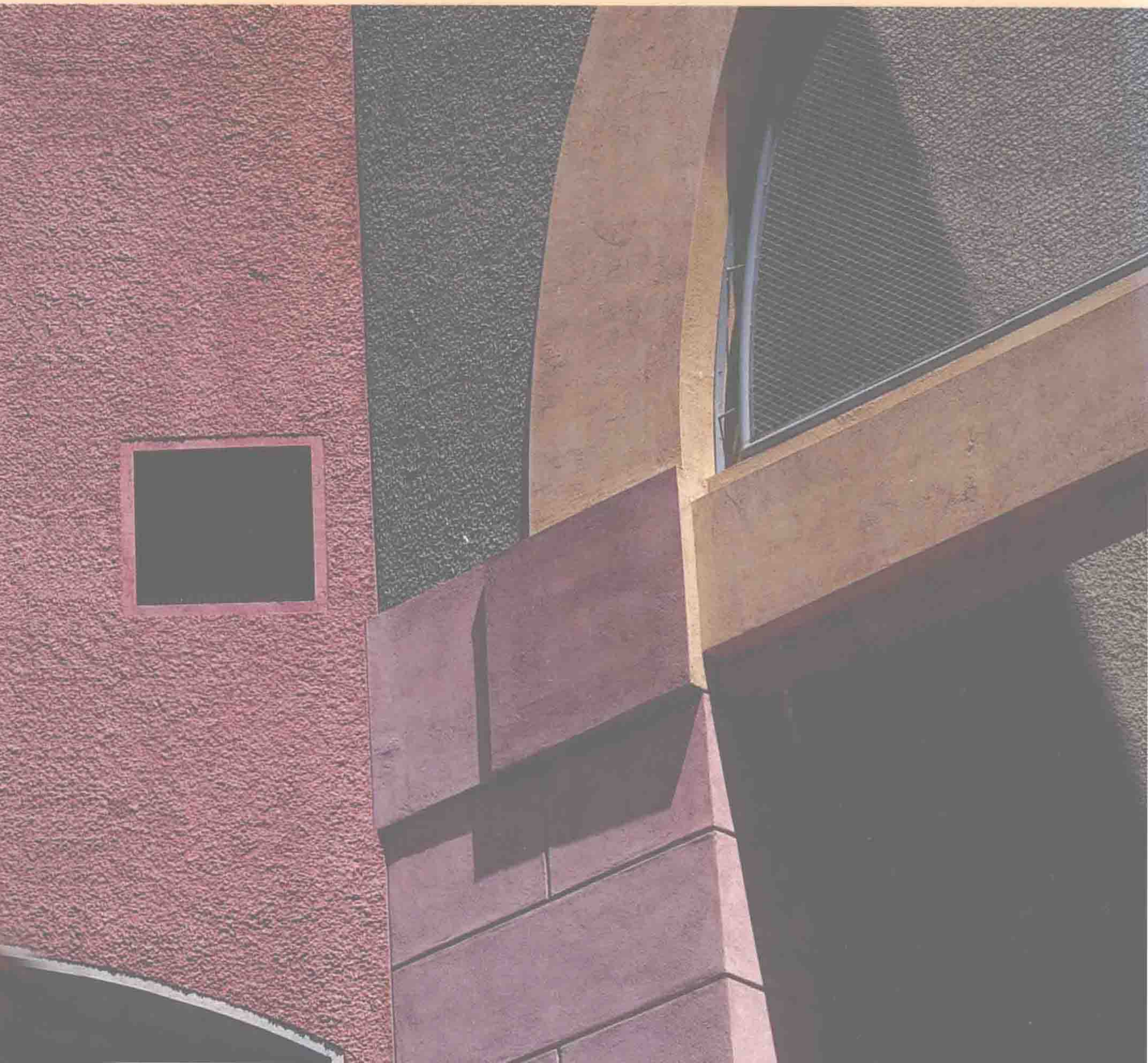
Finite Mathematics

for Management, Life, and Social Sciences

Harshbarger · Reynolds

Third Edition



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Finite Mathematics

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Preface

To paraphrase Alfred North Whitehead, the purpose of education is not to fill a vessel but to kindle a fire. This desirable goal is not always an easy one to realize with students whose primary interest is in an area other than mathematics. The purpose of this text, then, is to present mathematical skills and concepts and to apply them to areas that are important to students in the management, life, and social sciences. The applications included allow students to view mathematics in a practical setting relevant to their intended careers. Almost every chapter of this book includes a section or two devoted to the applications of mathematical topics. An index of these applications on the inside covers demonstrates the wide variety used in examples and exercises.

