) = \inf(|\alpha| - |\beta|) \quad \text{over all nonzero } a_{\beta\alpha}.$$

Note str $\delta = s$ implies $\delta(m^k) \subset m^{k+s}$, where m is the maximal ideal of the local ring.

(E) Using Jaffee's ideas, Bloom **[6]** and Vigué **[28]** independently proved that, for irreducible curves, Gr Der (V) is finitely generated by operators of degree $\leqq 2e + 1$. Vigué proved and Bloom showed by example that, if codim Sg $V \geqq 2$ and V has nonsingular normalization, then Gr Der (V) is not finitely generated. In addition Vigué gave such an example with $C(V)$ representable and Der (V) finitely generated.

(F) Kantor **[13]** has shown that if V is the quotient of $\boldsymbol{C}^n$ by a finite group of automorphisms of $\boldsymbol{C}^n$ then Gr Der (V) is finitely generated. It is interesting to note that, for $n = 2$, Brieskorn **[7]** has shown that all such quotients have only curves of germs zero in the exceptional sets of their minimal resolutions.

(G) Bernstein, Gel'fand, and Gel'fand **[4]** have shown that, for the cubic cone $x^3 + y^3 + z^3 = 0$ in $\boldsymbol{C}^3$, Der (V) is not finitely generated and that every $D \in$ Der (V) has nonnegative strength. This also gives an everywhere locally irreducible example with $C(V)$ not representable because $\partial/\partial x \in C(V)$ and has strength $+1$. This paper is published in Russian; see **[29]** for a French translation. The basic technique is the following: Since $\mathcal{O}(V)$ is a homogeneous ring, the differential operators can be decomposed into sums of operators of homogenous degree and strength. Such operators may be considered to be acting on the dehomogenization of the cone to the projective curve $x^3 + y^3 + 1 = 0$, which has genus one. The Riemann-Roch

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