A TREATISE ON ELECTRICITY AND MAGNETISM

VOLUME II

Third Edition

A TREATISE

ON

ELECTRICITY AND MAGNETISM

BY

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VOL. II
THIRD EDITION

OXFORD UNIVERSITY PRESS LONDON: GEOFFREY CUMBERLEGE FIRST EDITION 1873
SECOND EDITION 1881
THIRD EDITION 1892; Reprinted 1904

REPRINTED LITHOGRAPHICALLY IN GREAT BRITAIN BY LOWE & BRYDONE, PRINTERS, LTD., LONDON FROM SHEETS OF THE THIRD EDITION 1937, 1946, 1955

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