Pedagogical Features

This text offers students and instructors a proven pedagogical program. Important features are the following.

Intuitive Viewpoint. The book is written from an intuitive viewpoint, with emphasis on concepts and problem solving rather than on mathematical theory. Each topic is carefully explained and examples illustrate the techniques involved. Exercises stress computation and drill, but there are enough challenging problems to stimulate students.

Flexibility. At different colleges or universities the coverage and sequencing of topics may vary according to the purpose of this course. To accommodate this, the text has a great deal of flexibility in the order of topics.

Chapter Warmups. A Warmup appears at the beginning of each chapter and invites students to test themselves on the skills needed for that chapter. The Warmups present several prerequisite problem types that are taken from parts of upcoming problems. Each prerequisite problem type is keyed to the upcoming section where that skill is needed, and students who have difficulty with any particular skill are directed to specific sections of the text for review. Instructors may find the Warmups useful in creating a syllabus.

Applications. We have found that offering applied topics such as cost, revenue, and profit functions in a separate section brings the preceding mathematical discussions into clear and concise focus. There are 7 such sections in this book. Beyond this, there are over 840 applied exercises and hundreds of applied examples covering a variety of disciplines throughout the text.

Objective Lists. Every section begins with a brief list of objectives that outlines the goals of that section for the student.

Procedure/Example Tables. Sprinkled throughout the text, Procedure/Example tables aid student understanding by giving step-by-step descriptions of important procedures with illustrative examples worked out beside the procedures.

Boxed Information. All important information is boxed for easy reference, and key terms are highlighted in boldface.

Review Exercises. At the end of each chapter, a set of Review Exercises offers the student extra practice on the topics in that chapter. These exercises are annotated with section numbers so that the student having difficulty can turn to the appropriate section for review.

Content in the Third Edition

A major focus of this edition was to improve the exercise sets. This was achieved by providing the following: (1) a better balance between the odd- and the even-numbered exercises, (2) a smoother progression from easy exercises to difficult exercises, (3) better coverage of the topics within each section, and (4) the inclusion of numerous new applications. The third edition contains over 2000 exercises, more than 840 of which are applications.

Other significant improvements in the third edition are the following.

In Chapter 1, the discussion of functions was expanded to include piecewise-defined functions. In addition, the simultaneous solution of three equations in three variables was added to this chapter.

In Chapter 3, the notation for the simplex method has been standardized so that it it is more consistent with other linear programming discussions. The simplex method was extended to cases where infinitely many and no solutions occurred.

Chapter 4 was rewritten with a stronger emphasis on the mathematics of finance. The discussion of sequences is now used to support the development of the mathematics of finance rather than as the central theme of the chapter. Deferred annuities were added to the section on ordinary annuities. The discussion of depreciation has been updated to reflect changes brought about by the Tax Reform Law of 1986. The notation used in this chapter has been made consistent with the notation used in business textbooks and all important formulas have been boxed.

A new chapter (Chapter 7) on game theory covers strictly determined games, games of mixed strategy, and the simplex solution to mixed strategy games.

Throughout the text, exercises that are best worked with a calculator are highlighted with the symbol.

Supplements

Instructor's Guide. This booklet contains two forms of a test for each chapter of the text, with answers provided. In addition, the answers to all even-numbered exercises of the text are included.

Selected Solutions Guide. In addition to an answer section at the end of the text, the solutions to all odd-numbered exercises are included in this supplementary booklet.

HeathTest +

Hardware requirements: IBM PC or compatible, two disk drives, IBM graphics-compatible dot-matrix printer or laser printer.

Items included: One program disk, test item disks, User Manual/Printed Test Item File.

This new, versatile test-generating program offers full graphics capability and allows instructors to customize tests for their own classes.

With this program, instructors can preview questions on-screen and then add each item to a test with one keystroke. Random generation of test items by chapter is possible. Additionally, instructors may edit existing items or add new items either to the database or to individual tests. Tests may be saved and then printed in multiple scrambled versions. Answer keys are automatically generated.

Related Texts

This book is one of three covering finite mathematics and applied calculus. All three texts heavily emphasize real-world applications of the mathematics featured as the students in these courses are typically majoring in management or the life or social sciences. The other texts in this series are:

Applied Calculus for Management, Life, and Social Sciences, Third Edition.

This text is designed for a one-term course covering a review of algebra, functions of one variable, derivatives, exponential and logarithmic functions, indefinite and definite integrals, and finally, functions of two or more variables. Sections on numerical methods of integration and double integrals have been added to this text.

Mathematical Applications for Management, Life, and Social Sciences, Third Edition.

This text is a combined version of *Finite Mathematics* and *Applied Calculus* and is designed for a one- or two-term course in finite math and calculus.

Acknowledgements

We would like to thank the many people who have helped us at various stages on this series. The encouragement, criticism, and suggestions that have been offered have been invaluable to us. We are especially indebted to Samuel Laposata, Virginia Electric Power Company, who provided ideas and encouragement, and to Frank Kocher, The Pennsylvania State University, who provided support and reviews throughout the first edition's evolution. Our special thanks are due Gordon Shilling, University of Texas at Arlington, who carefully examined the exercise sets; and to Stanley Chadick, Northwestern State University; James Runyon, Rochester Institute of Technology; and Ellen Wood, Stephen F. Austin State University; each of whom reviewed manuscript and made many helpful comments. Survey respondents offered many valuable suggestions; our thanks to Patricia Blitch, Lander College; Louis Bush, San Diego City College; Nancy Fisher, University of Alabama; Mary Guffey, Auburn University; Ronald Rule, Georgia College; and Gary Taka, Santa Monica College. Our thanks also to Bill Bolem, Glenn Ostrowski, Patti Stacey, and Sheryl Rutkowski for their help with manuscript preparation. We would also like to express our appreciation to the editorial staff at D. C. Heath for their continued enthusiasm and support.

Ronald J. Harshbarger
James J. Reynolds

Contents

Finite Mathematics

for Management, Life, and Social Sciences

1

Sets, Linear Equations, and Functions

A wide variety of problems from business, the social sciences, and the life sciences may be solved by using sets or equations. Managers and economists use equations and their graphs to study costs, sales, national consumption, or supply and demand. Social scientists may plot demographic data or try to develop equations that predict population growth, voting behavior, or learning and retention rates. Life scientists use equations to model the flow of blood or the conduction of nerve impulses and to test theories or develop new ones by plotting experimental evidence.

In this chapter we begin with a discussion of sets and their applications. We will investigate linear equations and their graphs. We will also define the concepts of relation and function, introduce functional notation, and then relate all of these back to equations and graphs.

Numerous applications of mathematics are given throughout the text, but all chapters contain special sections emphasizing business and economics applications. In particular, this chapter introduces three important applications: national consumption; supply and demand as functions of price (market analysis); and total cost, total revenue, and total profit as functions of the quantity produced or sold (theory of the firm).

Linear equations are used by most businesses to predict such things as future revenues and costs. Linear functions are used in economics, biology, and sociology to relate data, and are also used to a large extent in accounting courses. Keynes used the linear function as the model for national consumption. In this chapter we discuss several important features of Keynesian analysis. We also formulate linear supply and demand functions and then determine the price and quantity at which market equilibrium occurs. We use linear revenue and cost functions to obtain profit functions and to find break-even points, and we solve problems involving linear equations from the social and life sciences.

1.1 Sets

Objectives

- ☐ To use set notation to describe sets
- ☐ To determine set membership
- ☐ To find relations between two sets
- ☐ To find the intersection, union, and difference of sets
- ☐ To find the complement of a set
- ☐ To represent sets with Venn diagrams
- ☐ To identify subsets of the real numbers
- ☐ To locate subsets of the real numbers on a number line

A set is a well-defined collection of objects. We may talk about a set of books, a set of dishes, or a set of students. We shall be concerned with sets of numbers. There are two ways to tell what a given set contains. One way is by listing the **elements (or members)** of the set (usually between braces). We may say that a set A contains 1, 2, 3, and 4 by writing $A = \{1, 2, 3, 4\}$. To say that 4 is a member of set A, we write $4 \in A$.

If all the members of the set can be listed, the set is said to be a **finite set.** $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$ are examples of finite sets. Although we cannot list all the elements of an **infinite set,** we can use three dots to indicate the unlisted members of such a set. For example, $N = \{1, 2, 3, 4, \ldots\}$ is an infinite set. This set N is called the set of **natural numbers.** Although they are not all listed, we know $10 \in N$, $1121 \in N$, and $15{,}331 \in N$, but $\frac{1}{2}$ is not a member of N (that is, $\frac{1}{2} \notin N$) because $\frac{1}{2}$ is not a natural number.

Another way to specify the elements of a given set is by description. For example, we may write $D = \{x: x \text{ is a Ford automobile}\}$ to describe the set of all Ford automobiles. $F = \{y: y \text{ is an odd natural number}\}$ is read "F is the set of all y such that y is an odd natural number." Thus $3 \in F$, $5 \in F$, and $7 \in F$ because they are odd natural numbers, and $6 \notin F$ because 6 is not an odd natural number.

EXAMPLE 1 Write the following sets in two ways.

(a) The set A of natural numbers less than 6.
(b) The set B of natural numbers greater than 10.
(c) The set C containing only 3.

Solution

(a) $A = \{1, 2, 3, 4, 5\}$ or $A = \{x: x \text{ is a natural number less than } 6\}$
(b) $B = \{11, 12, 13, 14, \ldots\}$ or $B = \{x: x \text{ is a natural number greater than } 10\}$
(c) $C = \{3\}$ or $C = \{x: x = 3\}$

Note that set C of Example 1 contains one member, 3; set A contains five members; and set B contains an infinite number of members. It is possible for a set to contain no members. Such a set is called the **empty set** or the **null set,** and it is denoted by $\varnothing$ or by $\{\ \}$. The set of living veterans of the War of 1812 is empty because there are no living veterans of that war. Thus

$$\{x: x \text{ is a living veteran of the War of 1812}\} = \varnothing.$$

Special relations that may exist between two sets are defined as follows.

Relations with sets

Definition	Example
1. Sets X and Y are **equal** if they contain the same elements.	1. If $X = \{1, 2, 3, 4\}$ and $Y = \{4, 3, 2, 1\}$, then $X = Y$.
2. $A \subseteq B$ if every element of A is an element of B. A is called a **subset** of B. The empty set is a subset of every set.	2. If $A = \{1, 2, c, f\}$ and $B = \{1, 2, 3, a, b, c, f\}$, then $A \subseteq B$.
3. If C and D have no elements in common, they are called **disjoint.**	3. If $C = \{1, 2, a, b\}$ and $D = \{3, e, 5, c\}$, C and D are disjoint.

In the discussion of particular sets, the assumption is always made that the sets under discussion are all subsets of some larger set, called the **universal set** U. The choice of the universal set depends upon the problem under consideration. For example, in discussing the set of all students and the set of all female students, we may use the set of all humans as the universal set.

We may use **Venn diagrams** to illustrate the relationships among sets. We use a rectangle to represent the universal set and closed figures inside the rectangle to represent the sets under consideration. Figure 1.1 shows such a Venn diagram.

Figure 1.1 shows that B is a subset of A; that is, $B \subseteq A$. In Figure 1.2, M and N are disjoint sets. In Figure 1.3, sets X and Y overlap; that is, they are not disjoint.

The shaded portion of Figure 1.3 indicates where the two sets overlap. The set containing the members that are common to two sets is said to be the **intersection** of the two sets.

Set Intersection The intersection of A and B, written $A \cap B$, is defined by

$$A \cap B = \{x: x \in A \text{ and } x \in B\}.$$

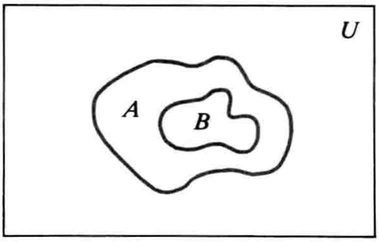

Figure 1.1

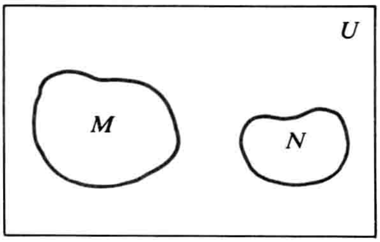

Figure 1.2

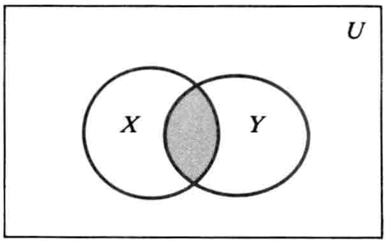

Figure 1.3

EXAMPLE 2 If $A = \{2, 3, 4, 5\}$ and $B = \{3, 5, 7, 9, 11\}$, find $A \cap B$.

Solution $A \cap B = \{3, 5\}$ because 3 and 5 are the common elements of A and B. Figure 1.4 shows the sets and their intersection.

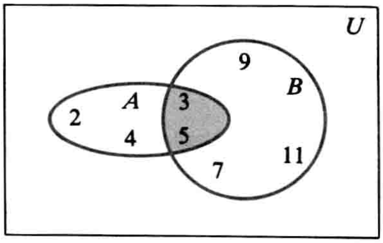

Figure 1.4

The **union** of two sets is the set that contains all members of the two sets.

Set Union The union of A and B, written $A \cup B$, is defined by

$$A \cup B = \{x: x \in A \text{ or } x \in B \text{ (or both)}\}.^*$$

EXAMPLE 3 If $X = \{a, b, c, f\}$ and $Y = \{e, f, a, b\}$, find $X \cup Y$.

Solution $X \cup Y = \{a, b, c, e, f\}$

EXAMPLE 4 Let $A = \{x: x \text{ is a natural number less than } 6\}$ and $B = \{1, 3, 5, 7, 9, 11\}$.
(a) Find $A \cap B$.
(b) Find $A \cup B$.

Solution (a) $A \cap B = \{1, 3, 5\}$
(b) $A \cup B = \{1, 2, 3, 4, 5, 7, 9, 11\}$

We can illustrate the intersection and union of two sets by the use of Venn diagrams. The shaded region in Figure 1.5 represents $A \cap B$, the intersection of A and B, while the shaded region in Figure 1.6 represents $A \cup B$.

All elements of the universal set that are not contained in a set A form a set called the **complement** of A.

*In mathematics, the word *or* means one or the other or both.

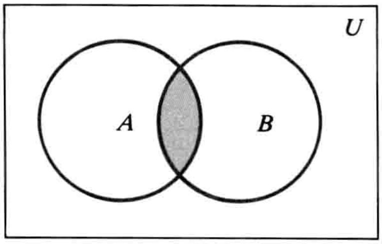

Figure 1.5

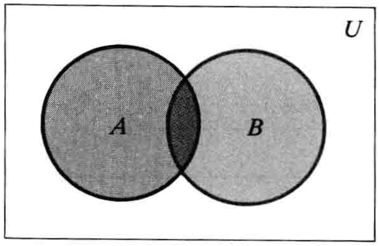

Figure 1.6

Set Complement The complement of A, written A', is defined by

$$A' = \{x: x \in U \text{ and } x \notin A\}.$$

EXAMPLE 5 If $U = \{x \in N: x < 10\}$, $A = \{1, 3, 6\}$, and $B = \{1, 6, 8, 9\}$, find the following.
(a) A' (b) B' (c) $(A \cap B)'$ (d) $A' \cup B'$

Solution (a) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ so $A' = \{2, 4, 5, 7, 8, 9\}$
(b) $B' = \{2, 3, 4, 5, 7\}$
(c) $A \cap B = \{1, 6\}$ so $(A \cap B)' = \{2, 3, 4, 5, 7, 8, 9\}$
(d) $A' \cup B' = \{2, 4, 5, 7, 8, 9\} \cup \{2, 3, 4, 5, 7\}$
$= \{2, 3, 4, 5, 7, 8, 9\}$

We can use a Venn diagram to illustrate the complement of a set. The shaded region of Figure 1.7 represents A' and the *un*shaded region of Figure 1.5 represents $(A \cap B)'$.

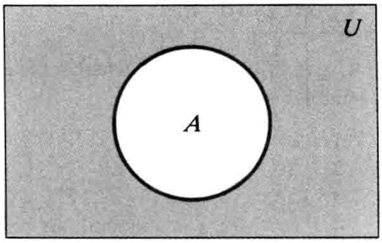

Figure 1.7

The universal set for our discussion in this text is the set of **real numbers.** Since there is exactly one point on a straight line for each real number, we can represent the real numbers along a line called the **real number line.** This number line is a picture, or graph, of the real numbers. Two numbers are said to be equal whenever they are represented by the same point on the number line. The equation $a = b$ (a equals b) means that the symbols a and b represent the same real number.